Differences in radial differential rotation inversions due to uncertainties in the stellar models

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rad ion

Testing Inversions

due the

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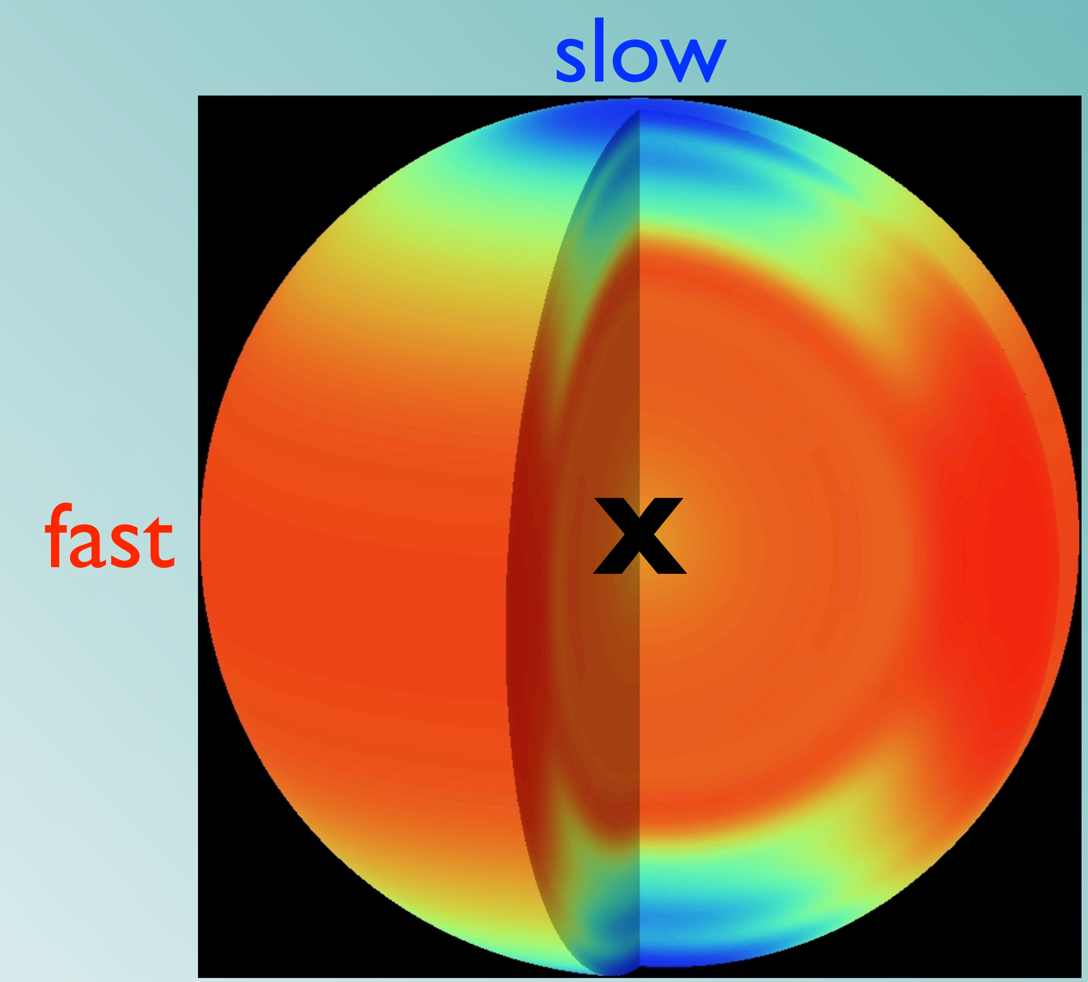
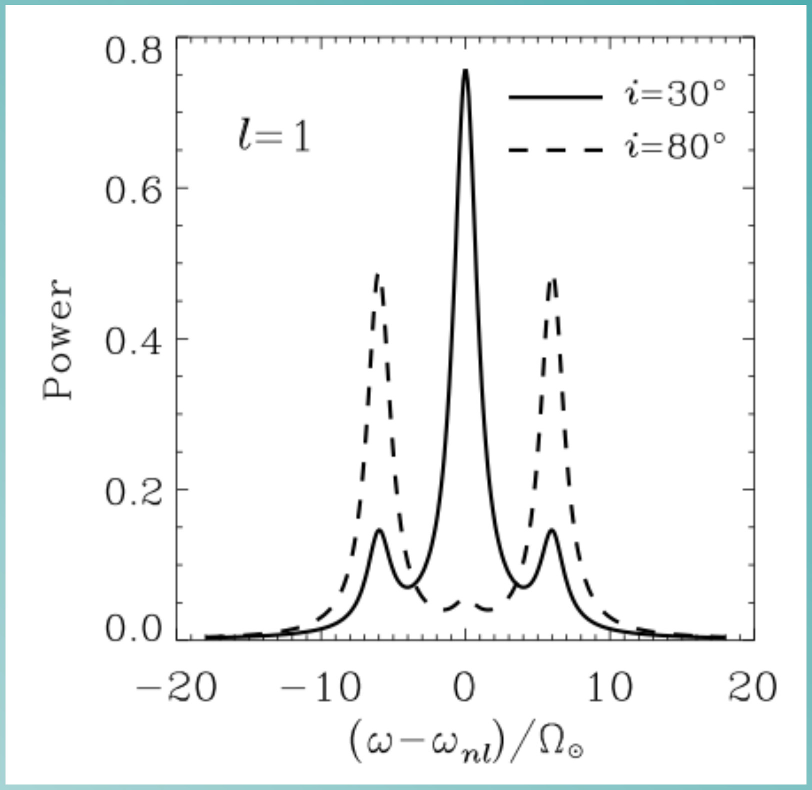
Internal rotation of the Sun



Schou et al. 1998

- The azimuthal modes of the Sun are sensitive to the rotation

- Observed as frequency 'splittings'



- Use solar model to determine the sensitivity of the modes

- Linear inversion to image the interior rotation

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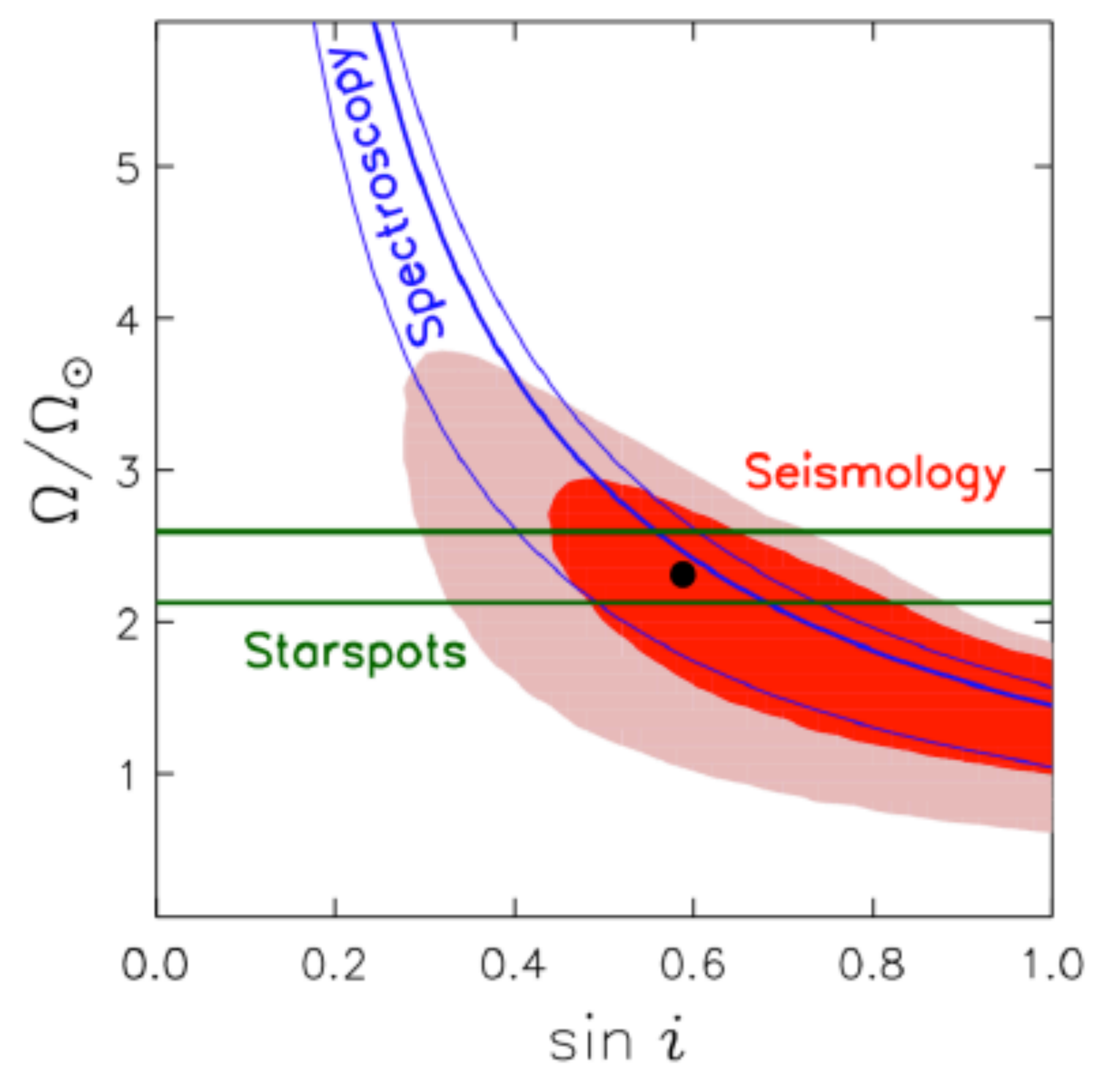
Rotation in Sun-like stars

Unambiguous seismic detection of rotation and axis of inclination in a Sun-like star

CoRoT ~4 months

HD52265



Ballot et al 2011; Gizon et al 2013

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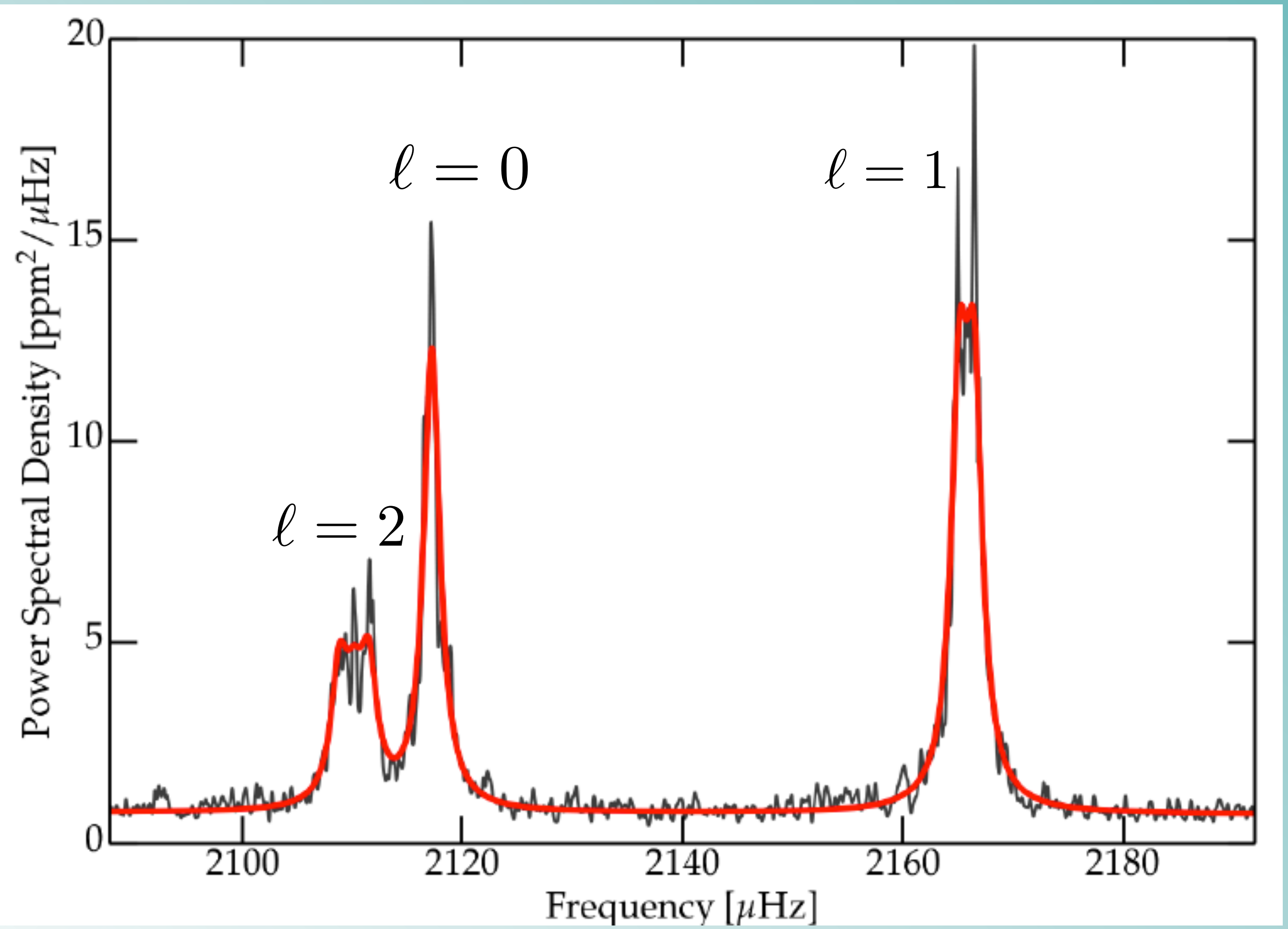
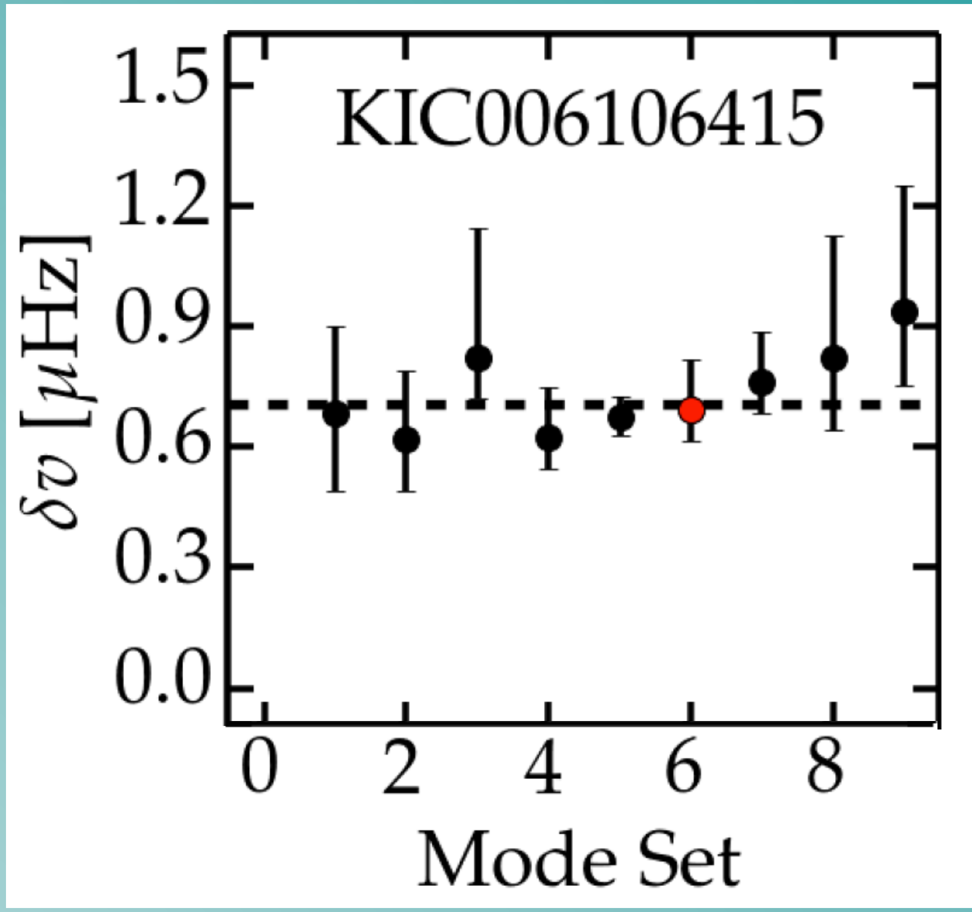
Rotation in Sun-like stars

Independent splittings measured for six Sun-like main sequence *Kepler* stars (~3 years)

Nielsen et al, submitted





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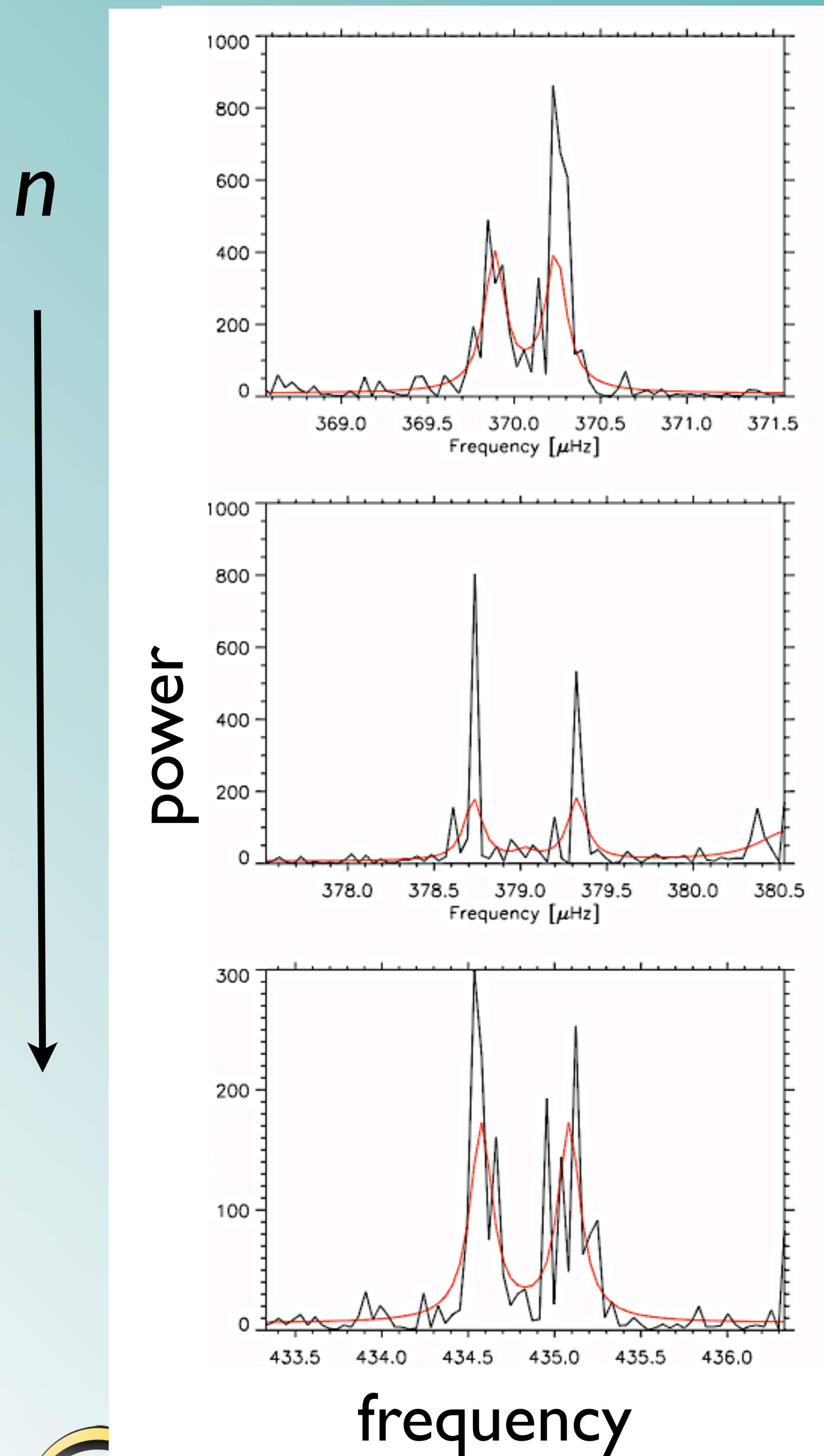
Radial differential rotation in sub-giants



courtesy T. Stahn

Seven sub-giants from *Kepler*

Deheuvels et al 2012; 2014

Rotational splitting varies with radial order (depth)

$\ell = 1, \quad m = \pm 1$

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Rotation and splittings

uncertainty in observations

observed

?

$$\delta\omega_i = \beta_i \int_0^R K_i(r)\Omega(r)\mathrm{d}r + \epsilon_i$$

from models

spectroscopic observations
+ physics

MPS

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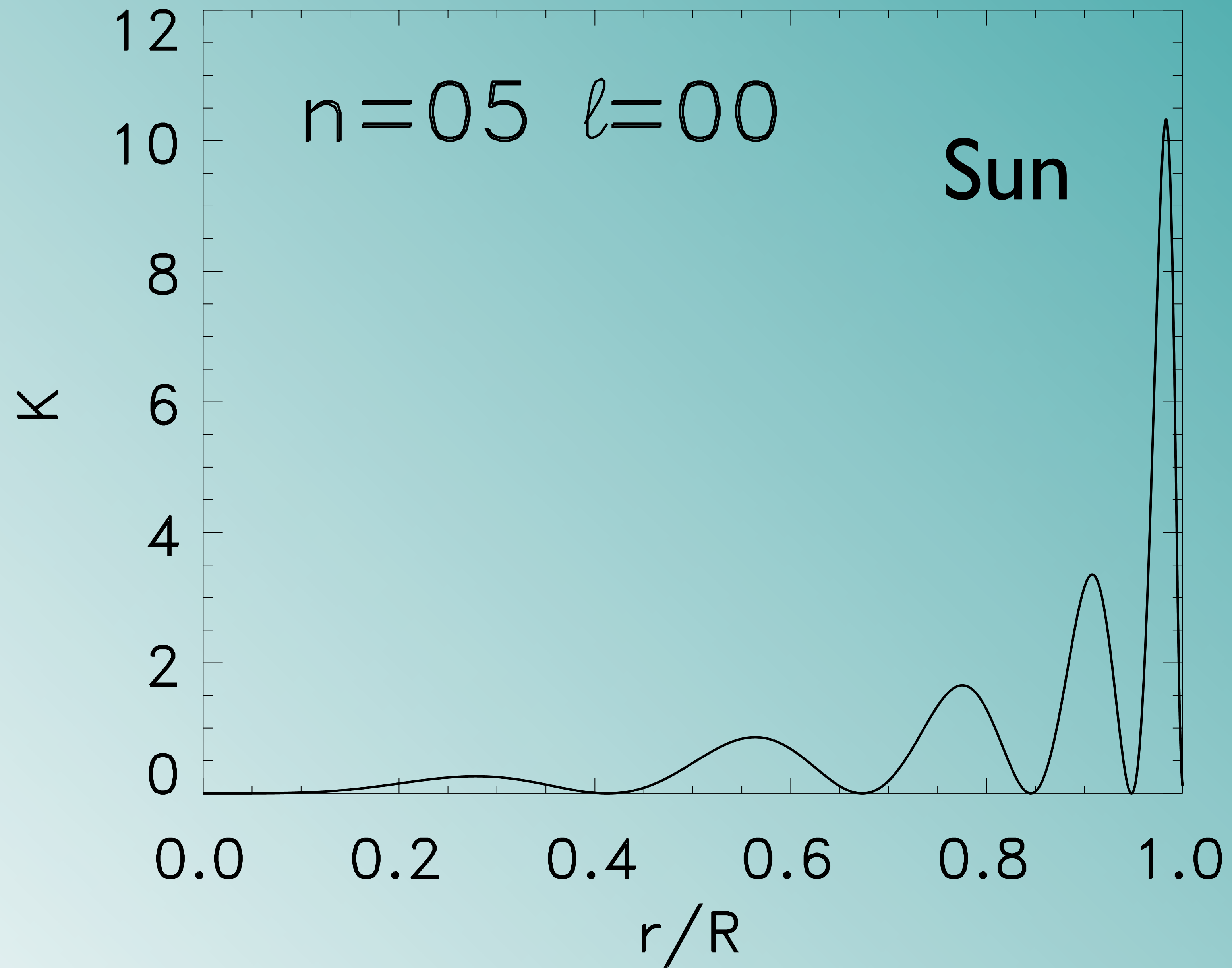
Rotation Kernels

$$K_{nl} = \frac{(\xi_r^2 + L^2\xi_h^2 - 2\xi_r\xi_h - \xi_h^2)r^2\rho}{\int_0^R(\xi_r^2 + L^2\xi_h^2 - 2\xi_r\xi_h - \xi_h^2)r^2\rho \mathrm{d}r}$$



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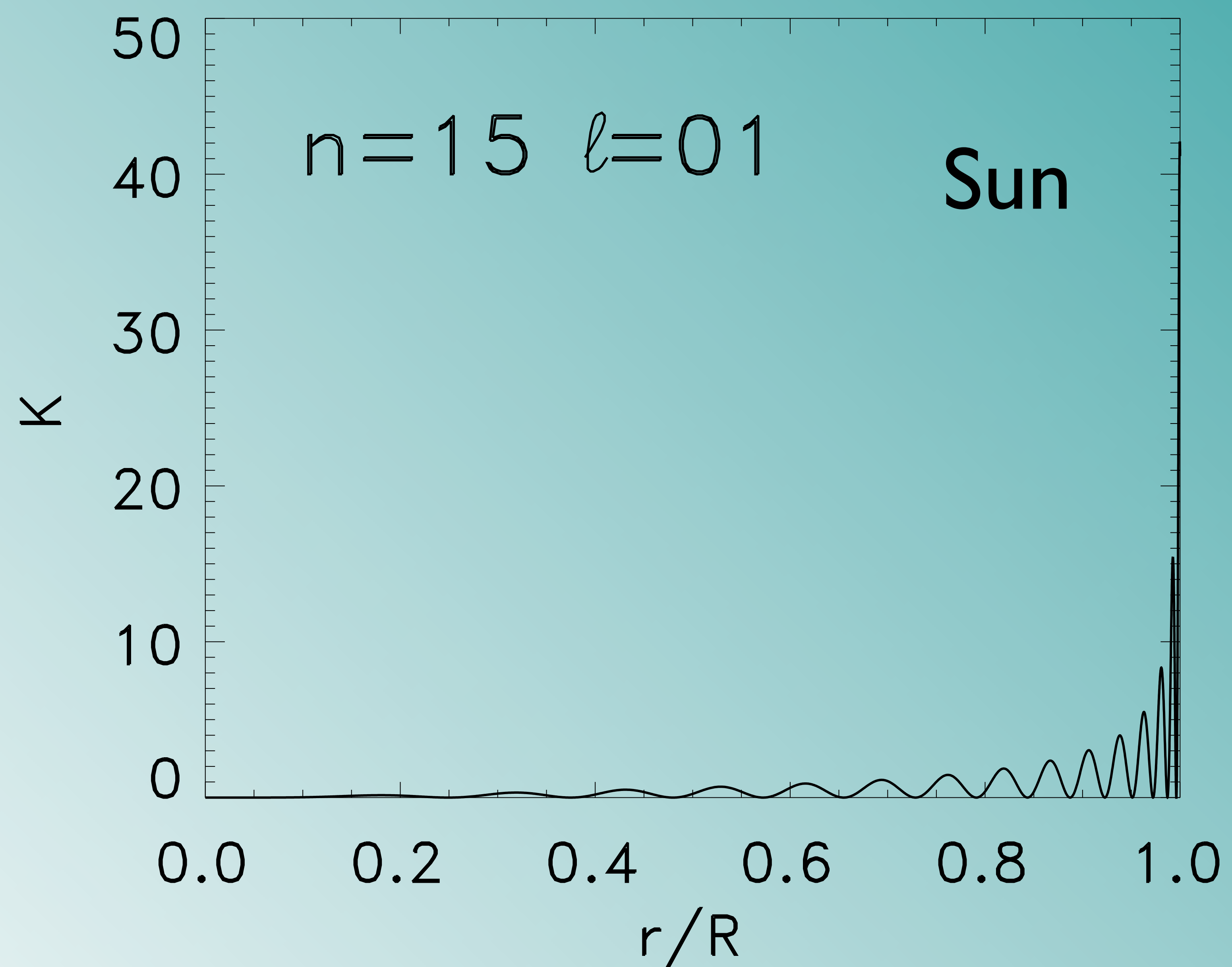
Rotation Kernels

$$K_{nl} = \frac{(\xi_r^2 + L^2\xi_h^2 - 2\xi_r\xi_h - \xi_h^2)r^2\rho}{\int_0^R (\xi_r^2 + L^2\xi_h^2 - 2\xi_r\xi_h - \xi_h^2)r^2\rho \mathrm{d}r}$$



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Linear inversions for radial rotation

observed — from models — ? — uncertainty in observations

$$\delta\omega_i = \beta_i \int_0^R K_i(r)\Omega(r)\mathrm{d}r + \epsilon_i$$

Linear inversions - need good **stellar models**!

e.g.

Regularised Least Squares
Optimally Localised Averages
Functional Fitting

RLS Inversions

(Tikhonov regularisation)

$$\sum_{i\in M}\frac{1}{\sigma_i^2}\left[\delta\omega_i-\sum_{j=1}^{N}\overline{\Omega}_j\int_0^R K_i(r)\phi_j(r)dr\right]^2+\mu F(\overline{\Omega}_j)$$

Minimise w.r.t $\overline{\Omega}_j$

$$\overline{\Omega}_j=\sum_{i=1}^{M}c_{ij}\delta\omega_i$$

How do the inversions for interior rotation depend on the accuracy of the stellar model?

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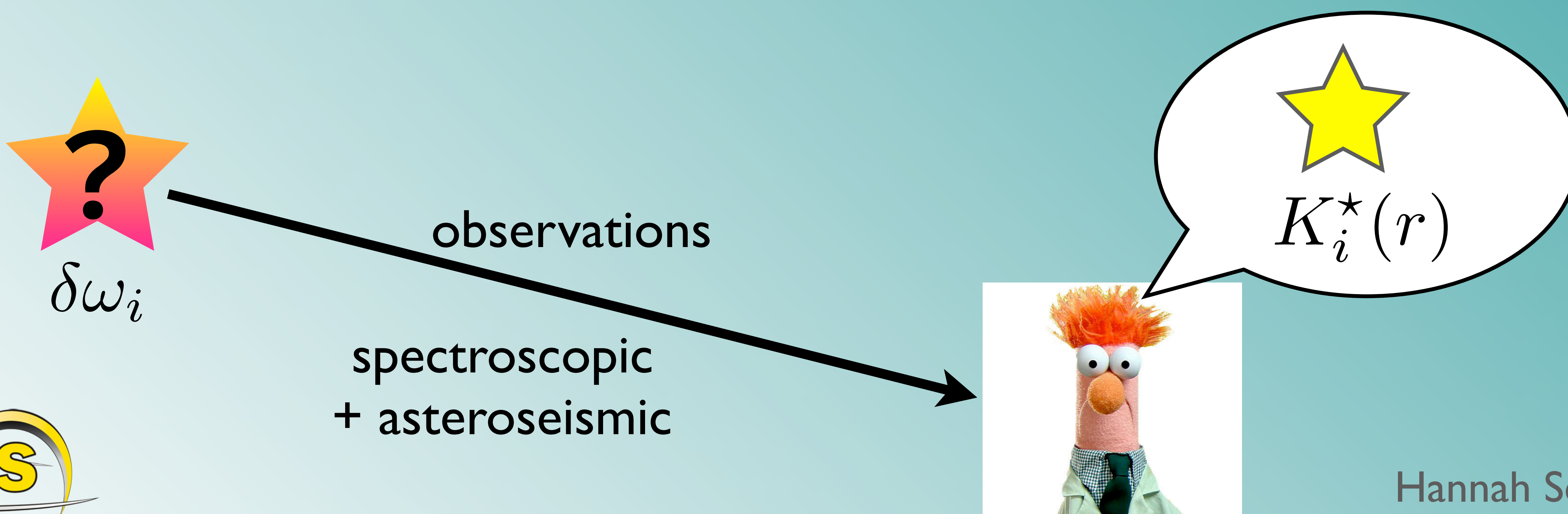
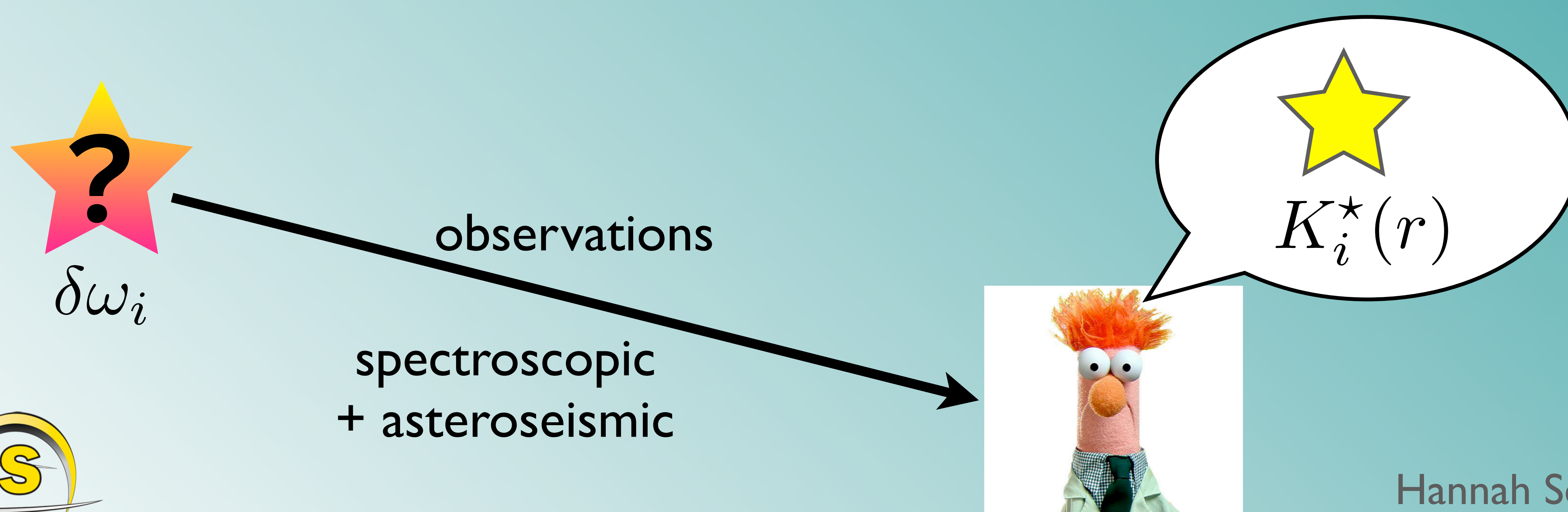
Experiment


?
$\delta\omega_i$
observations
spectroscopic
+ asteroseismic
$K_i^\star(r)$

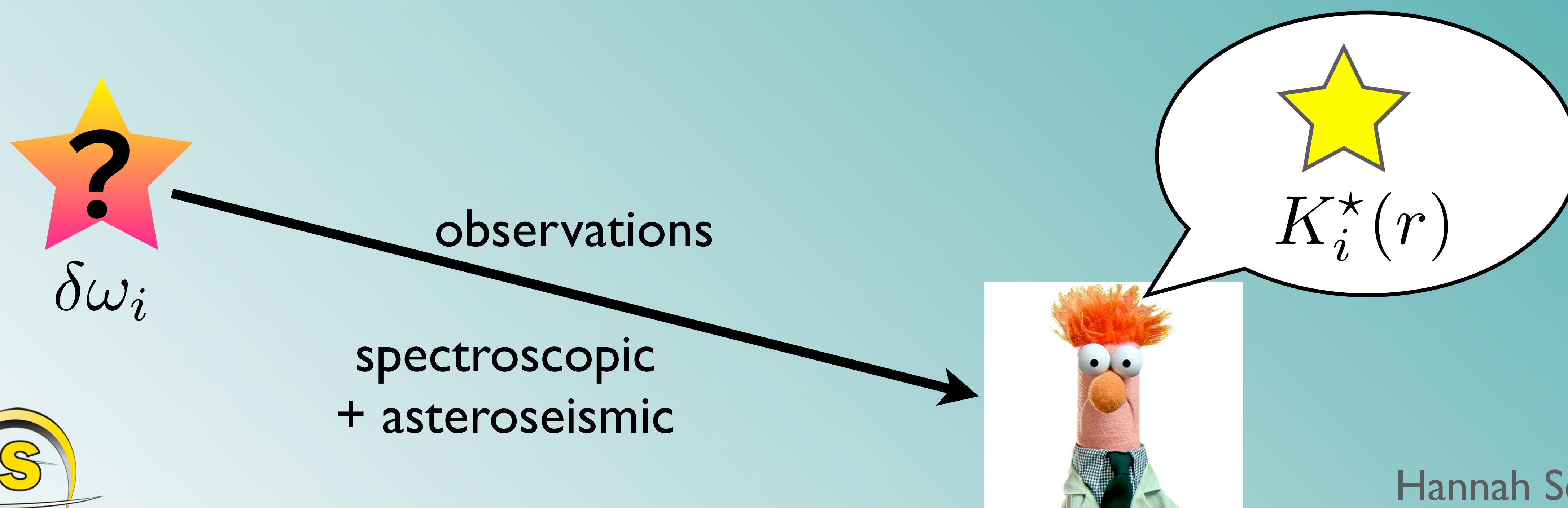
Experiment

- Compute a *reference* stellar model that best matches the observed spectroscopic parameters (Teff, logg, Fe/H) and frequencies of a star



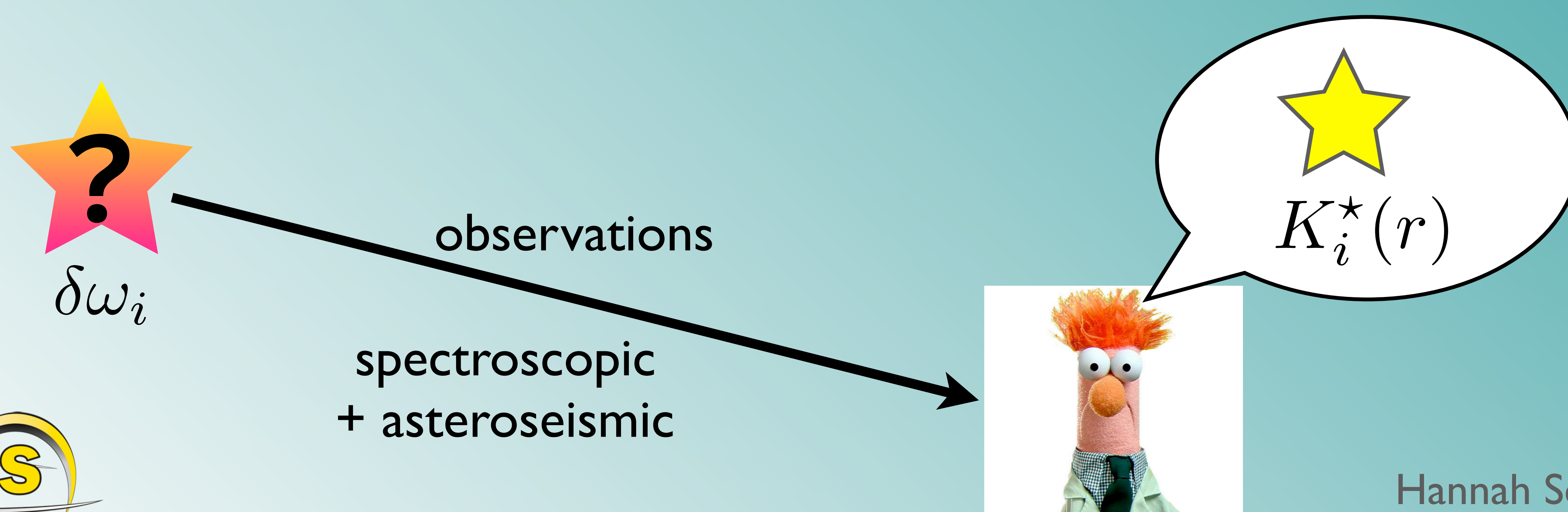
Experiment

- Compute a *reference* stellar model that best matches the observed spectroscopic parameters (Teff, logg, Fe/H) and frequencies of a star
- Compute *perturbed* stellar models for perturbations to the spectroscopic parameters and frequencies within the uncertainties



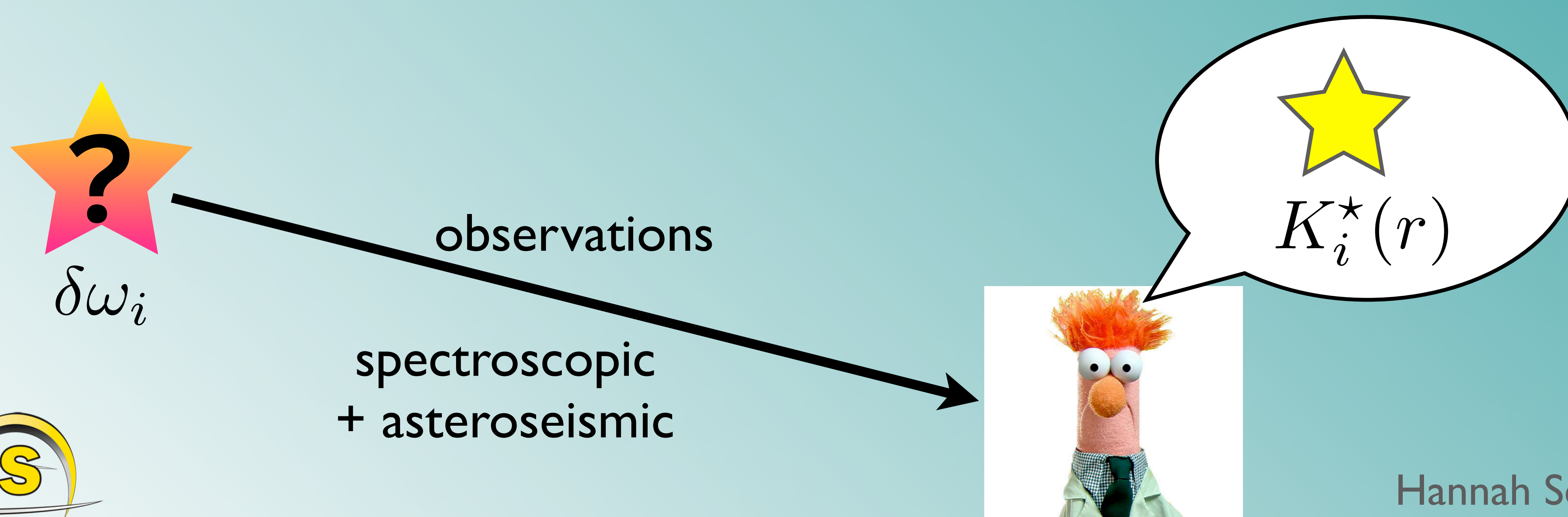
Experiment

- Compute a *reference* stellar model that best matches the observed spectroscopic parameters (Teff, logg, Fe/H) and frequencies of a star
- Compute *perturbed* stellar models for perturbations to the spectroscopic parameters and frequencies within the uncertainties
- Specify a synthetic rotation profile



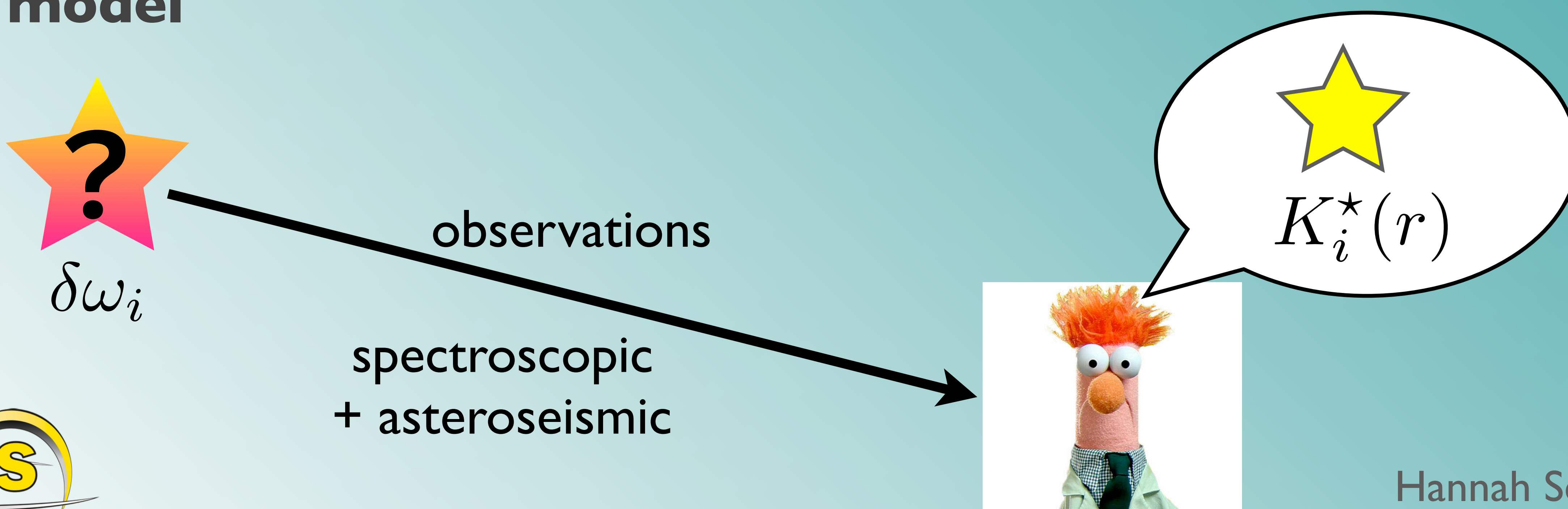
Experiment

- Compute a *reference* stellar model that best matches the observed spectroscopic parameters (Teff, logg, Fe/H) and frequencies of a star
- Compute *perturbed* stellar models for perturbations to the spectroscopic parameters and frequencies within the uncertainties
- Specify a synthetic rotation profile
- Compute the rotational splittings (plus some noise) for each stellar model



Experiment

- Compute a *reference* stellar model that best matches the observed spectroscopic parameters (Teff, logg, Fe/H) and frequencies of a star
- Compute *perturbed* stellar models for perturbations to the spectroscopic parameters and frequencies within the uncertainties
- Specify a synthetic rotation profile
- Compute the rotational splittings (plus some noise) for each stellar model
- Invert for the rotation profile using **splittings from each perturbed model** but using only **kernels from the reference model**



Stellar modelling constraints

Sun-like star
HD52265

Ballot et al. 2011; Gizon et al. 2013

$$[M/H] = 0.19 \pm 0.05 \text{ dex}$$

$$T_{\text{eff}} = 6100 \pm 60\text{K}$$

$$\log g = 4.35 \pm 0.9$$

Observed mode frequencies

$\ell = 0, 1, 2 \quad 1.6 \leq \nu \leq 2.5 \text{ mHz}$

Sub-giant star
KIC7341231 (Otto)

Appourchaux et al 2012; Deheuvels et al. 2012

$$[M/H] = -1.64 \pm 0.05 \text{ dex}$$

$$T_{\text{eff}} = 5233 \pm 50\text{K}$$

$$\log g = 3.55 \pm 0.03$$

Observed mode frequencies

$\ell = 0, 1, 2 \quad 0.31 \leq \nu \leq 0.54 \text{ mHz}$

Stellar models

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Stellar models

- MESA code to generate the best-fit stellar model for reference and perturbed parameters

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- ADIPLS to compute the eigenmodes

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- Correction for surface effects $\nu^3/\mathcal{I}$

  Ball & Gizon et al submitted

Stellar models

- MESA code to generate the best-fit stellar model for reference and perturbed parameters
- ADIPLS to compute the eigenmodes
- Correction for surface effects $\nu^3/\mathcal{I}$

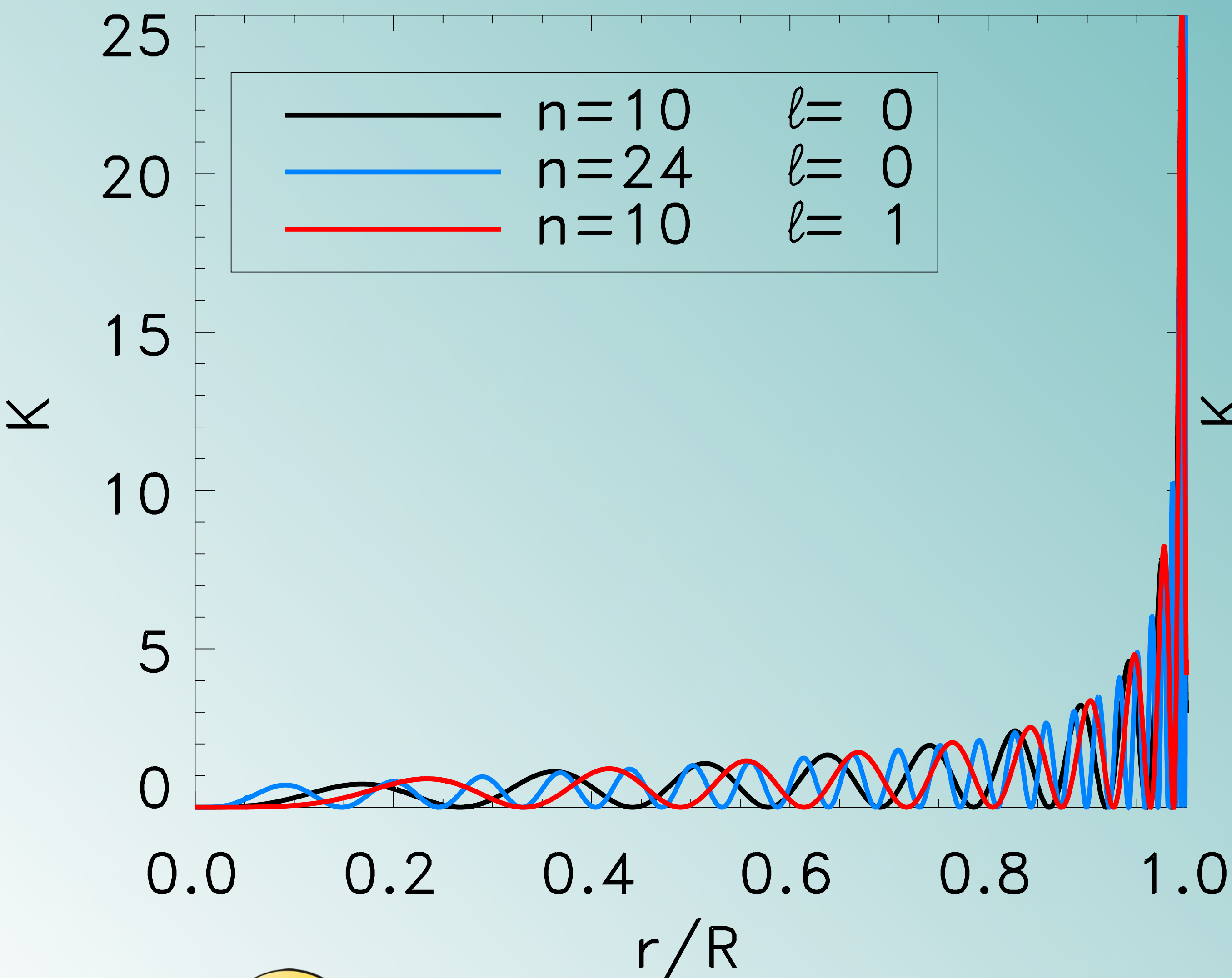
  Ball & Gizon et al submitted

- HD52265: 100 perturbed models
- Otto: 20 perturbed models

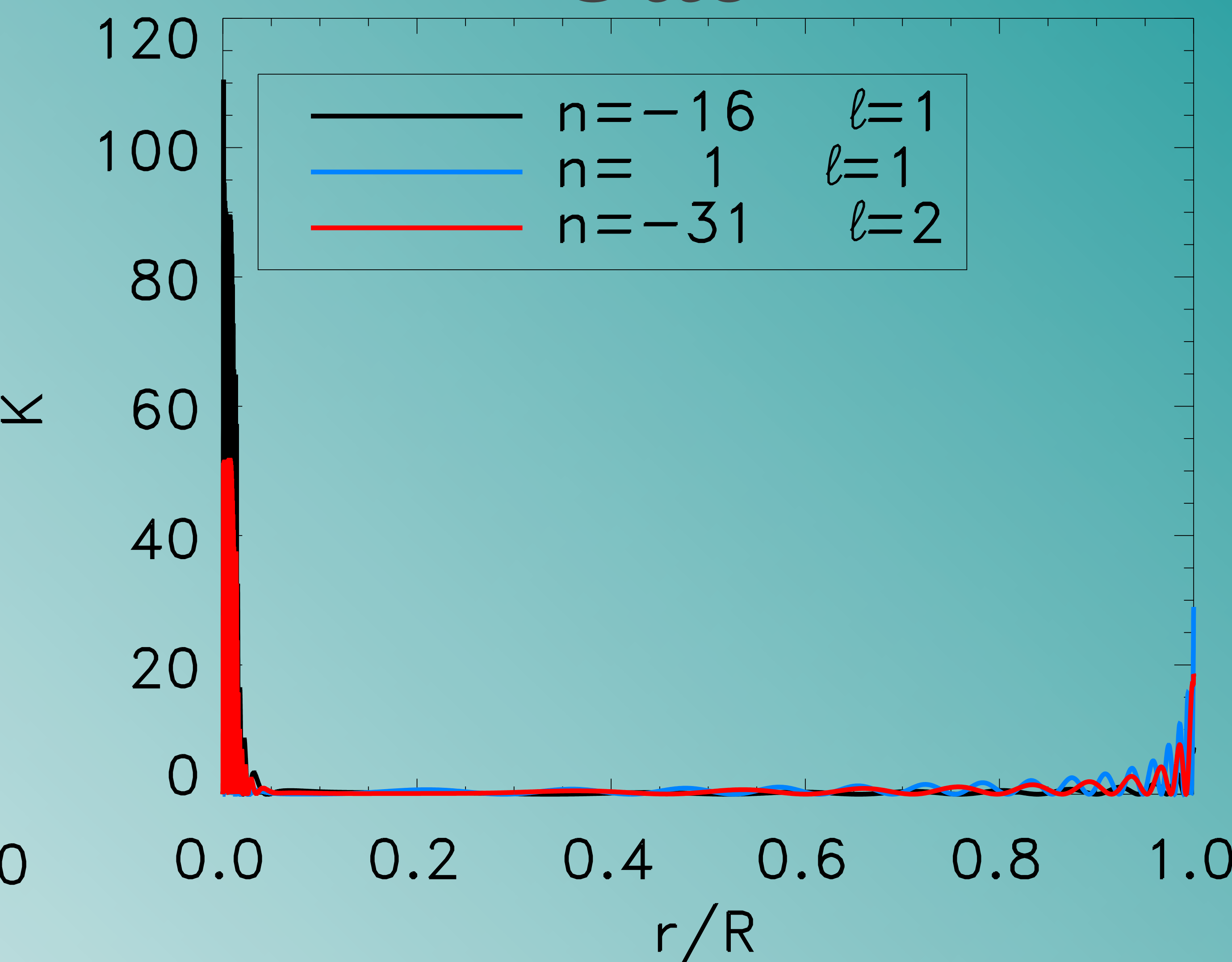
Rotation Kernels

$$K_{nl} = \frac{(\xi_r^2 + L^2\xi_h^2 - 2\xi_r\xi_h - \xi_h^2)r^2\rho}{\int_0^R (\xi_r^2 + L^2\xi_h^2 - 2\xi_r\xi_h - \xi_h^2)r^2\rho \mathrm{d}r}$$

HD52265



Otto



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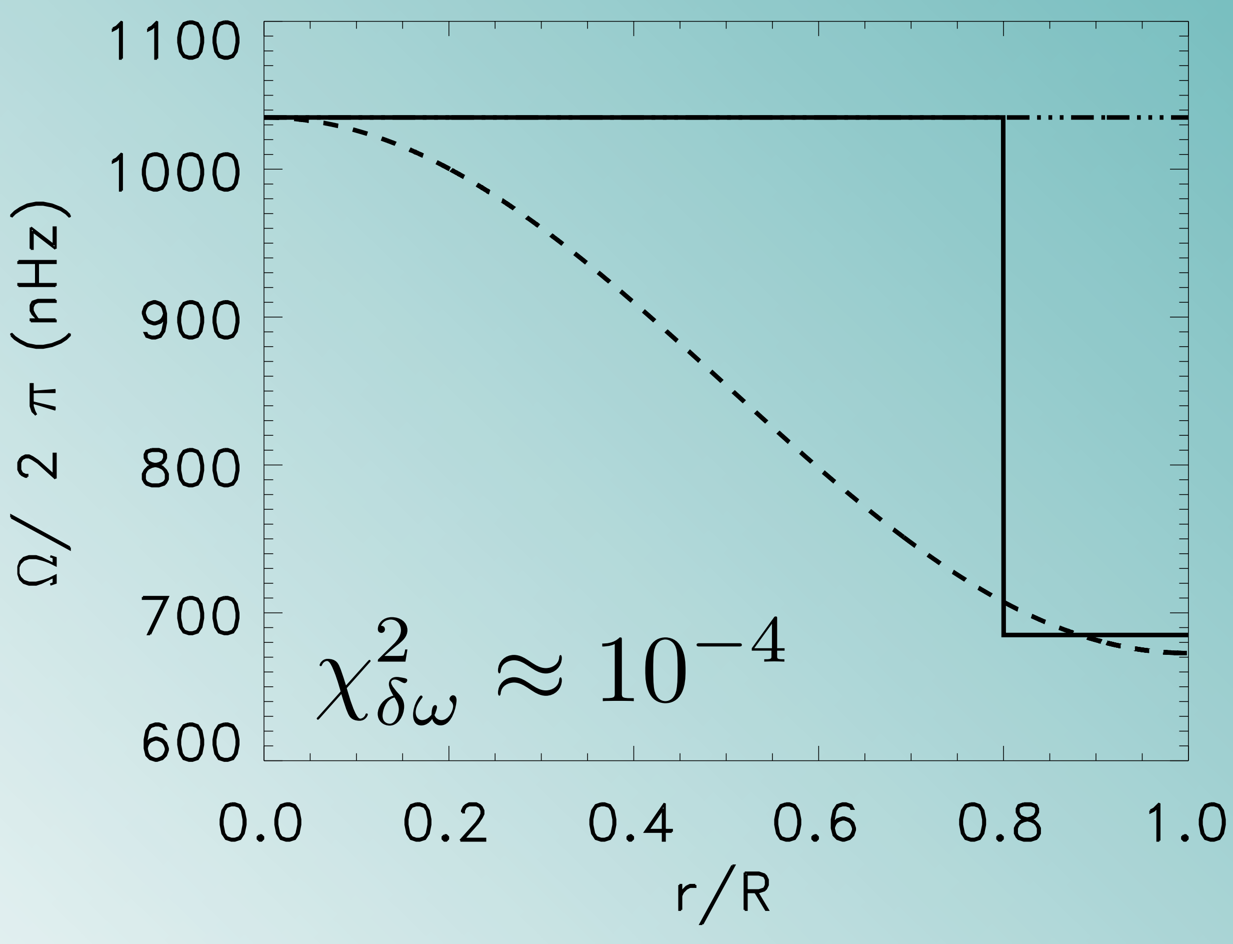
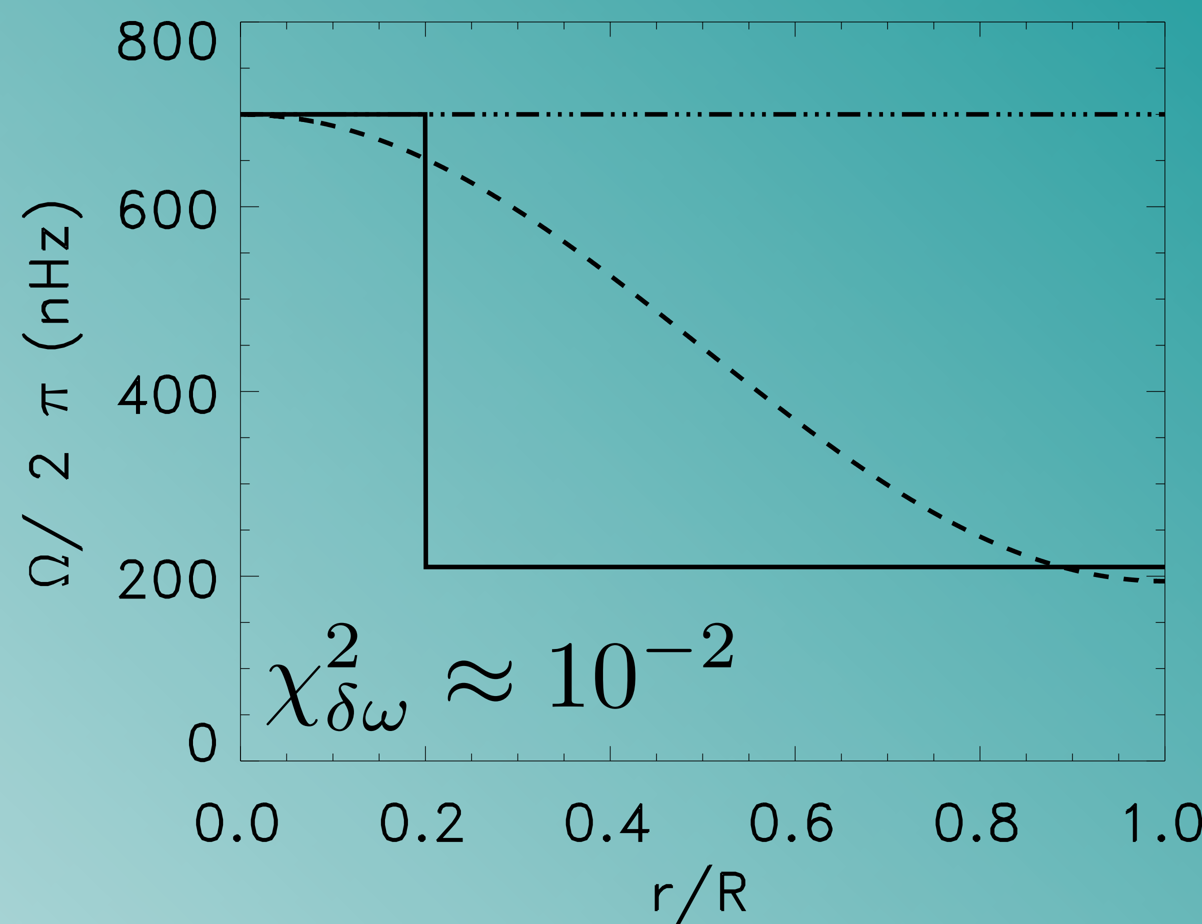
Synthetic rotation profiles

HD52265 $v \sin i = 3.6^{+0.3}_{-0.1} \mathrm{kms}^{-1}$



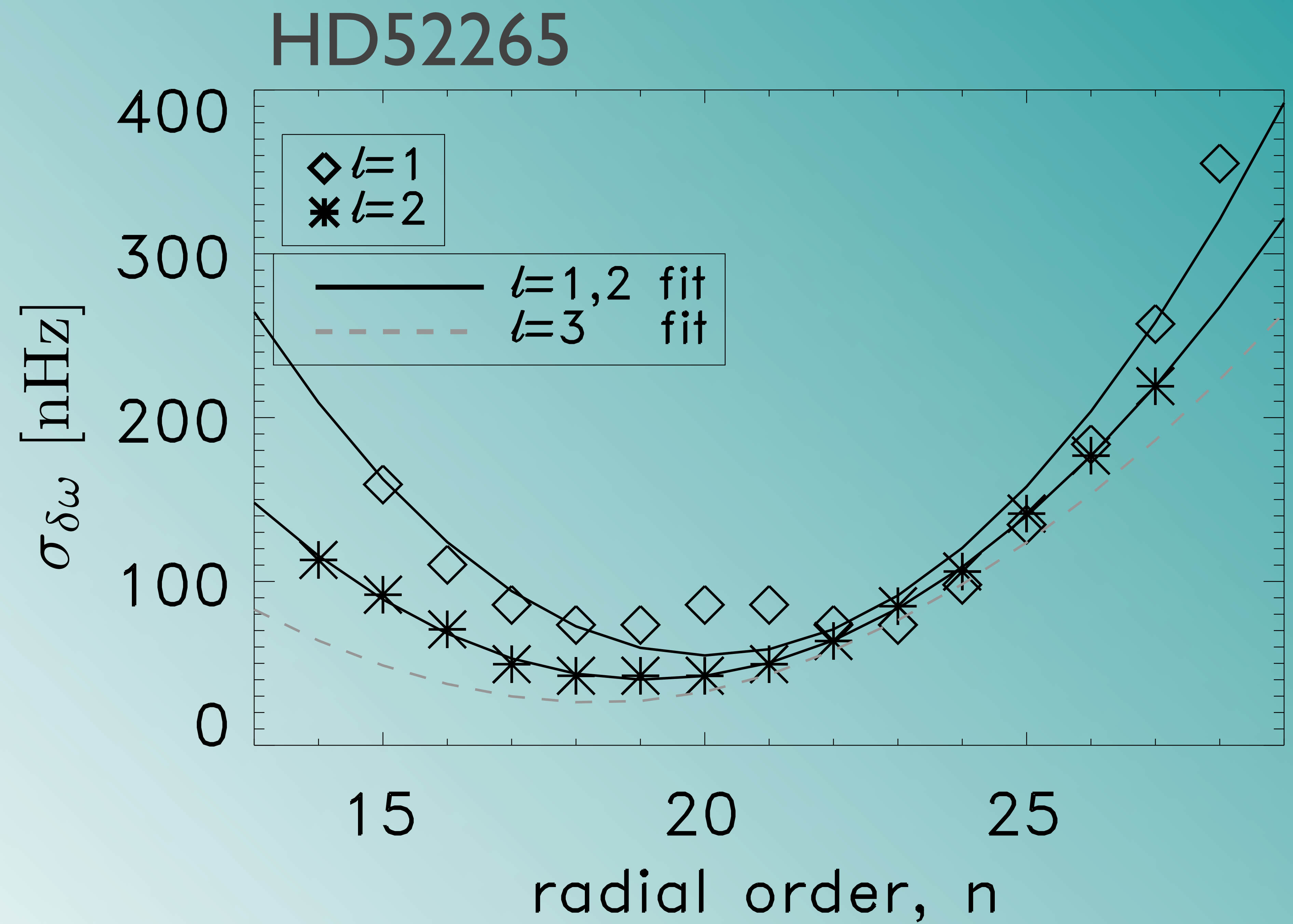
Otto $v \sin i < 1.0 \pm 0.5 \mathrm{kms}^{-1}$



$$\delta\omega_i = \int_0^R K_i(r)\Omega(r)dr$$

Noise model

T~ 4 months

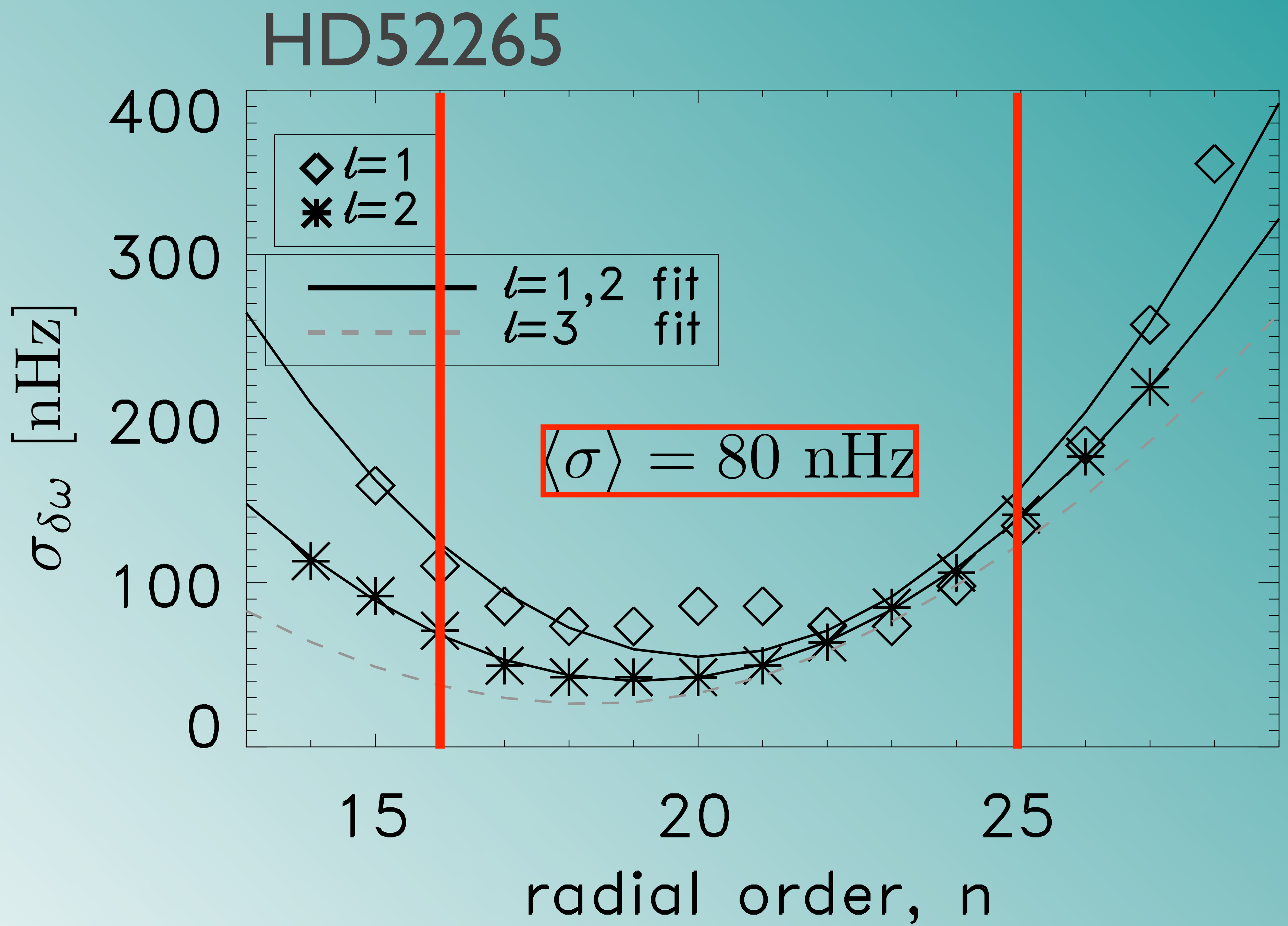


Stahn 2011

$$\sigma_{\delta\omega} = \frac{\sigma_\omega}{\sqrt{1/3\ell(\ell+1)}}$$

Noise model

T~ 4 months



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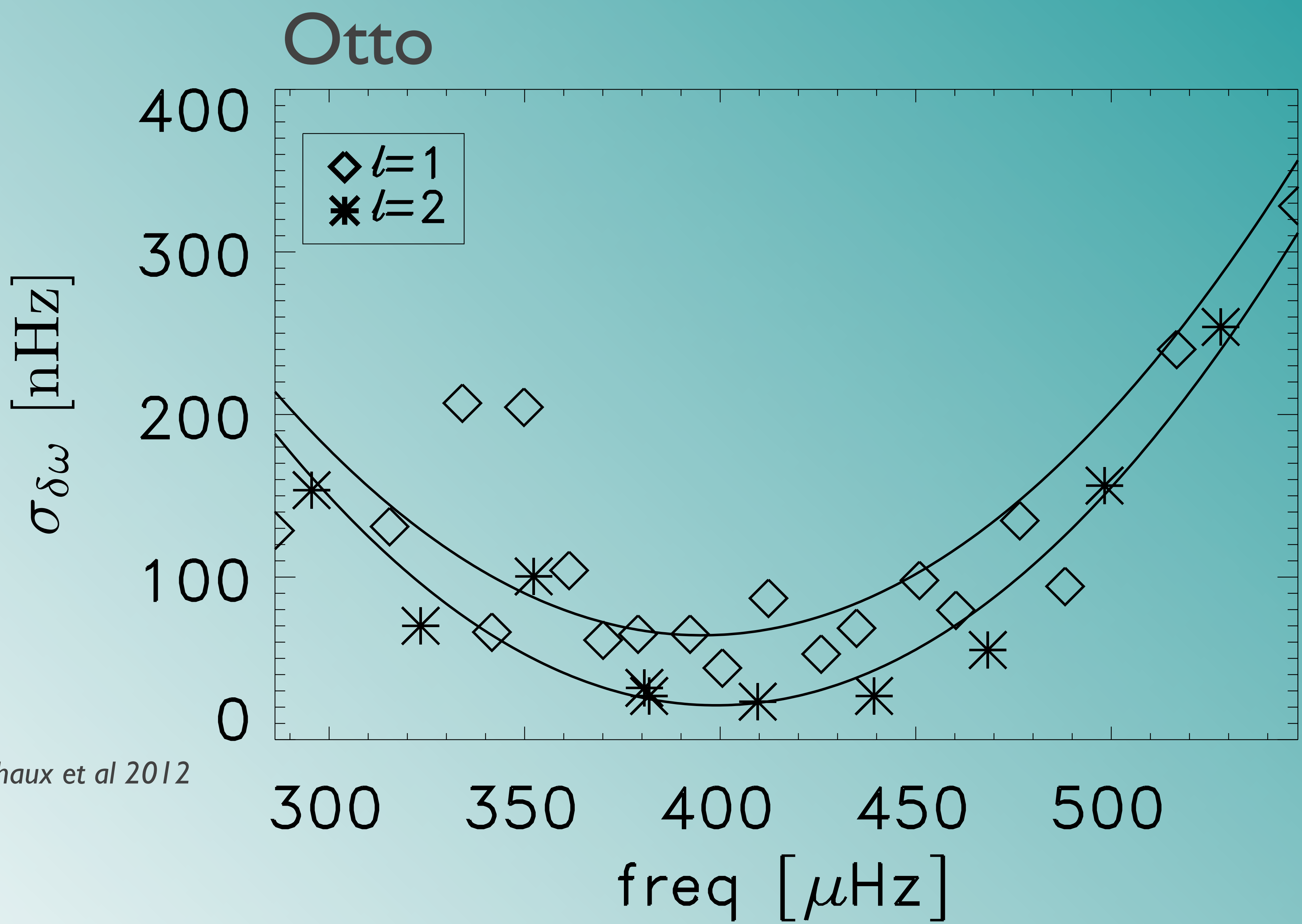

Appourchaux et al 2012

$$\sigma_{\delta\omega} = \frac{\sigma_\omega}{\sqrt{1/3\ell(\ell+1)}}$$

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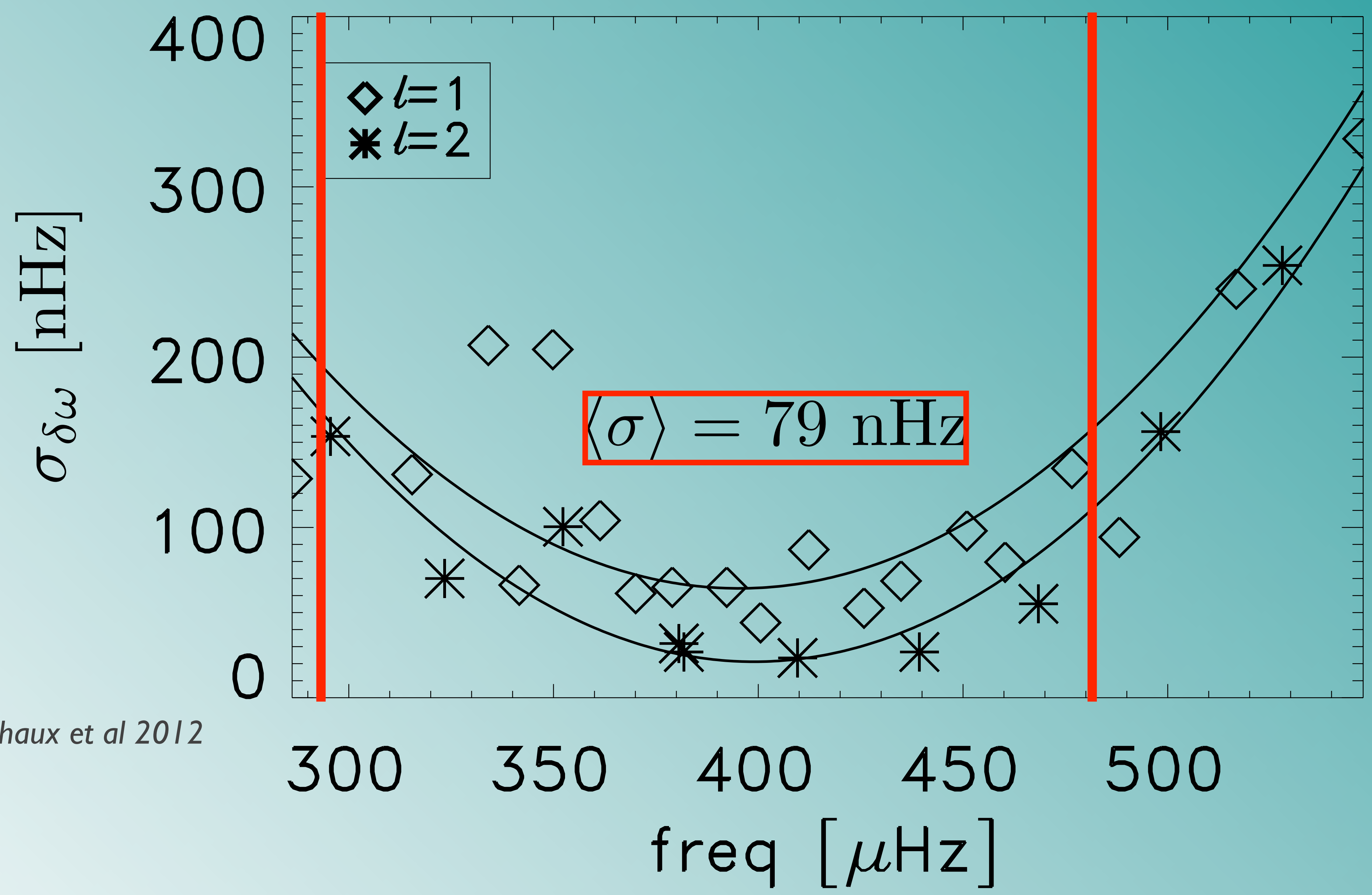
Noise model

T~ 4 months

Otto



Appourchaux et al 2012

$$\sigma_{\delta\omega} = \frac{\sigma_\omega}{\sqrt{1/3\ell(\ell+1)}}$$

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Method

forward problem

$$K_i, \Omega(r) \longrightarrow \delta\omega_i$$

$$\delta\omega_i \downarrow$$

inverse problem

$$\overline{\Omega}(r) \longleftarrow \delta\omega_i, \epsilon_i, K_i^\star$$

Method

forward problem

$$K_i, \Omega(r) \longrightarrow \delta\omega_i$$

$$\delta\omega_i \downarrow$$

inverse problem

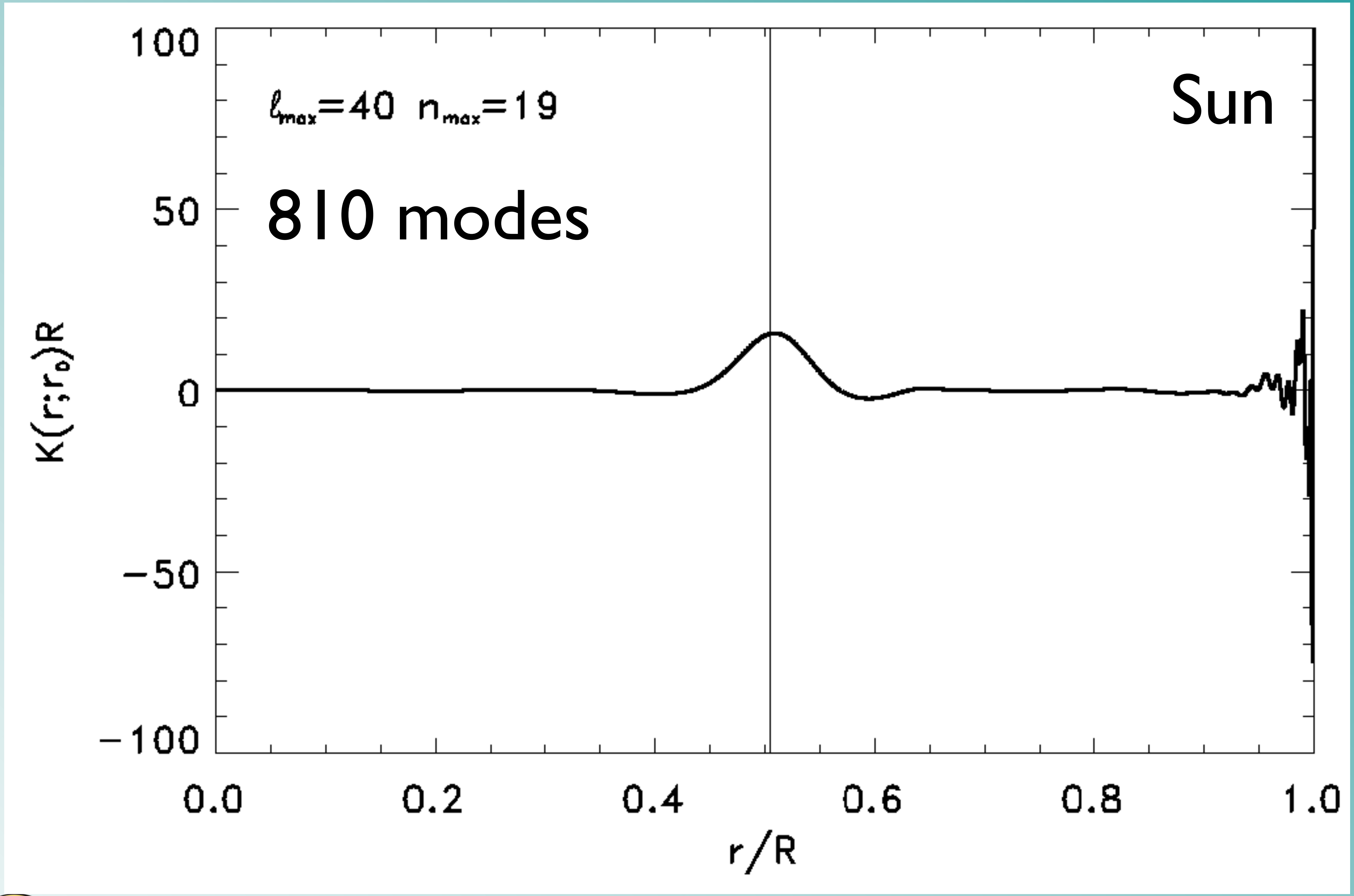
$$\overline{\Omega}(r) \longleftarrow \delta\omega_i, \epsilon_i, K_i^\star$$

RLS

$$\sum_{i \in M} \frac{1}{\sigma_i^2} \left[\delta\omega_i - \sum_{j=1}^{N} \overline{\Omega}_j \int_0^R K_i(r)\phi_j(r)dr \right]^2 + \mu F(\overline{\Omega}_j)$$

Averaging Kernels

$$\mathbf{K}(r_0; r) = \sum_i c_i(r_0) K_i(R)$$



Averaging Kernels

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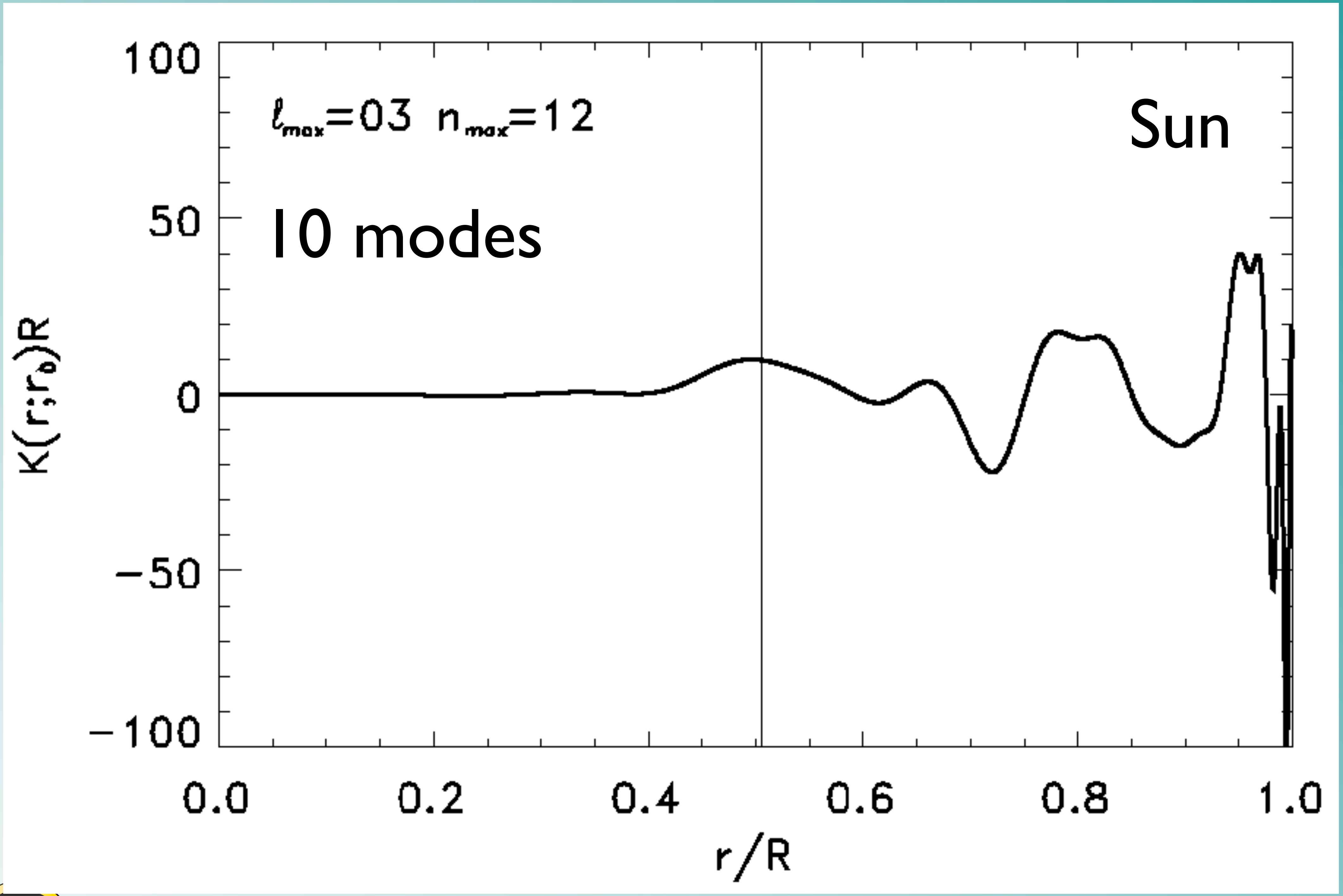

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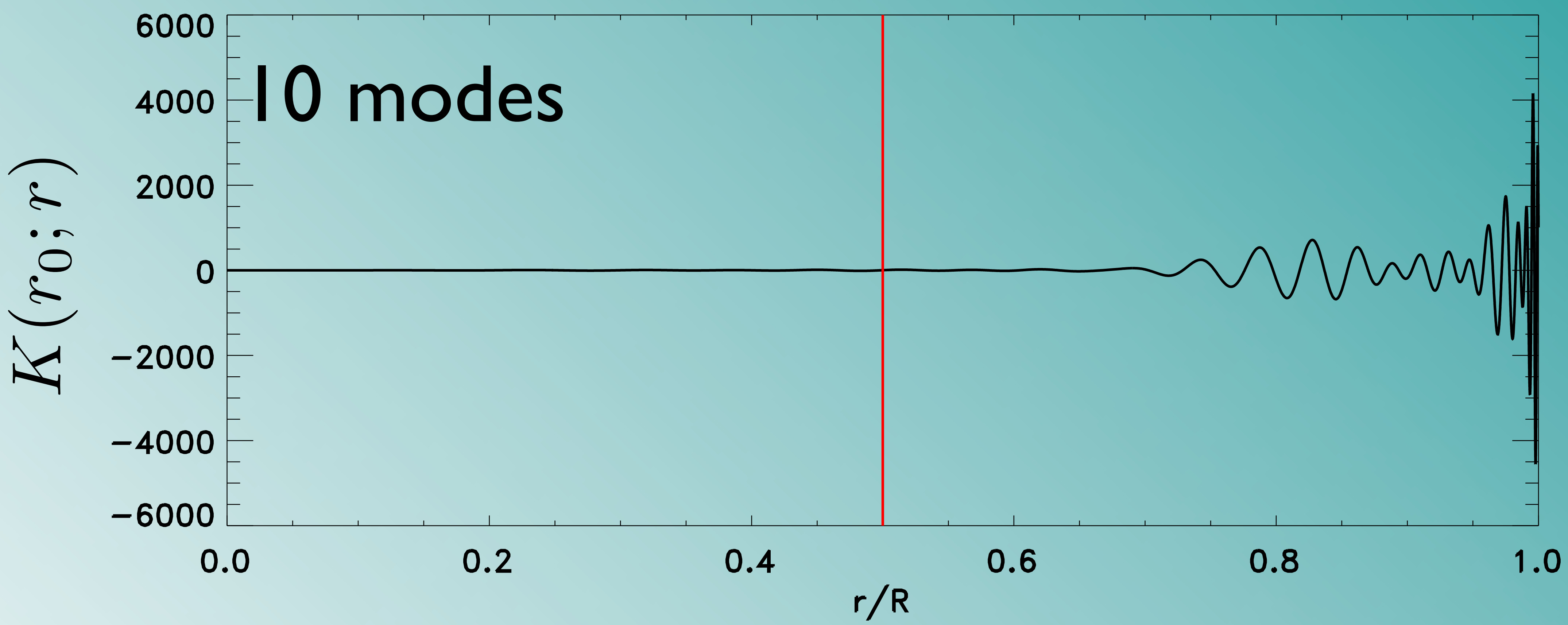
Averaging Kernels

$$\mathbf{K}(r_0; r) = \sum_i c_i(r_0) K_i(R)$$

HD52265



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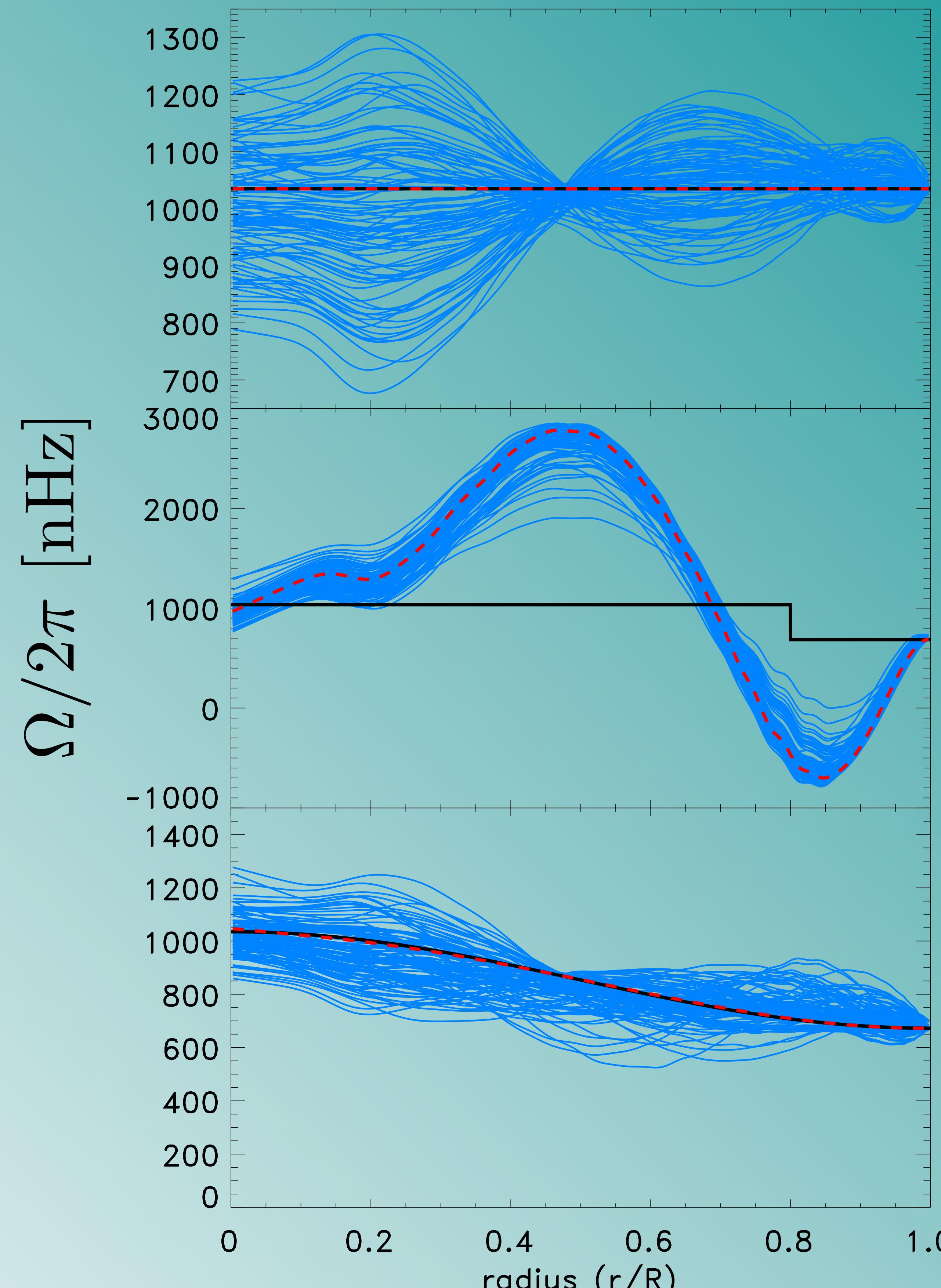
Inverted rotation profiles

HD52265

no noise

$\mu = 10^3$



synthetic
reference
perturbed

!

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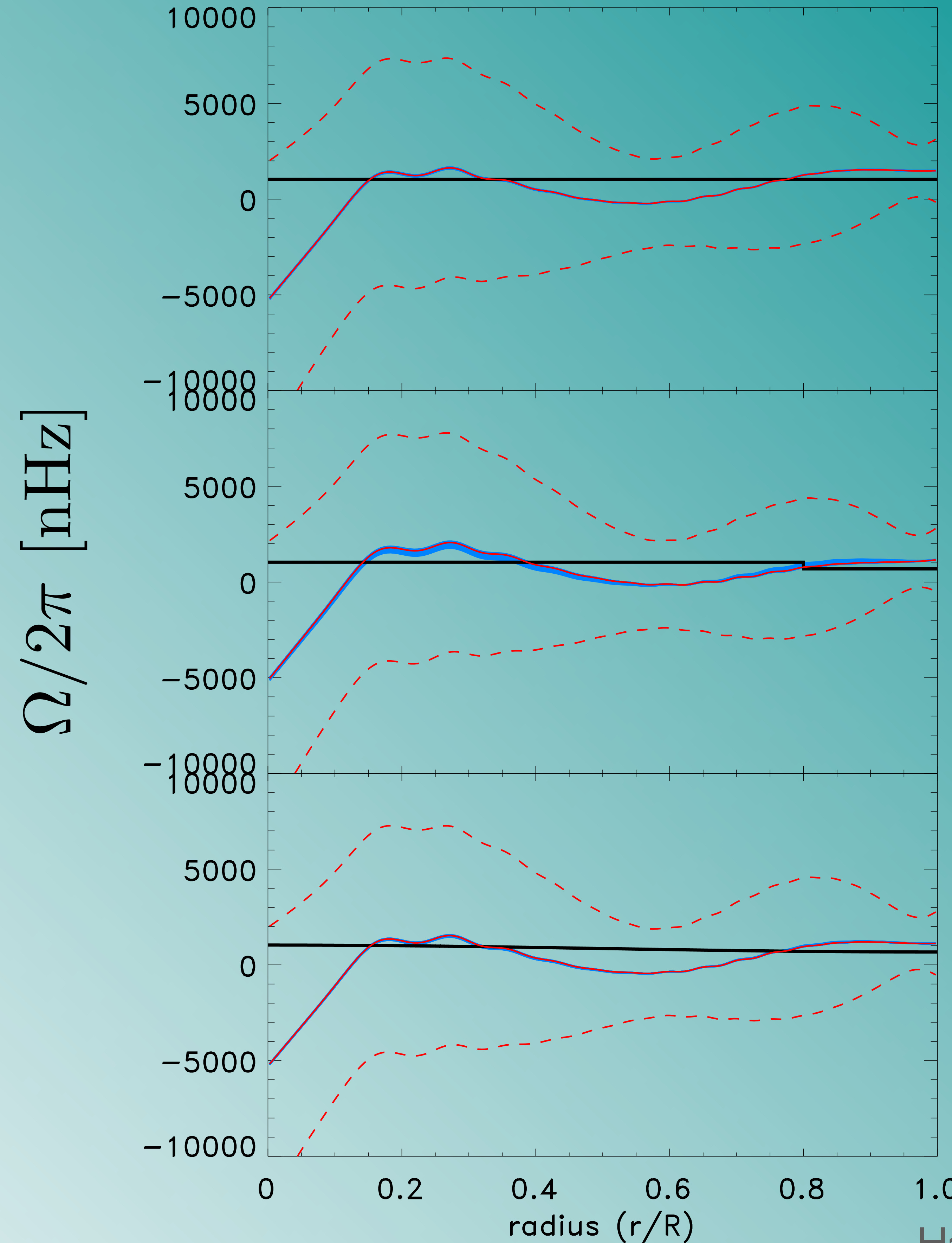
Inverted rotation profiles

HD52265

$$\langle \sigma \rangle = 80 \text{ nHz}$$

$$\mu = 10^7$$

$$\sigma_\Omega(r_0) = \sqrt{\sum_{i=1}^{M} [c_i(r_0)\sigma_i]^2}$$



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Sensitivity is confined to the surface

Sun-like stellar models are well enough constrained

Sub-giants

Effect of Noise

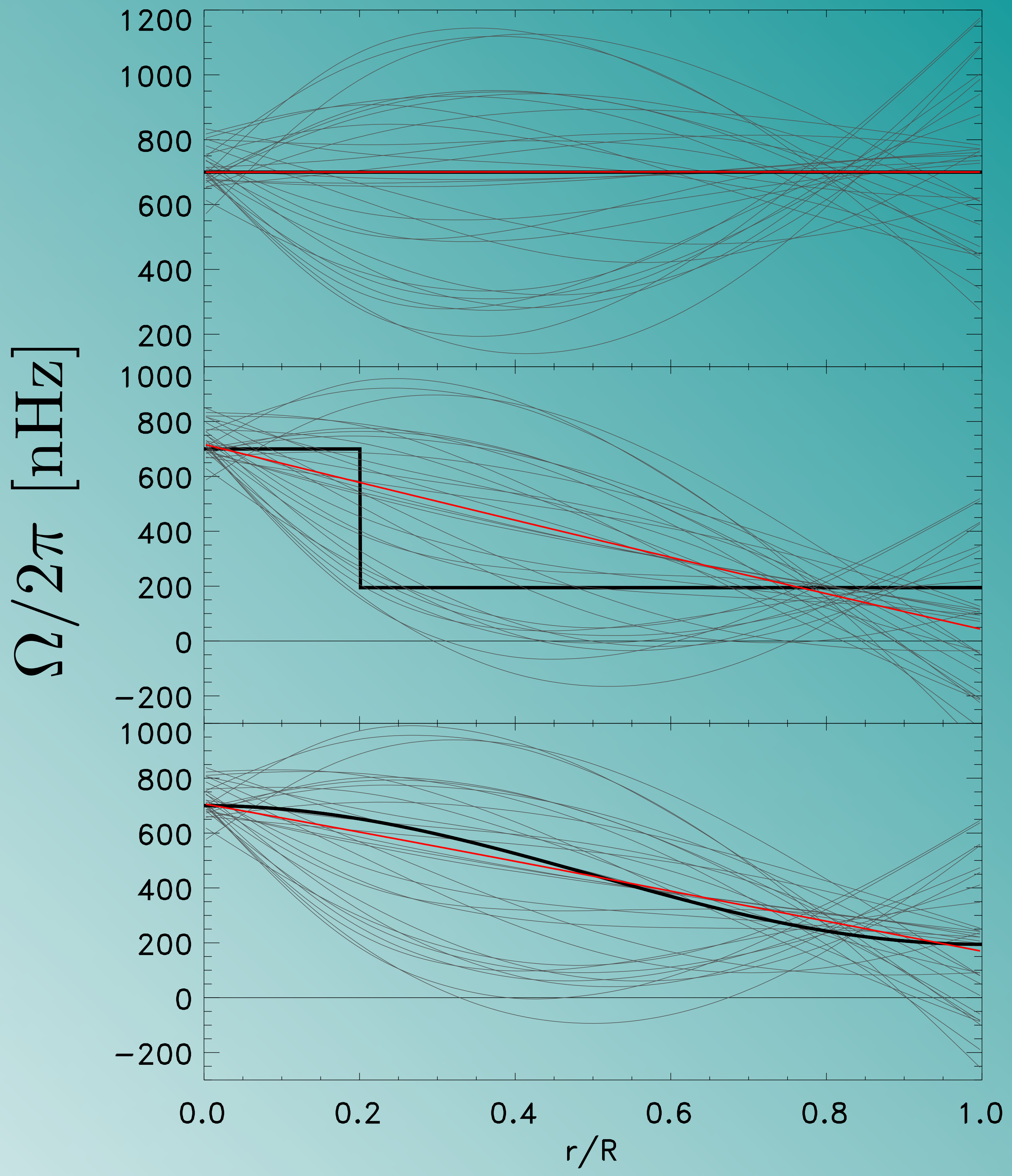
Otto

30 random noise realisations on the splittings

$$\langle \sigma \rangle = 80 \text{ nHz}$$

(one stellar model only)

$$\frac{M}{\sum_{i=1}^{M} \sigma_i^2} \mu = 1.1 \times 10^{10}$$



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Inverted rotation profiles

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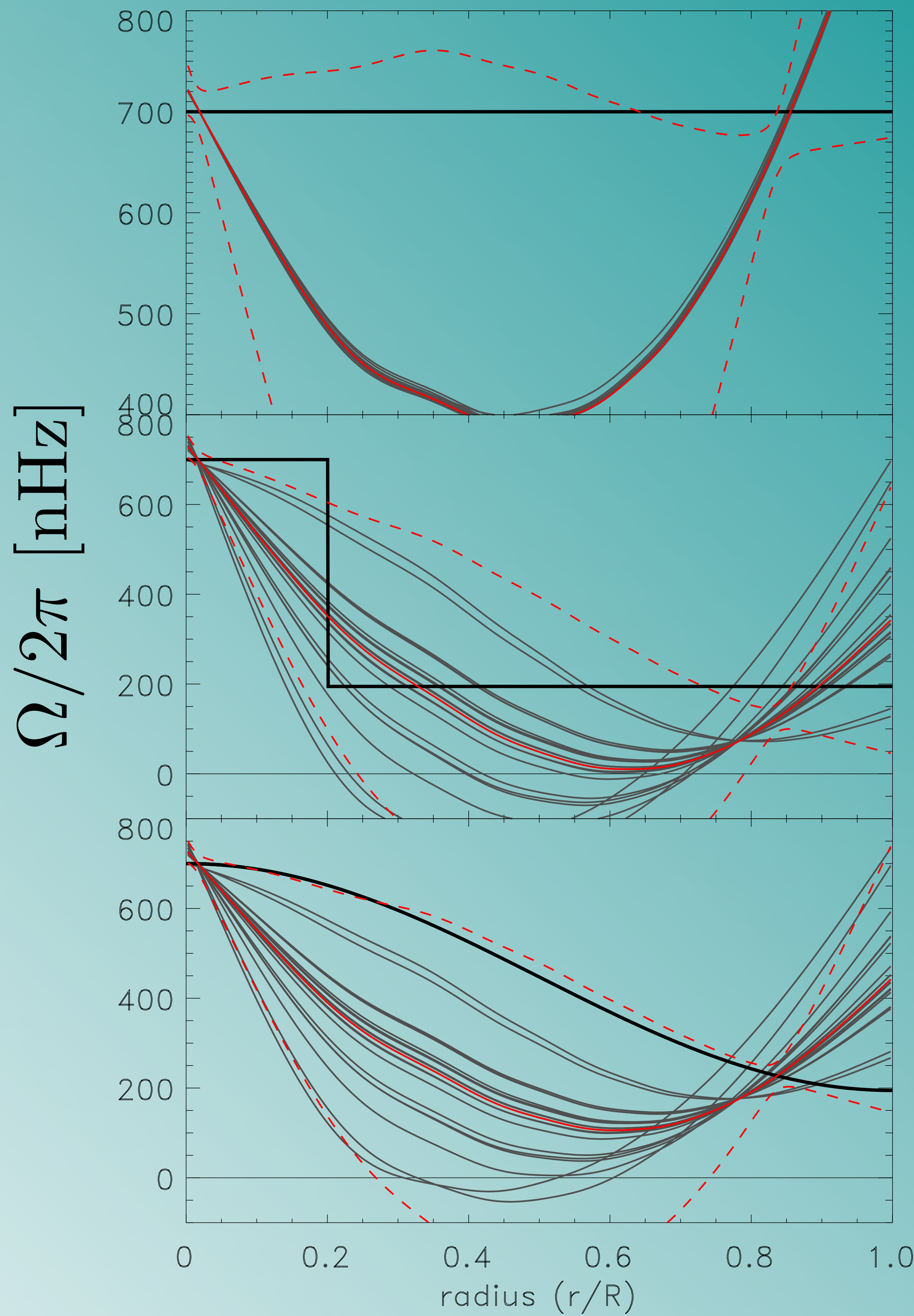
$\ell = 1, 2$

$\langle\sigma\rangle = 79 \text{ nHz}$

$$\frac{M}{\sum_{i=1}^{M} \sigma_i^2}\mu = 1.3 \times 10^{10}$$



synthetic
reference
perturbed

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Need to be careful which stellar model we use when inverting for sub-giant stars

Ensemble Inversions

$$\sum_{i\in M}\frac{1}{\sigma_i^2}\left[\delta\omega_i-\sum_{j=1}^{N}\overline{\Omega}_j\int_0^R K_i(r)\phi_j(r)dr\right]^2+\mu F(\overline{\Omega}_j)$$

Many stars of similar type

More modes

Assume similar rotation profiles

-- sign of the shear

-- magnitude of the shear

MPS

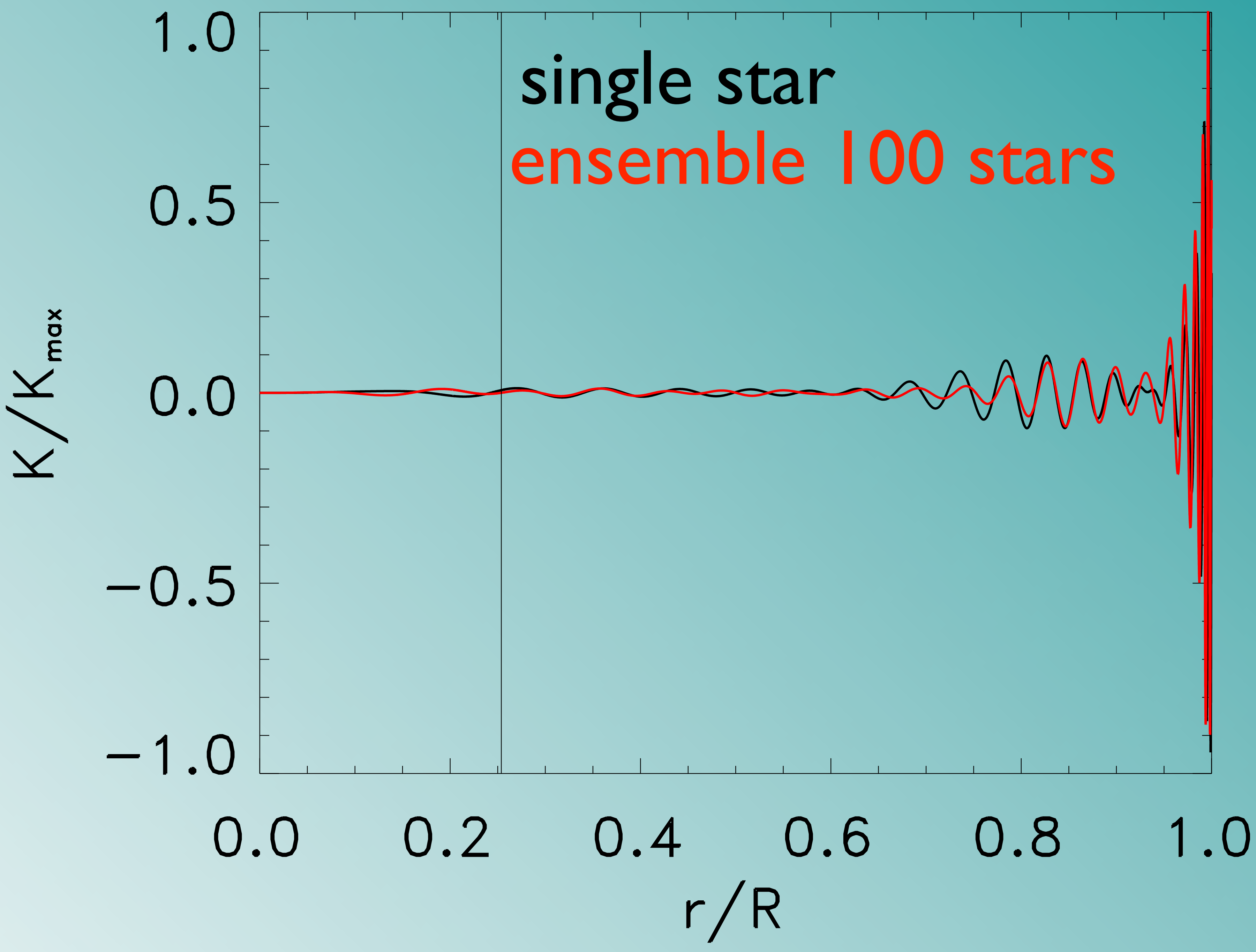
Ensemble Inversions

$$\mathbf{K}(r_0; r) = \sum_i c_i(r_0) K_i(R)$$

HD52265

$\mu = 10^7$

$\mu = 10^9$



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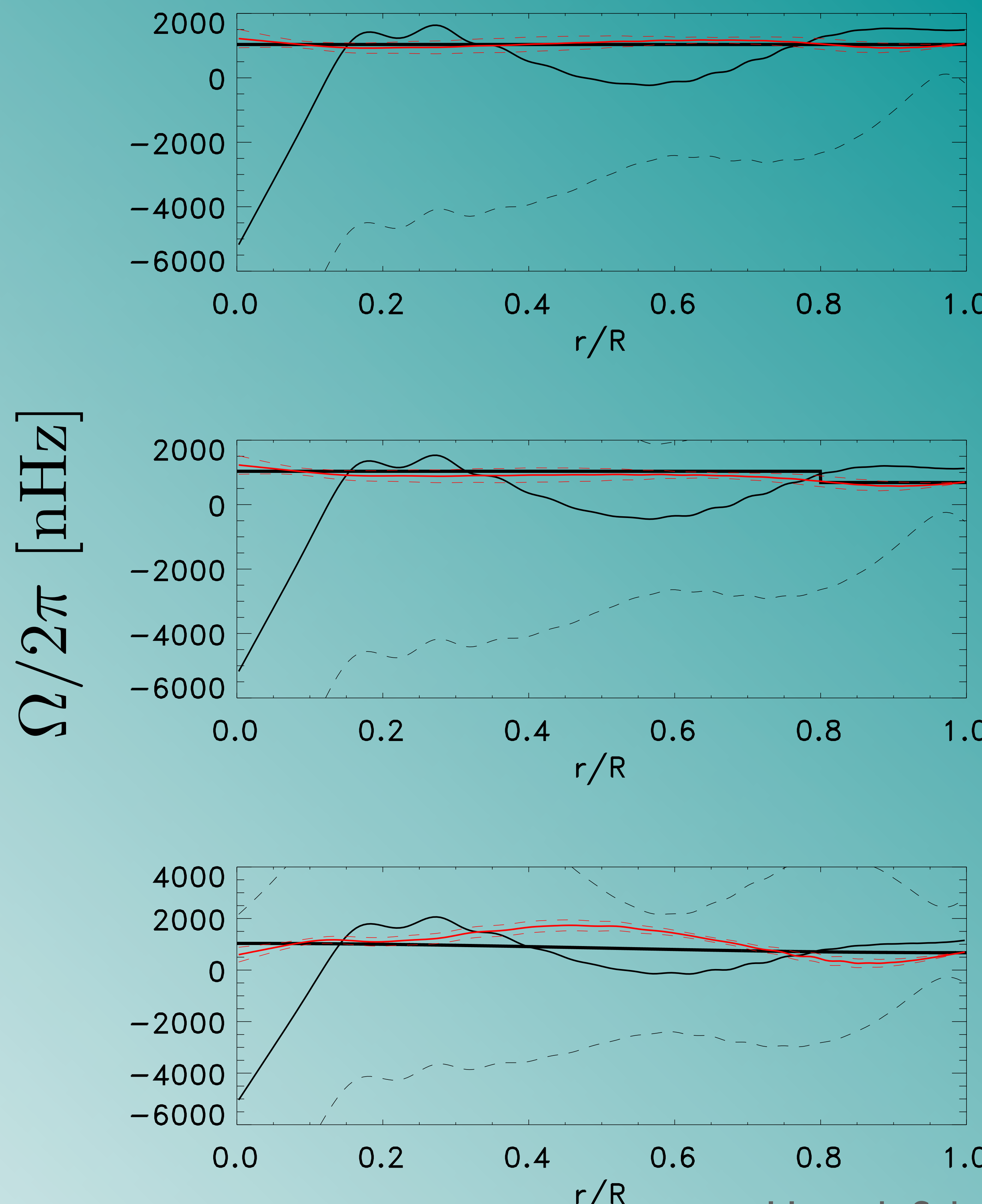
Ensemble Inversions

single star
$\mu = 10^7$

ensemble 100 stars
$\mu = 10^9$



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Ensemble Inversions

single star
$\mu = 10^7$

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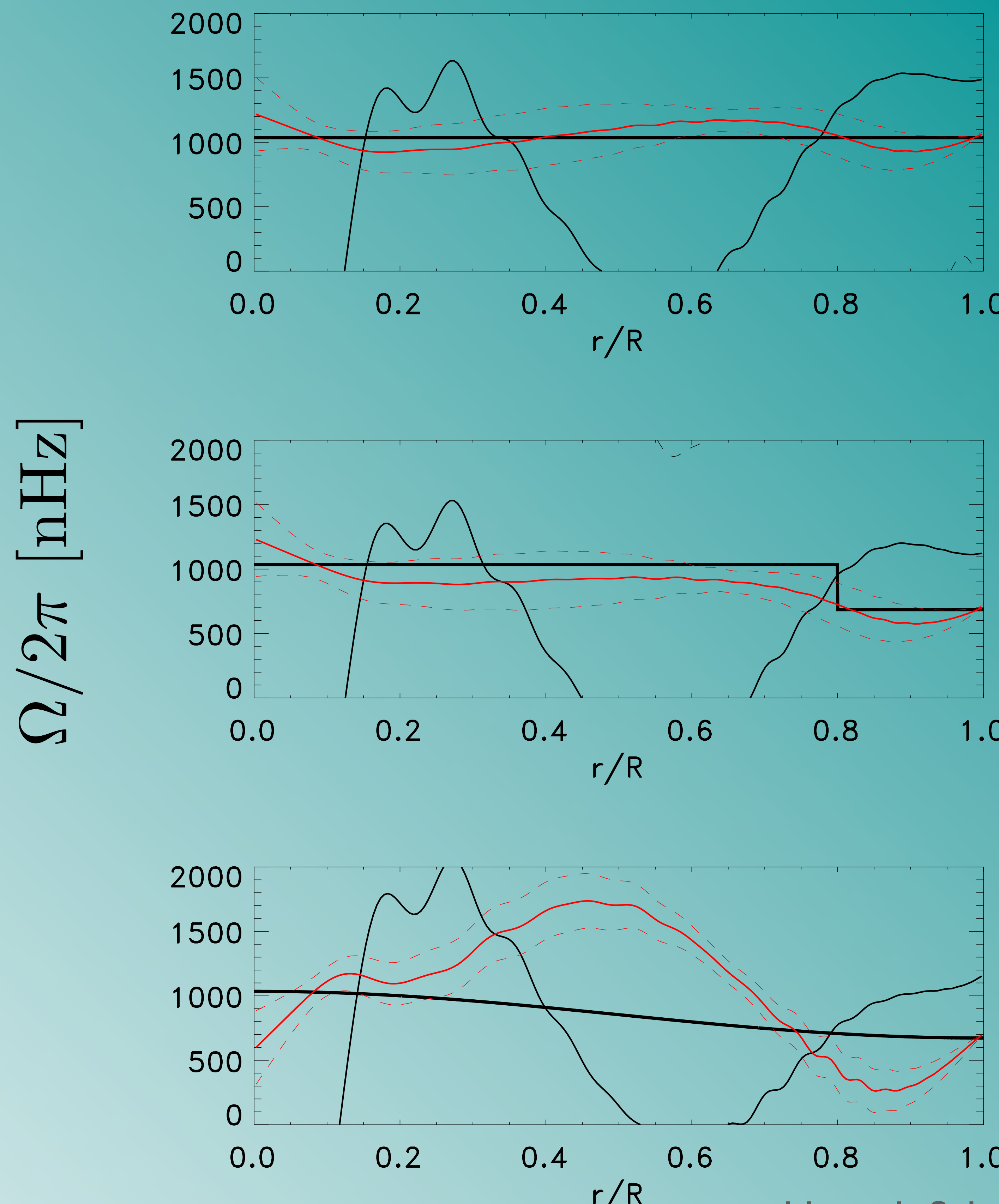

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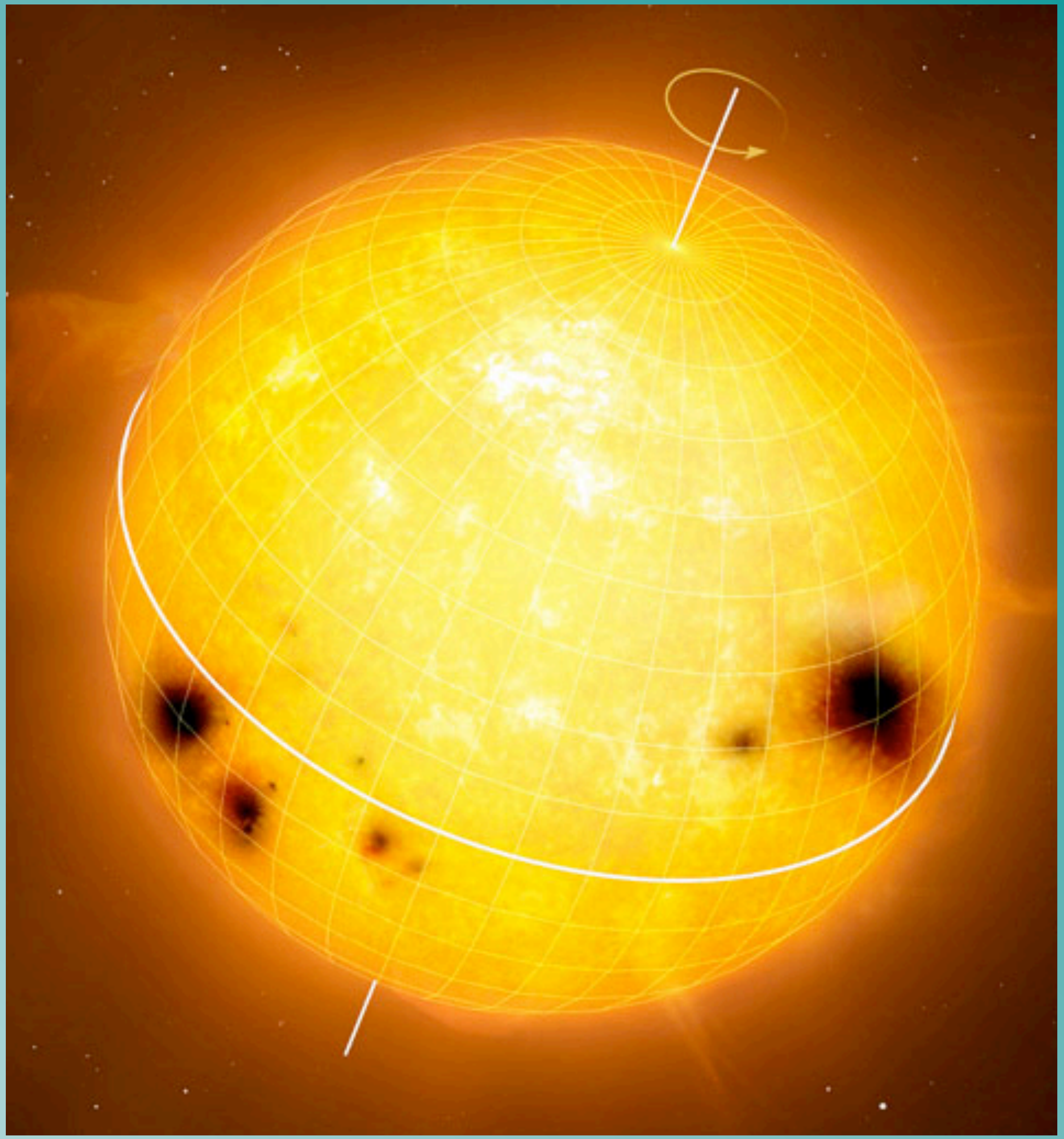
- Ensemble inversions for sign & magnitude of shear $\pm\frac{\mathrm{d}\Omega}{\mathrm{d}r}$

- Need to understand capabilities of linear inversions

- Need more constraints

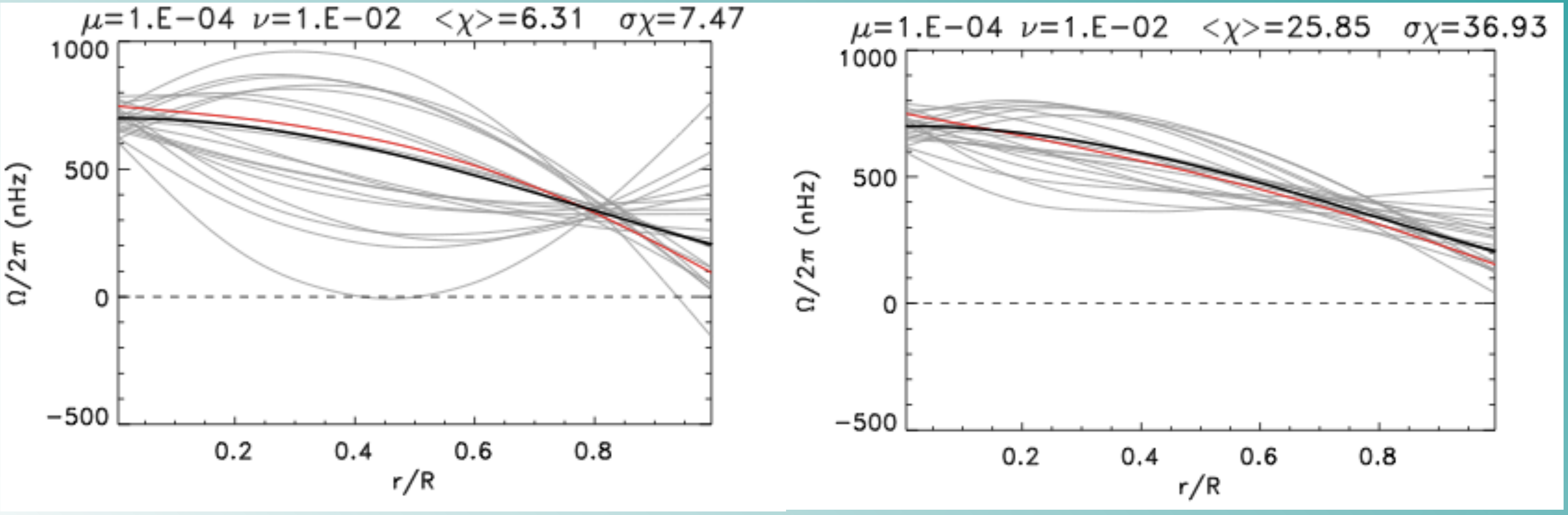
- Latitudinal differential rotation?
  Lund et al. 2014 *Poster #120*

- PLATO - more stars
  SONG - more modes



Surface constraints: starspot rotation

$$\sum_{i\in M}\frac{1}{\sigma_i^2}\left[\delta\omega_i-\sum_{j=1}^{N}\overline{\Omega}_j\int_0^R K_i(r)\phi_j(r)\ \mathrm{d}r\right]^2+\mu F(\overline{\Omega_j})+\nu(\Omega_S-\overline{\Omega}_N)^2$$



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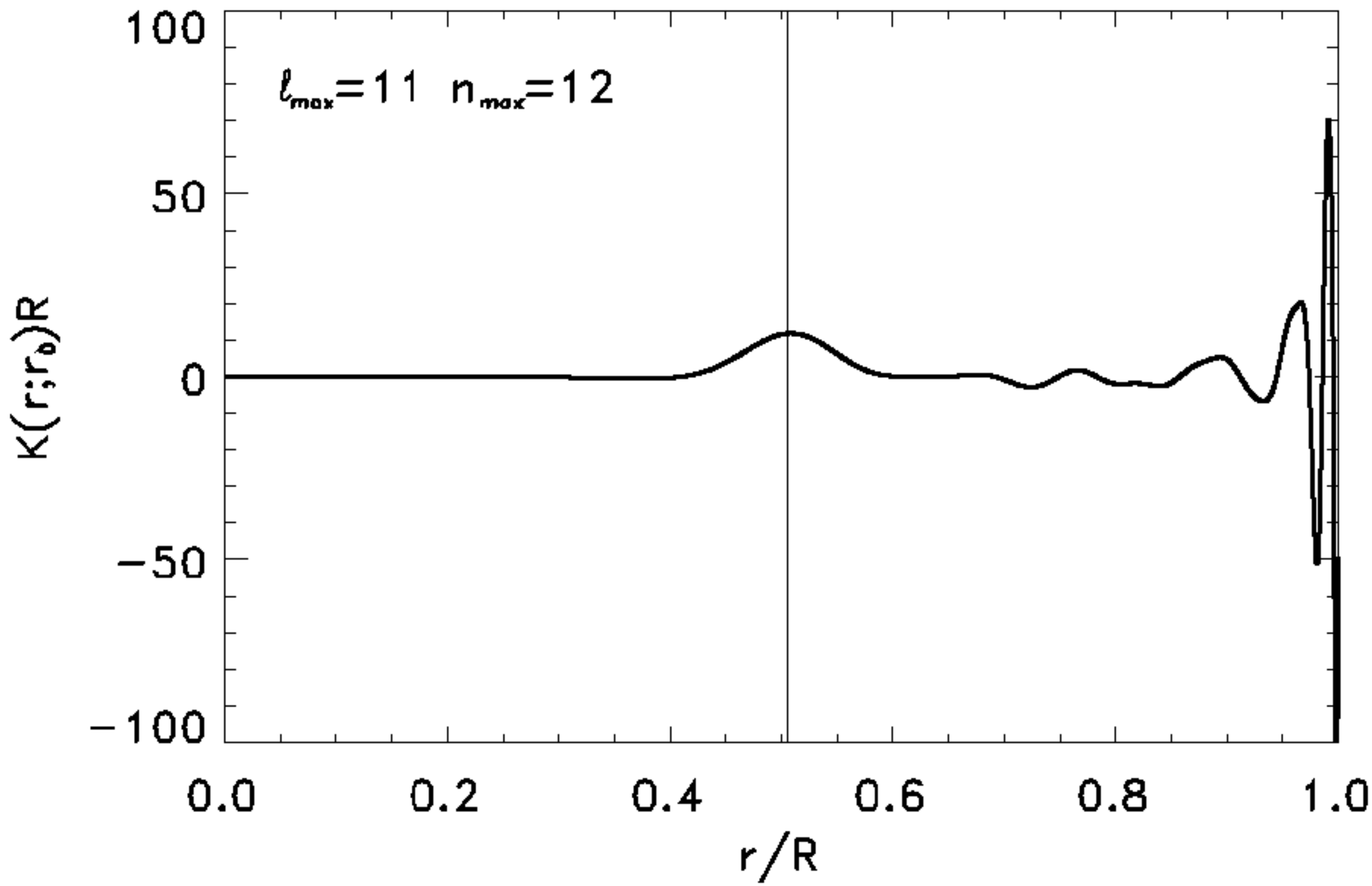
Averaging kernel for the Sun ~100 modes

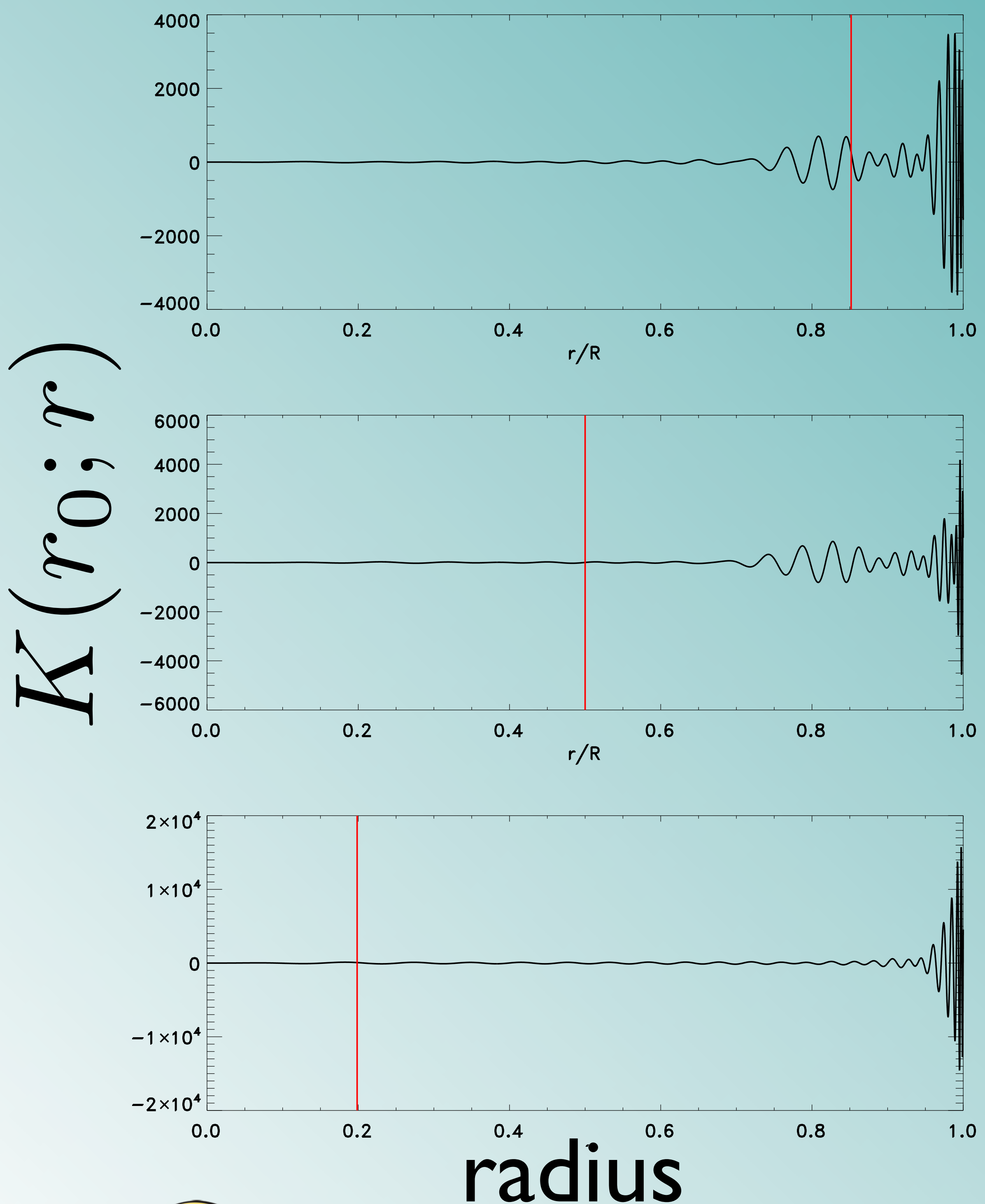


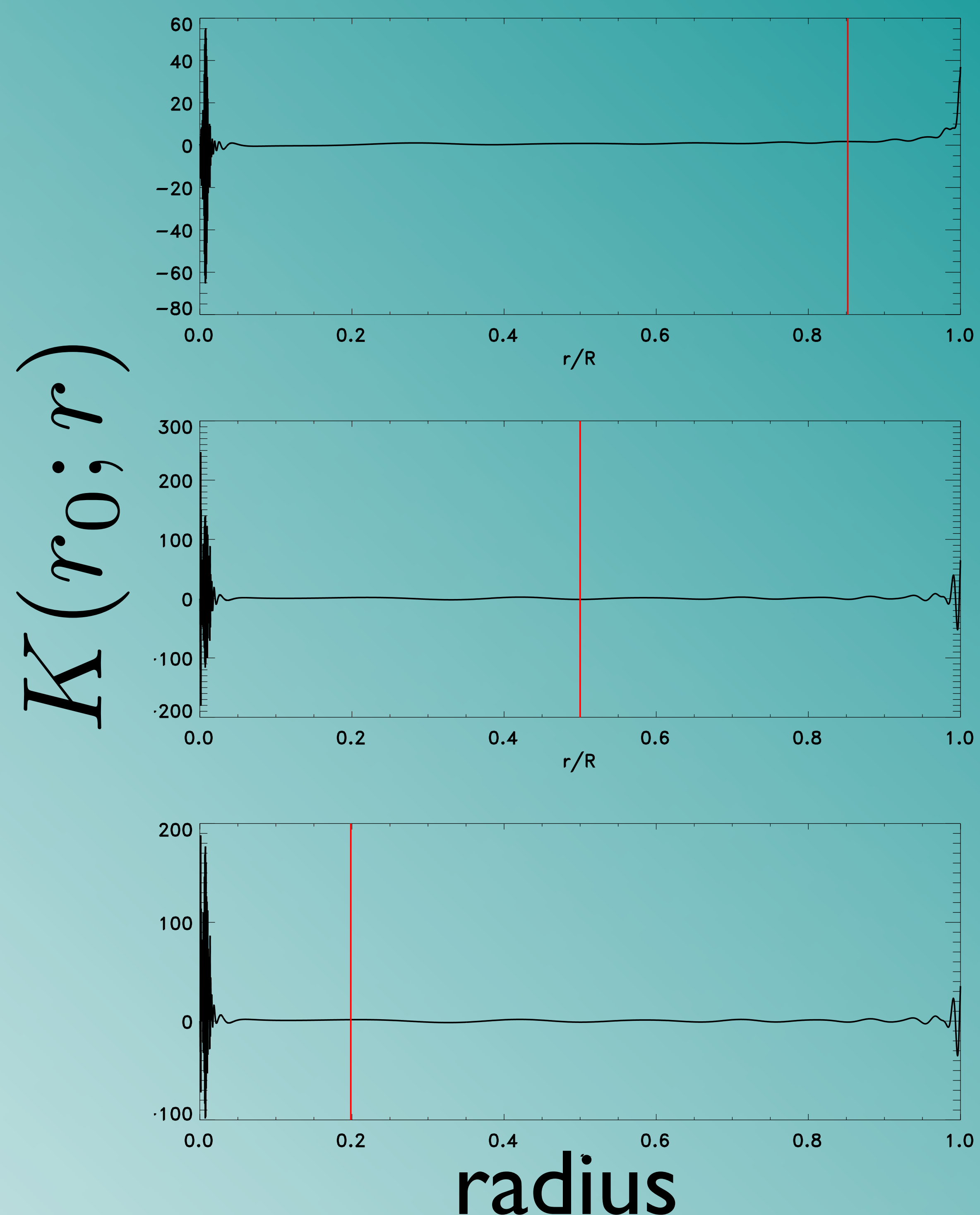
Averaging Kernels

HD52265

$$\mathbf{K}(r_0; r) = \sum_i c_i(r_0) K_i(R)$$

Otto





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Stellar models

HD52265

rms difference

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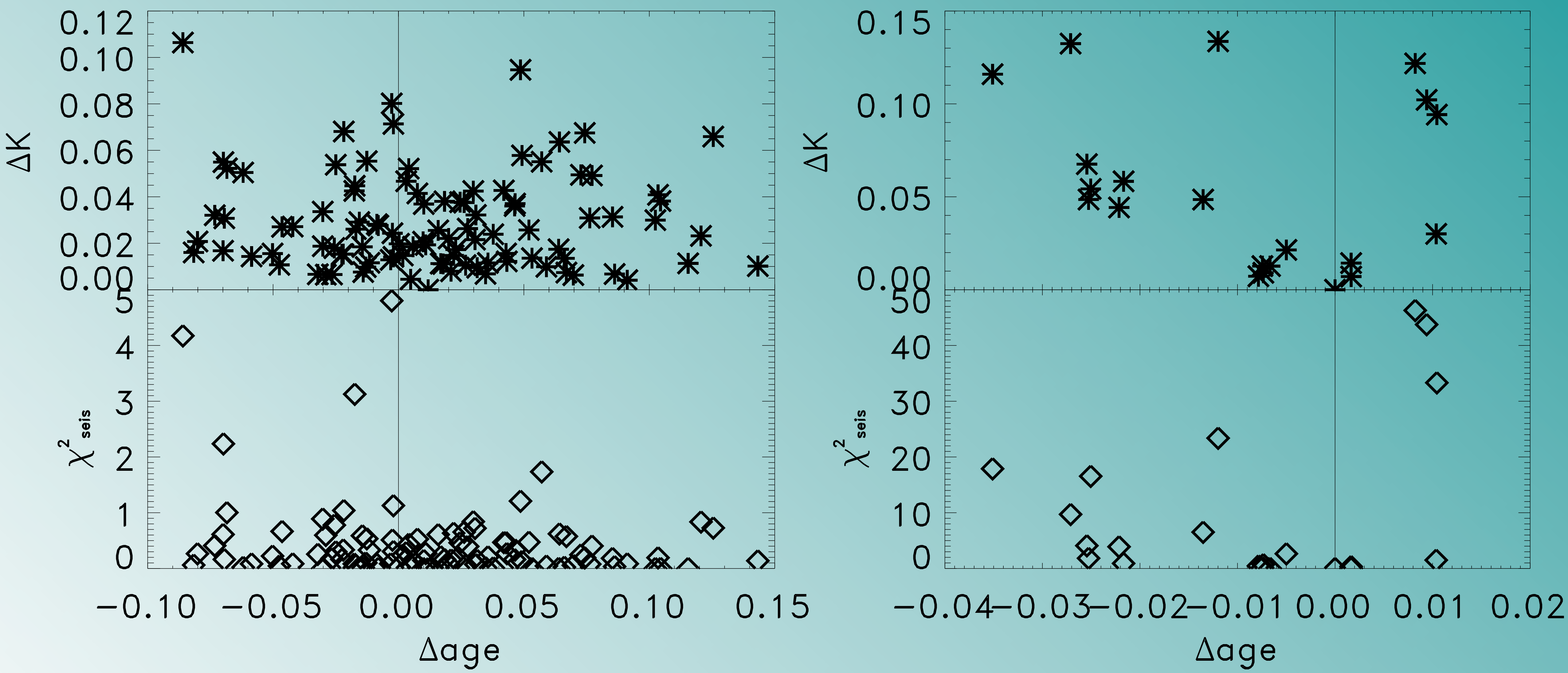
$$\Delta K = \frac{1}{M}\sum_{i=1}^{M}\int_0^R |K_i^\star(r) - K_i(r)|\mathrm{d}r$$



$$\chi^2_{\mathrm{seis}} = \frac{1}{M}\sum_{i=1}^{M}\frac{(\omega_i^\star - \omega_i)^2}{\sigma_i^2}$$

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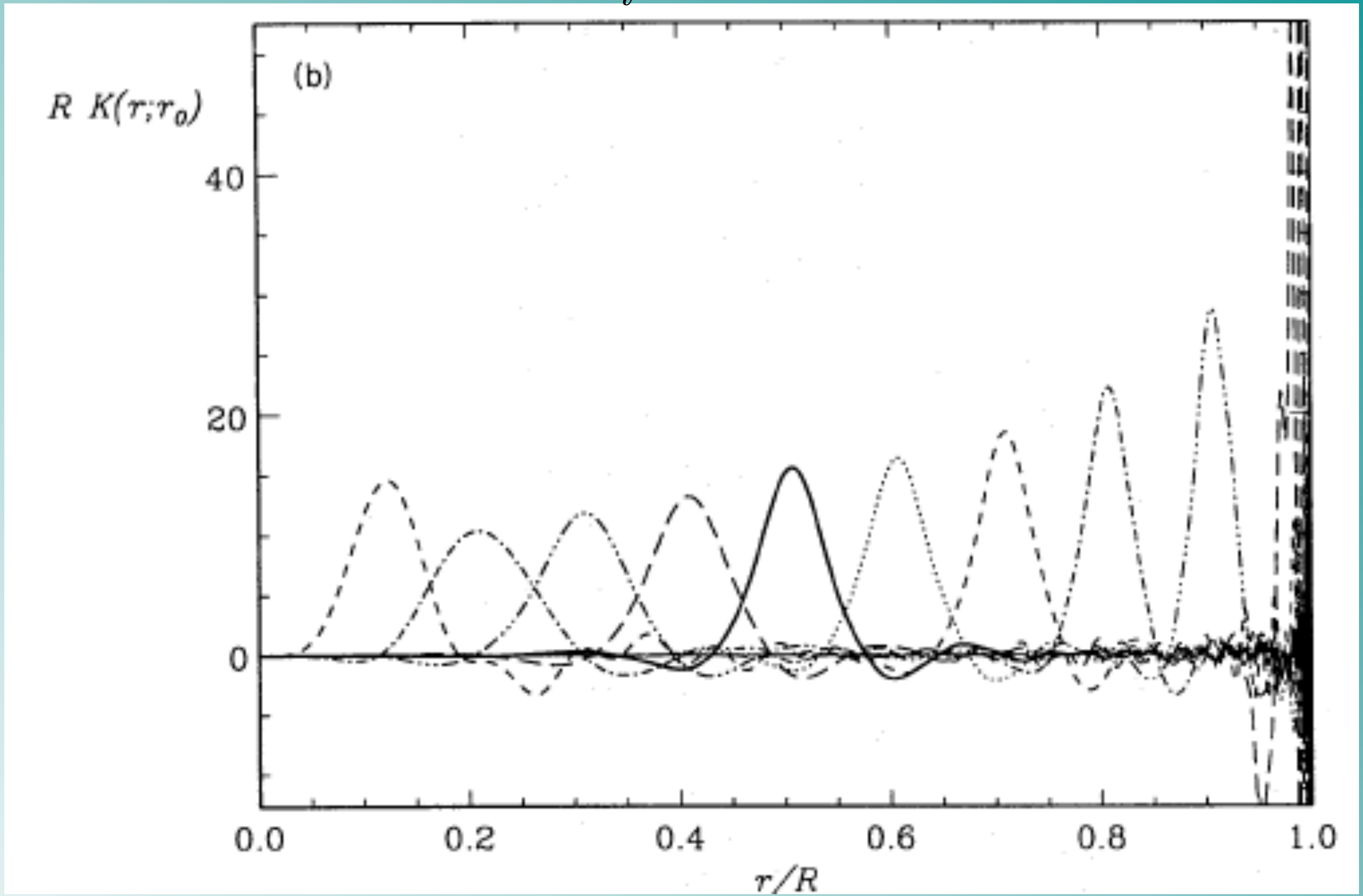
Averaging kernels for the Sun ~843 modes

$$\mathbf{K}(r_0; r) = \sum_i c_i(r_0) K_i(R)$$



Christensen-Dalsgaard et al 1990

Each model was fit to both seismic and non-seismic constraints, by optimizing the total χ^2 with respect to the stellar model's age t, mass M, initial metallicity [Fe/H], initial helium abundance Y_i, and mixing-length parameter α. That is, we minimize

$$\chi^2_{\rm mod} = \sum_{i=1}^{N} \left(\frac{x_{i,\rm mod} - x_{i,\rm obs}}{\sigma_i} \right)^2 \qquad (1)$$

where $x_{i,\rm obs}$ represent all of the N seismic and non-seismic and σ_i their corresponding uncertainties. The non-seismic constraints were the effective temperature $T_{\rm eff}$, surface gravity $\log g$, metallicity $[Fe/H]$, and luminosity $\log L_*$, with the former determined from high-resolution spectroscopy and the latter from Hipparcos observations. The seismic constraints were oscillation frequencies for 10 radial, 10 dipole, and 8 quadratic modes as reported by Ballot et al. (2011). We omitted the lowest order modes of each degree, which were noted as less reliable. The surface effects – known systematic discrepancies between observed and modelled frequencies of high-order modes – were fit using a function of the form $\nu^3/\mathcal{I}$, where ν and $\mathcal{I}$ are a mode's frequency and inertia, normalized at the photosphere (Ball & Gizon 2014). Mode frequencies and inertiae were computed with the Aarhus