Rosette modes of oscillations in rotating stars as a new aspect of rotation-pulsation interaction

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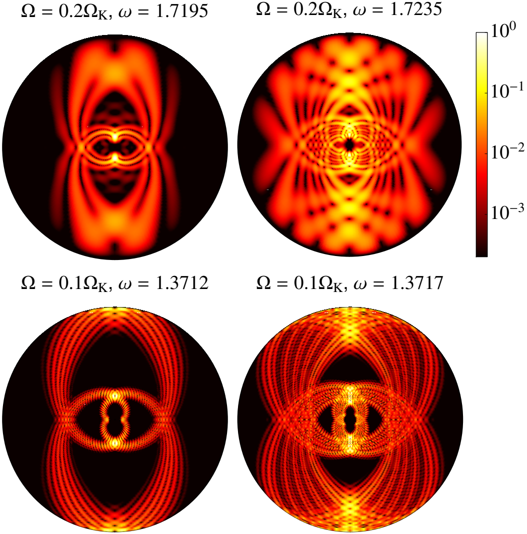
Introduction: what are rosette modes?

discovered by *Ballot et al. (2012)* in numerical calculations

- a class of eigenmodes found in rotating stars in the g-mode frequency range outside of the inertial domain ($\sigma > 2\Omega$)

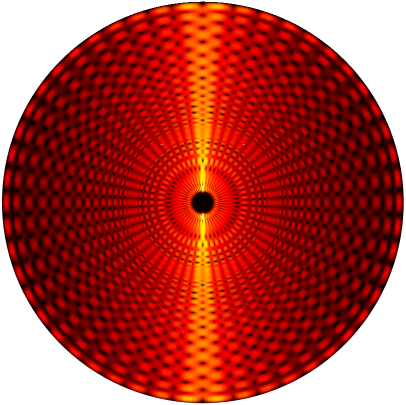
- rosette pattern: characteristic structure of the kinetic-energy density distribution on the meridional plane



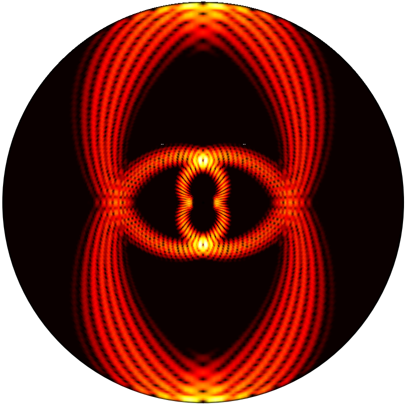
(polytropic model with index 3)

Ordinary modes vs. rosette modes

ordinary modes	rosette modes
$\Omega = 0.0\Omega_K,\ \omega = 1.3645$	$\Omega = 0.1\Omega_K,\ \omega = 1.3712$



$\ell = 25,\ n = -37,\ m = 0$



distributions of the kinetic-energy density on the meridional plane multiplied by r^2

How are rosette modes formed?

Takata & Saio (2013)

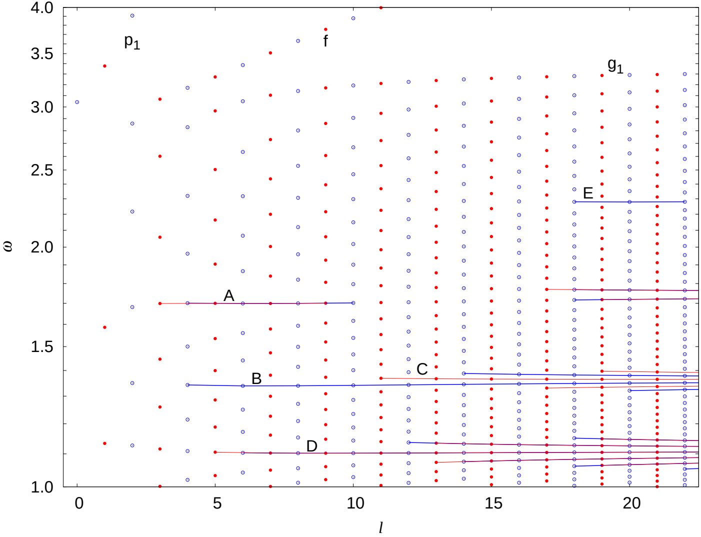
- close degeneracies among high-degree g modes (in the absence of rotation)

  asymptotic formula: $\omega_{n,\ell} \propto \frac{\sqrt{\ell(\ell+1)}}{|n|}$

  ($\omega_{n,\ell} \approx$ const. if ℓ/n is fixed for $\ell \gg 1$.)
- interaction among the closely-degenerate eigenmodes (with the same parity of ℓ) caused by the second-order effects of the Coriolis force
- described by quasi-degenerate *perturbation* theory: eigenfunctions of rosette modes can be expressed by linear combinations of eigenfunctions in the non-rotating limit.

Close degeneracies

ℓ–ω diagram of the polytropic model with index 3



A : $n + \ell = 0$
($K = 2$)

B : $2n + 3\ell = 0$
($K = 3$)

C : $2n + 3\ell = 1$
($K = 3$)

D : $n + 2\ell = 0$
($K = 4$)

E : $2n + \ell = 0$
($K = 1$)

Asymptotic analysis of rosette modes (1)

Takata (2014)

- close degeneracies:

$$an + b\ell = f$$

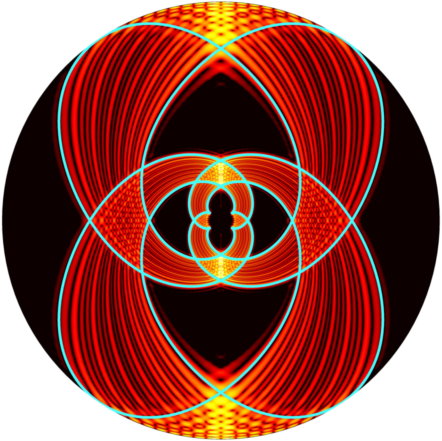
(a, b, f: integers)

- rosette structure:

$$\frac{K\pi}{2} - Z(r) \pm \theta \pm \frac{q\pi}{2} = 0,\ \pi$$

where

  - $Z(r) = \int_{r_1}^{r} \frac{1}{s} \left[\frac{N^2(s)}{\sigma^2} - 1 \right]^{1/2} \mathrm{d}s$
  - $K = \frac{2b}{a}$: integer
  - q: real number ($0 < q < 1$)



$\Omega = 0.15\Omega_{\mathrm{K}}$, $\omega = 1.3804$
$K = 3$, $q = 0.373$

Asymptotic analysis of rosette modes (2)

rosette patterns and close degeneracies

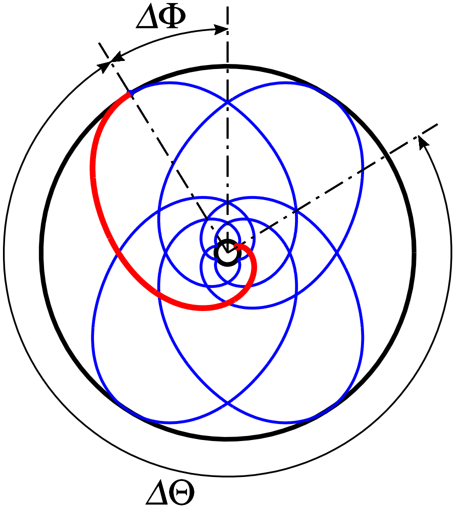
two parameters K and q to characterize the rosette patterns:

- $K = \dfrac{2b}{a}$: integer

  $\Rightarrow \quad \Delta\Theta = \dfrac{K\pi}{2}$

- q: real number ($0 < q < 1$)

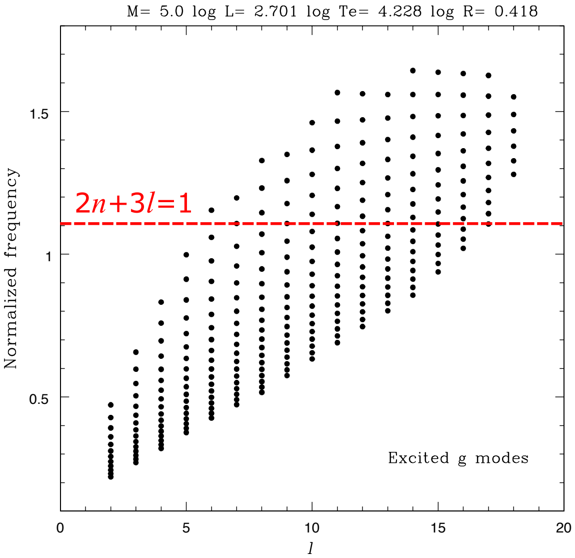
  $\Rightarrow \quad \Delta\Phi = \dfrac{q\pi}{2}$



($K = 3$, $q = 0.35$)

Rosette modes in a realistic stellar model (1)

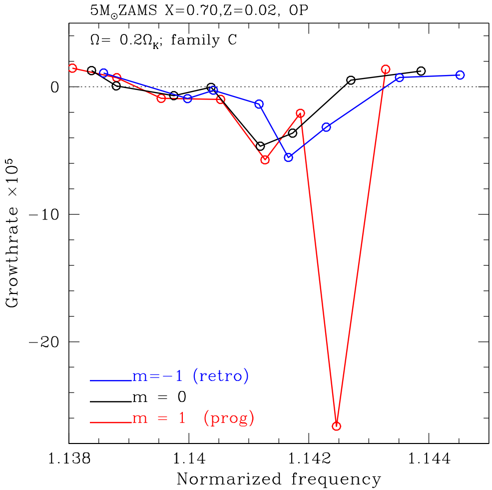
- 5 $M_\odot$ ZAMS model with $X = 0.70$ and $Z = 0.02$ (a model of SPB stars)
- many high-degree g modes excited by the iron opacity bump at $T \approx 2 \times 10^5$ K without rotation (the OP opacity is used)
- a family of the close degeneracy that satisfies $2n + 3l = 1$ ($K = 3$) (modes with $\ell = 7, 9, \cdots, 17$ are unstable)



Rosette modes in a realistic stellar model (2)

nonadiabatic mode analysis with rotation [cf. *Lee & Baraffe (1995)*]

- unstable rosette modes found for $\Omega \lesssim 0.25\Omega_K$ ($v_{eq} \lesssim 150$ km)
- only modes with considerable contribution from unstable components (in the non-rotation limit) are excited.
- the damping is caused by stable components (particularly those with high degrees).



Rosette modes in a realistic stellar model (3)

angular momentum transport by waves [cf. *Lee (2013)*]

- angular momentum conservation (Lagrangian mean-flow treatment):

$$\langle \rho \rangle \frac{d \langle l_z \rangle}{dt} = -\mathrm{div} \left\langle \vec{F} \right\rangle$$

  - l_z: specific angular momentum about the rotation axis
  - $\vec{F}$: angular momentum flux

$$\vec{F} = \frac{\partial p'}{\partial \phi} \vec{\xi} + \frac{\partial \Phi'}{\partial \phi} \left(\langle \rho \rangle \vec{\xi} + \frac{\nabla \Phi'}{4\pi G} \right)$$

  - $\langle f \rangle$: azimuthal average in the mean-flow coordinates

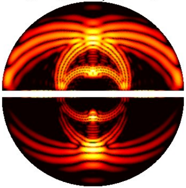
- effects on the local angular momentum:

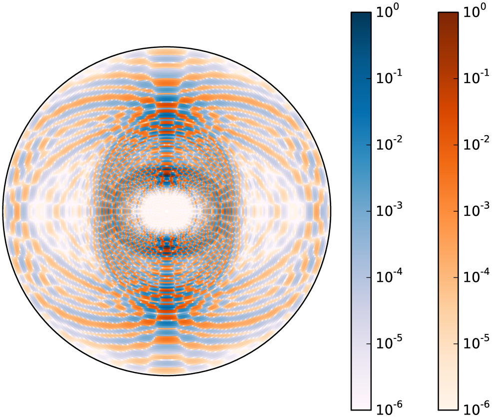
	damping	excitation
prograde modes	$+$	$-$
retrograde modes	$-$	$+$

Rosette modes in a realistic stellar model (4)

- no significant difference in the spherical averages of $\left\langle \vec{F} \right\rangle$ between rosette and ordinary modes
- conspicuous latitudinal dependence of div $\left\langle \vec{F} \right\rangle$

$\omega = 1.1445, m = -1$





distribution of $-\mathrm{div} \left\langle \vec{F} \right\rangle$
[blue (red) for positive (negative) values]

Summary

- formation: rosette modes are formed by the interaction of closely-degenerate eigenmodes with the same parity of the spherical degree; the interaction is caused by the second-order effect of the Coriolis force.
- asymptotic analysis: a simple relation can be derived to describe the rosette structures in terms of parameters that specify the close degeneracy.
- nonadiabatic calculation & angular momentum transport: some unstable rosette modes have been found in a model of SPB stars; those modes might contribute to the angular momentum transport in the stars in a unique way.

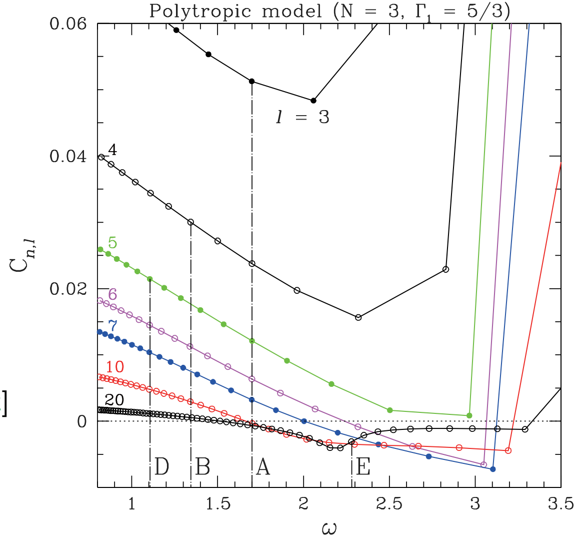
Nonaxisymmetric rosette modes (1)

Saio & Takata (2014)

- the first-order effect of the Coriolis force need to be small for nonaxisymmetric rosette modes to form:

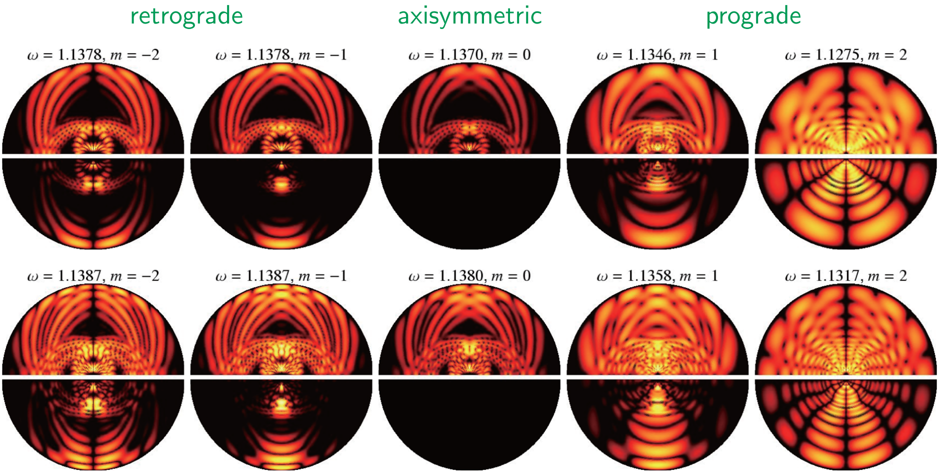
$$|C_{n,\ell}| \ll 1$$

[cf. $\Delta\sigma = m\,(1 - C_{n,\ell})\,\Omega$]



Nonaxisymmetric rosette modes (2)

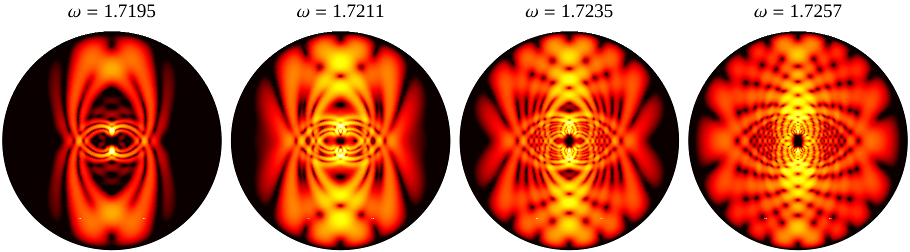
rosette modes that arise from family D



clearer rosette patterns in retrograde modes ($m < 0$) than in prograde modes ($m > 0$)

Gallery of rosette modes

multiple rosette modes arise from a family of the close degeneracy



distributions of the kinetic-energy density on the meridional plane multiplied by r^2 for family A (with odd ℓ) of the polytropic model with index 3 at $\Omega = 0.2\Omega_{\mathrm{K}}$

References

- Ballot, J. et al., 2012, ASP Conf. Ser., 462, 389
- Lee, U., 2013, PASJ, 65, 122
- Lee, U. & Baraffe, I., 1995, A&A, 301, 419
- Saio, H. & Takata, M., 2014, PASJ, 66, psu027
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- Takata, M. & Saio, H., 2013, PASJ, 65, 68