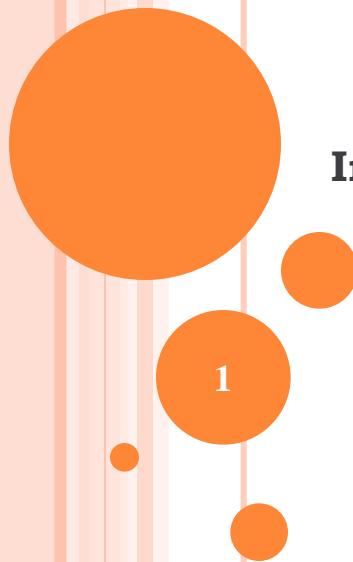
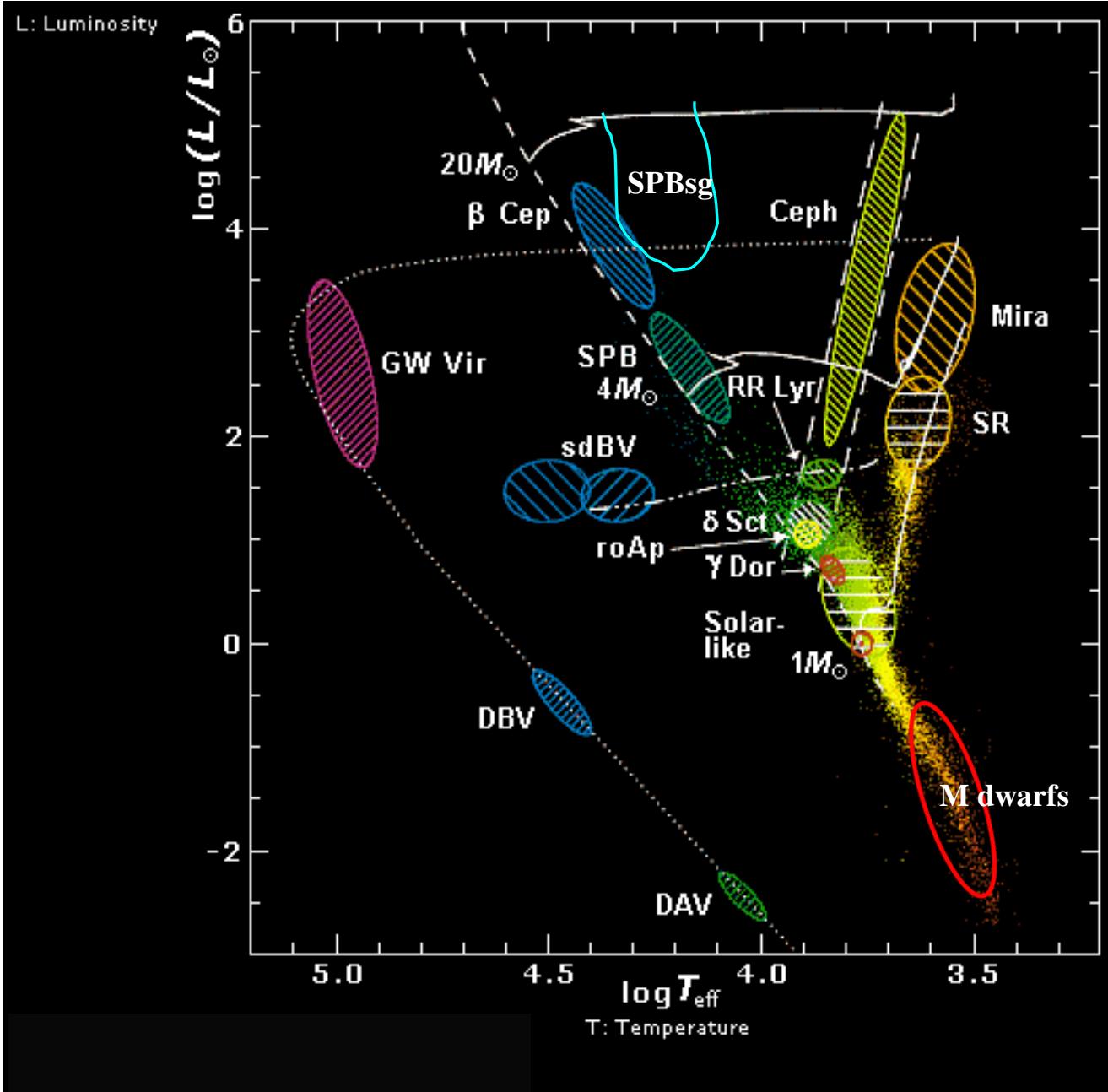


# Energetic properties of stellar pulsations across the HR diagram

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Instytut Astronomiczny, Uniwersytet Wrocławski, Poland





# **Energetic properties:**

**EXCITATION OF PULSATATIONAL MODES**

**KINETIC ENERGY OF A MODE**

**MODE AMPLITUDE, HEIGHT, LINETHICKNESS**

# EXCITATION OF PULSATATIONAL MODES

**Self-excitation (heat engine)  
(+convection)**

thermal  
processes

**Convective blocking**

**Turbulent convection**

dynamical  
processes

**Tidal effect**

## SELF-EXCITATION

**Work integral – the net energy gained by a mode during one cycle**

$$W = \oint \frac{dE}{dt} dt = \frac{\pi}{\omega} \int_0^M \left[ \frac{\delta T}{T} \delta \varepsilon_N - \frac{\delta T}{T} \delta \left( \frac{1}{\rho} \text{div} \mathbf{F} \right) \right] dM_r$$

$$\mathbf{F} = \mathbf{F}_R + \mathbf{F}_C$$

## $\delta\varepsilon$ - $\varepsilon$ -mechanism

$$\int_0^M \frac{\delta T}{T} \delta\varepsilon dM_r = \int_0^M \varepsilon \left( \varepsilon_T + \frac{\varepsilon_\rho}{\Gamma_3 - 1} \right) \left( \frac{\delta T}{T} \right)^2 dM_r$$

$$\varepsilon_T = \left( \frac{\partial \ln \varepsilon}{\partial \ln T} \right)_\rho \approx 4 - 30$$

$$\varepsilon_\rho = \left( \frac{\partial \ln \varepsilon}{\partial \ln \rho} \right)_T \approx 1 - 2$$

$$\Gamma_3 - 1 \approx \frac{2}{3}$$

**$\varepsilon_T$  and  $\varepsilon_\rho$  are always positive  $\rightarrow$  a positive contribution to  $W$  but usually negligible**

**If  $\delta\varepsilon \approx 0$  and  $\delta F_C \approx 0$**

$$W = - \int d^3x \nabla_{\text{ad}} \oint dt \operatorname{Re} \left[ \left( \frac{\delta P}{P} \right)^* \delta \operatorname{div} \mathbf{F}_R \right]$$

**In the diffusion approximation**

$$\delta \operatorname{div} \mathbf{F}_R = \frac{1}{4\pi r^2} \frac{d \delta L_r}{dr}$$

$$\frac{\delta L_r}{L_r} = \frac{dr}{d \ln T} \frac{d}{dr} \left( \frac{\delta T}{T} \right) - \frac{\delta \kappa}{\kappa} + 4 \left( \frac{\delta T}{T} + \frac{\delta r}{r} \right)$$



**1**

**Radiative dissipation**



**2**

**$\kappa$ -effect**



**3**

**$\gamma$ -effect**

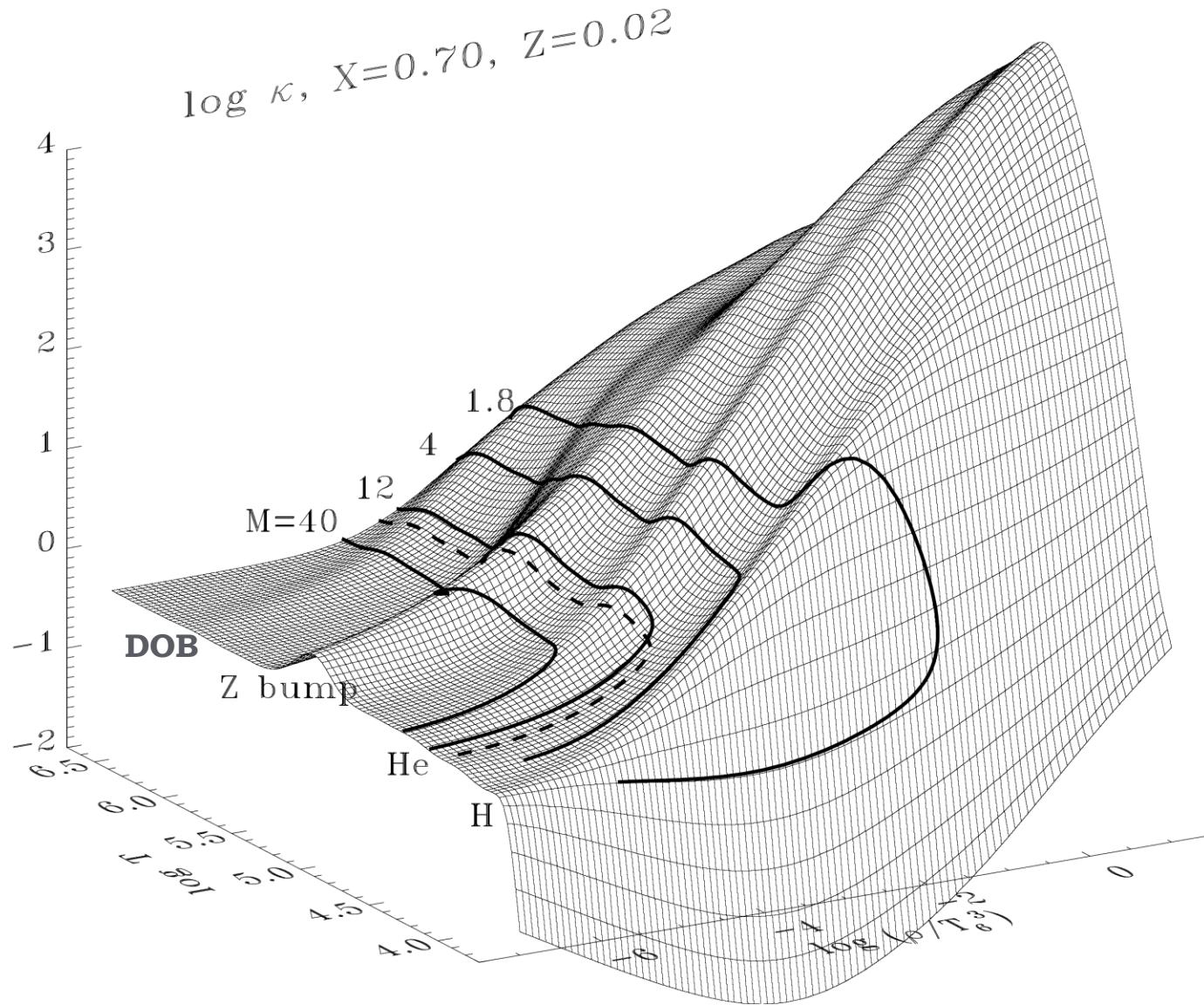


**4**

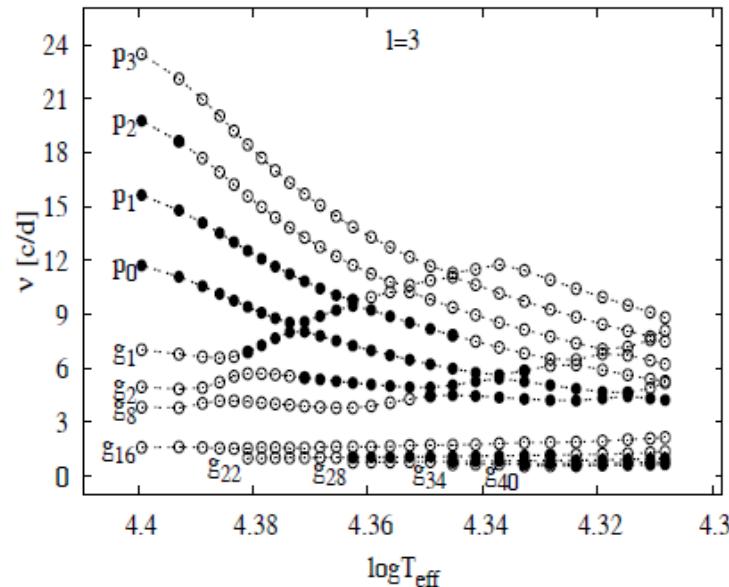
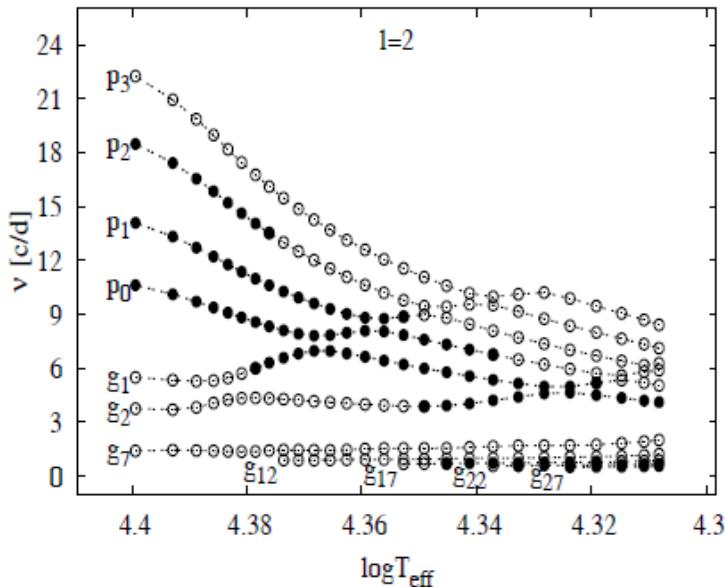
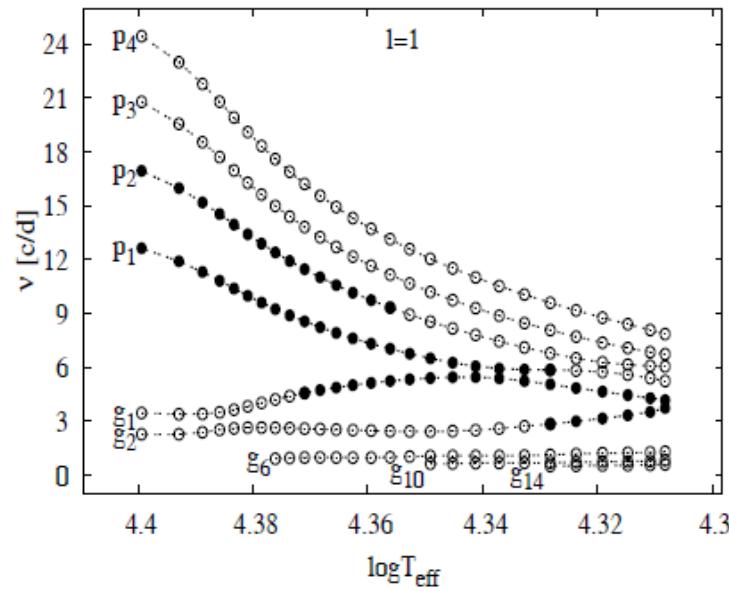
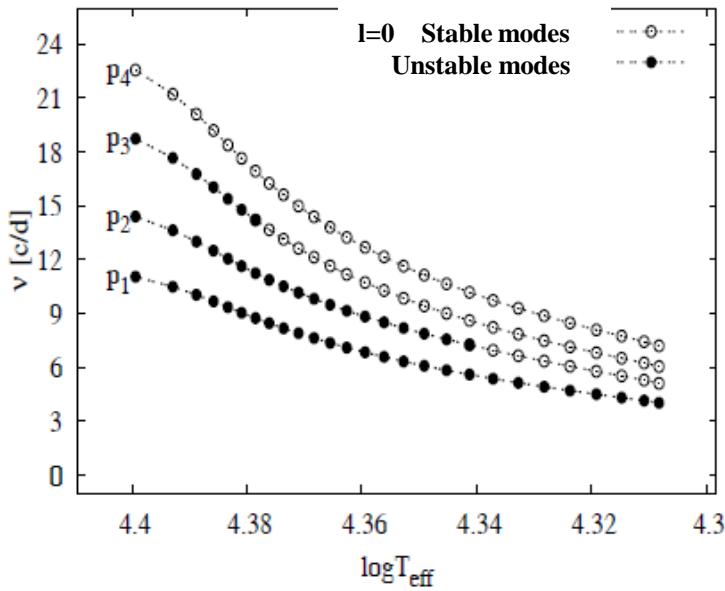
**$r$ -effect**

# $\kappa(+\gamma)$ -mechanism

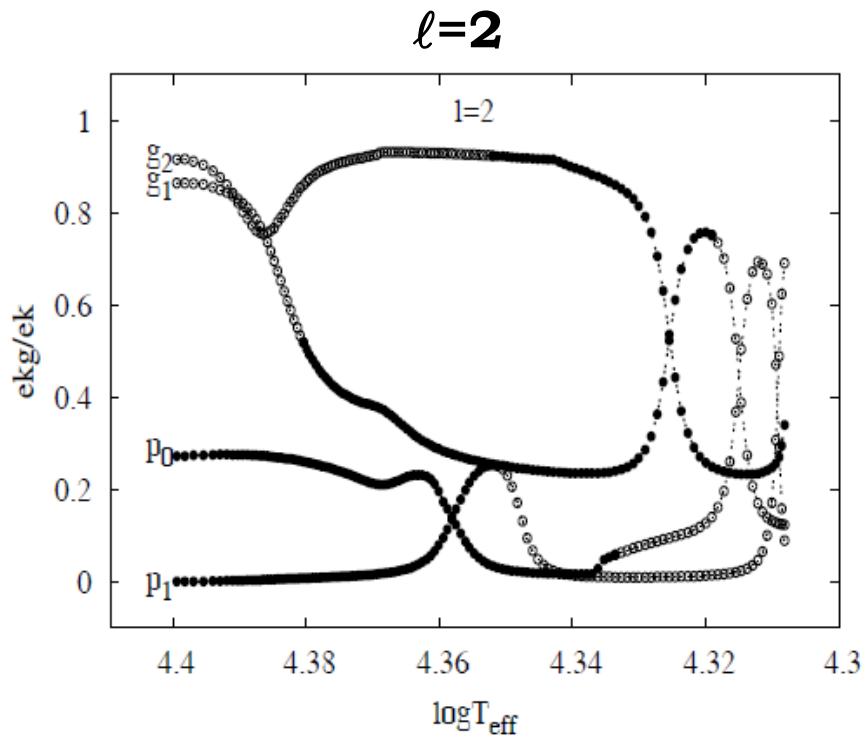
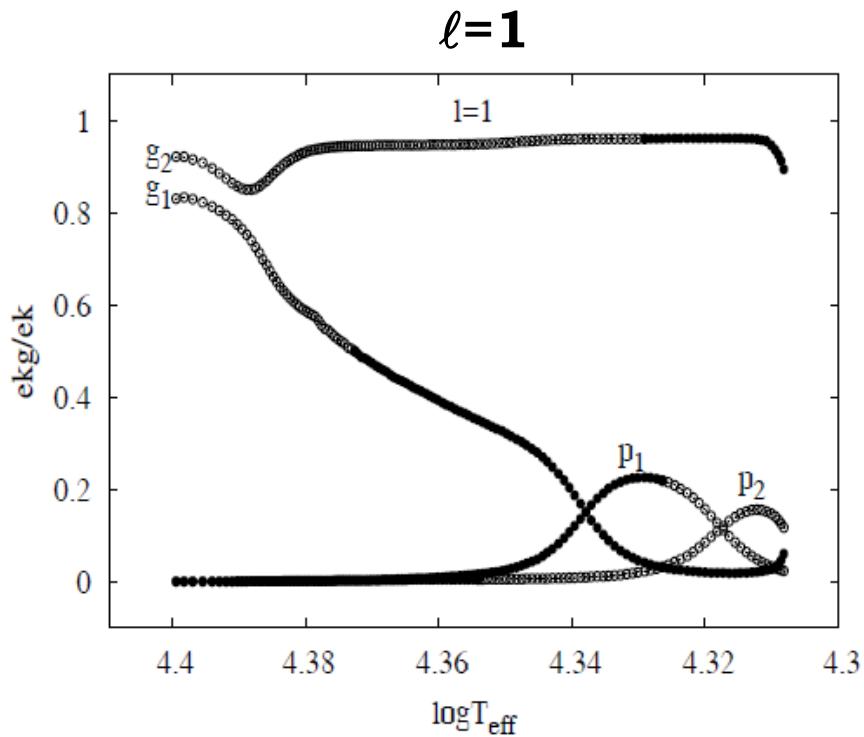
## $\kappa(\text{OPAL})$ as a function of $\log T$ and $\log \rho / T_6^3$ ( $T_6 = T / 10^6$ )



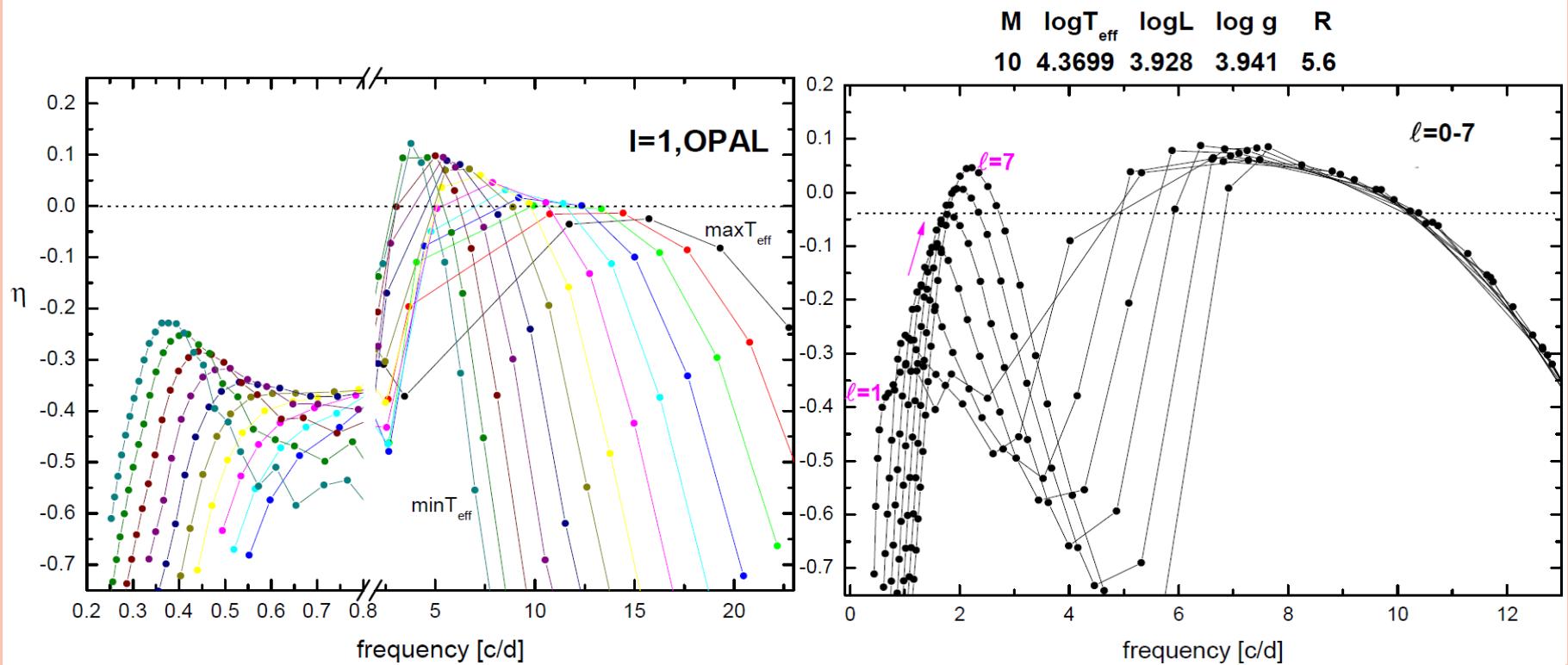
# 10 M<sub>⦿</sub> OP, Z=0.02



## A ratio of the gravity energy to the total one for selected modes in the $10M_{\odot}$ stellar model as a function of $T_{\text{eff}}$

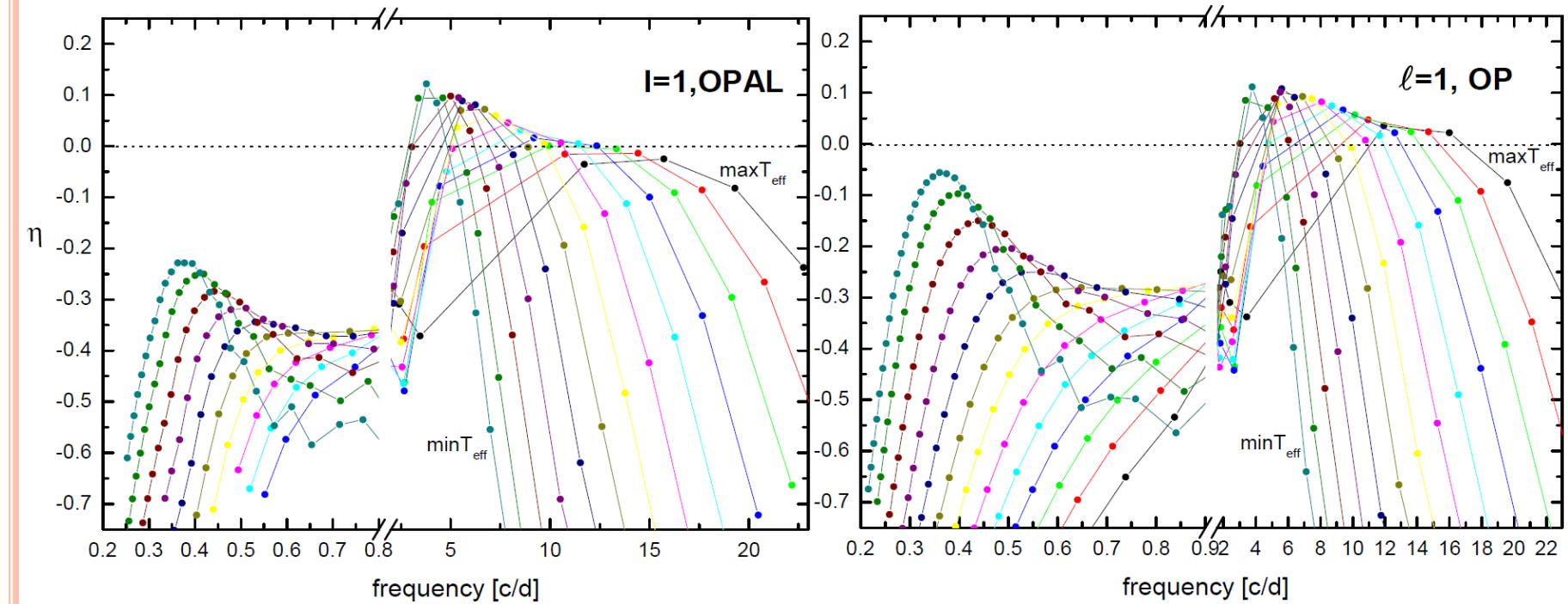


# 10 M<sub>⊙</sub> OPAL η vs. frequency



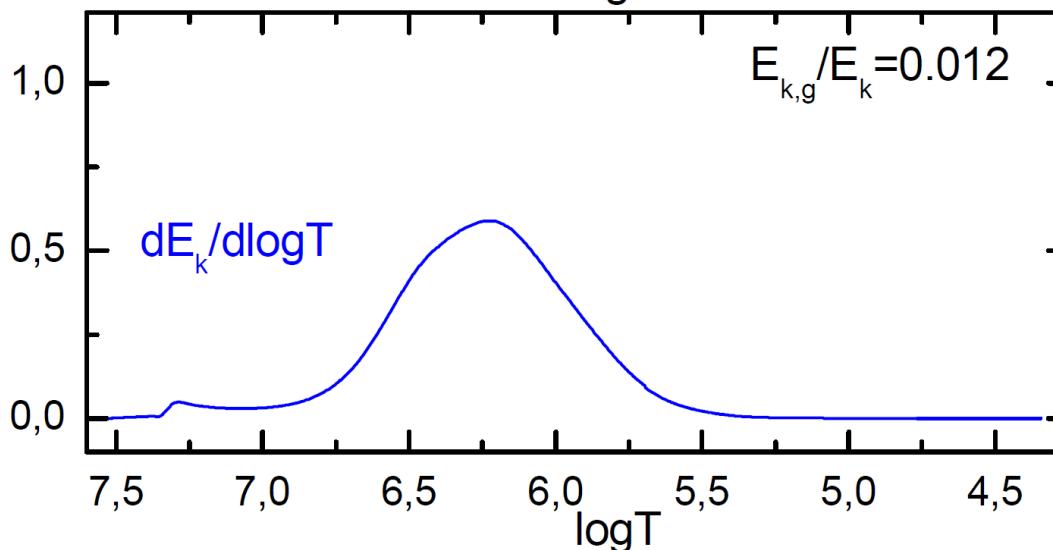
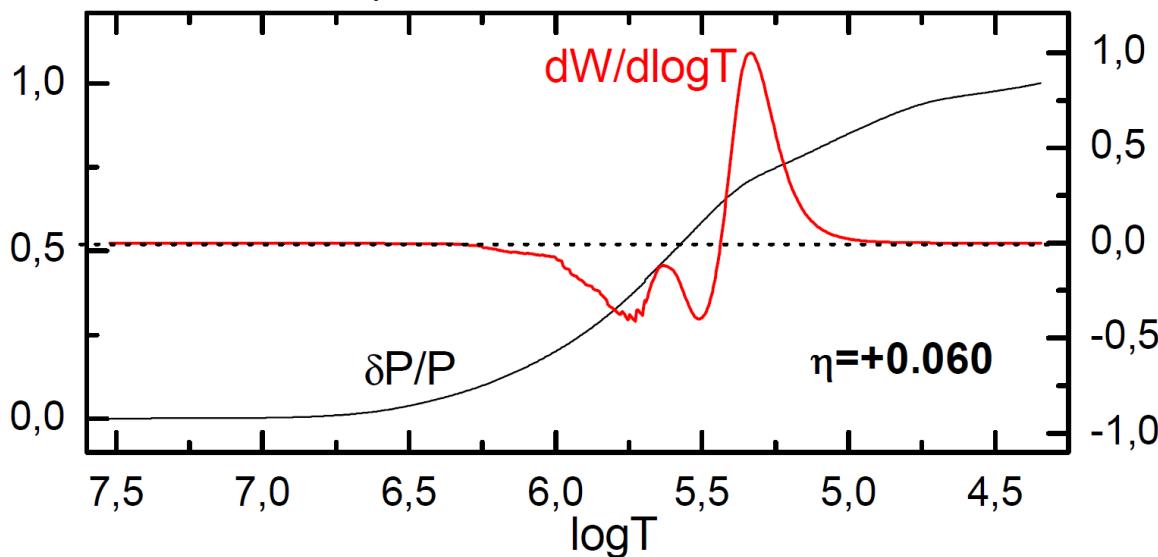
Instability up to  $\ell \approx 30$

# 10 M<sub>⊙</sub> OPAL vs. OP η vs. frequency



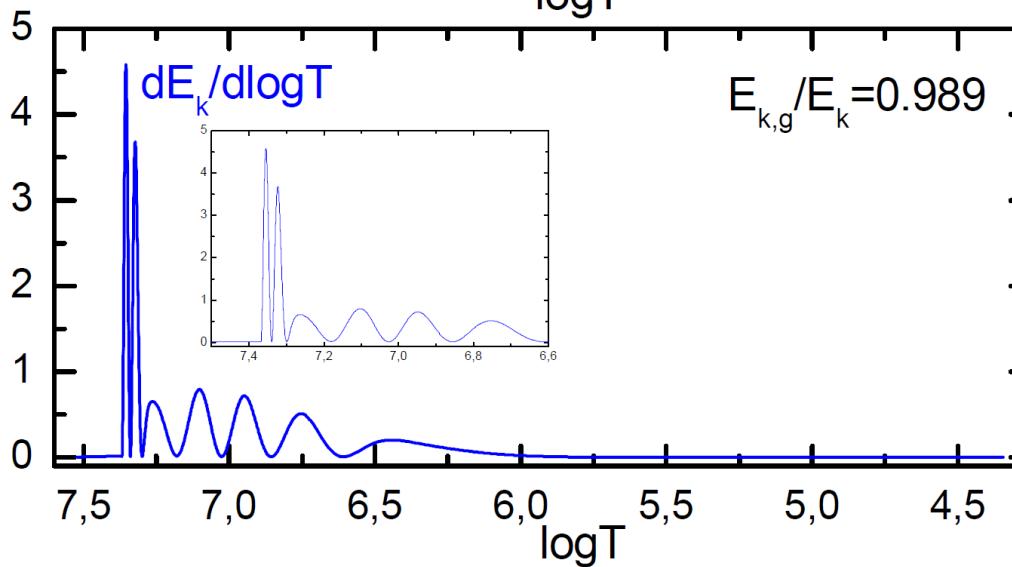
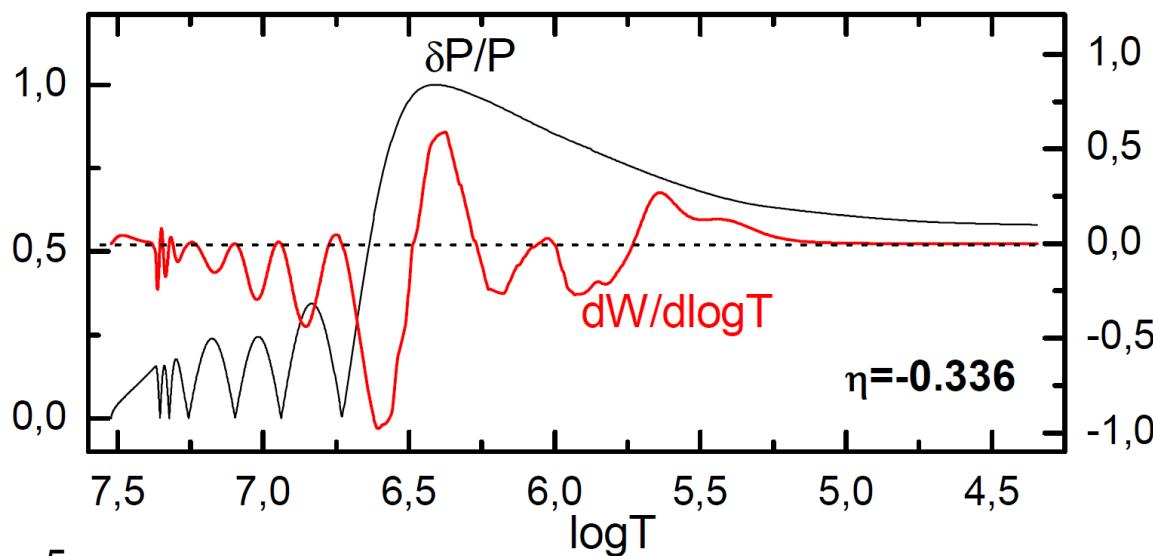
M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
10	4.3699	3.928	3.94	5.6

$I=1, p_1, P=0.137 \text{ [d]}, \nu=7.276 \text{ [c/d]}$



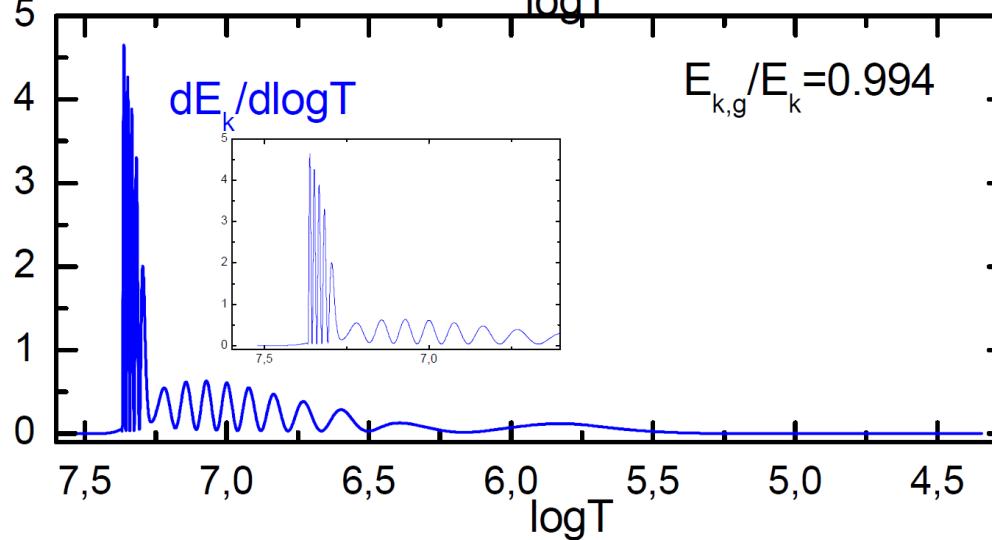
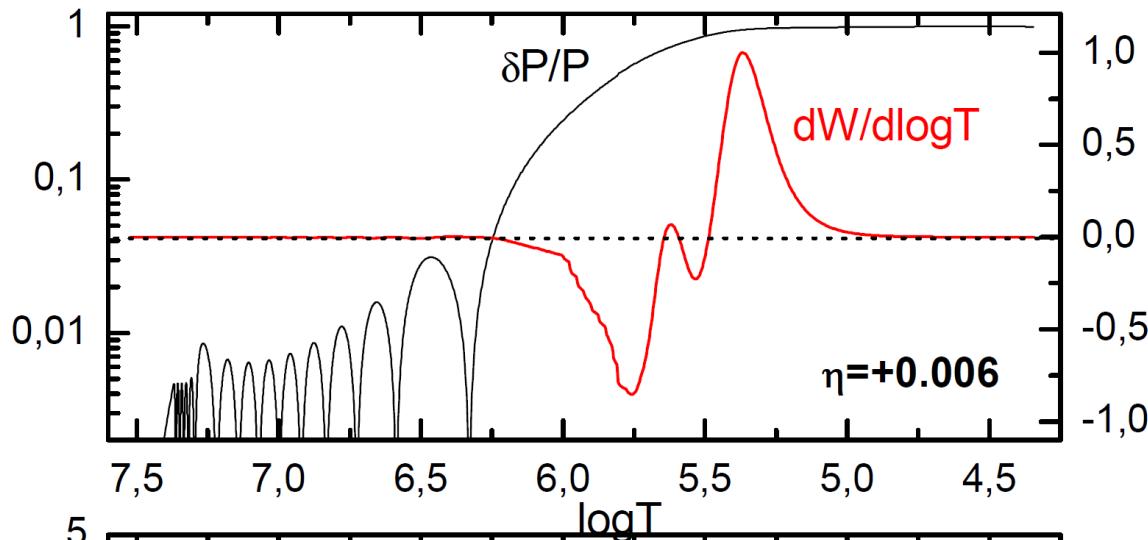
M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
10	4.3699	3.928	3.94	5.6

$I=1, g_7 \quad P=1.090 \text{ [d]} \quad \nu=0.917 \text{ [c/d]}$

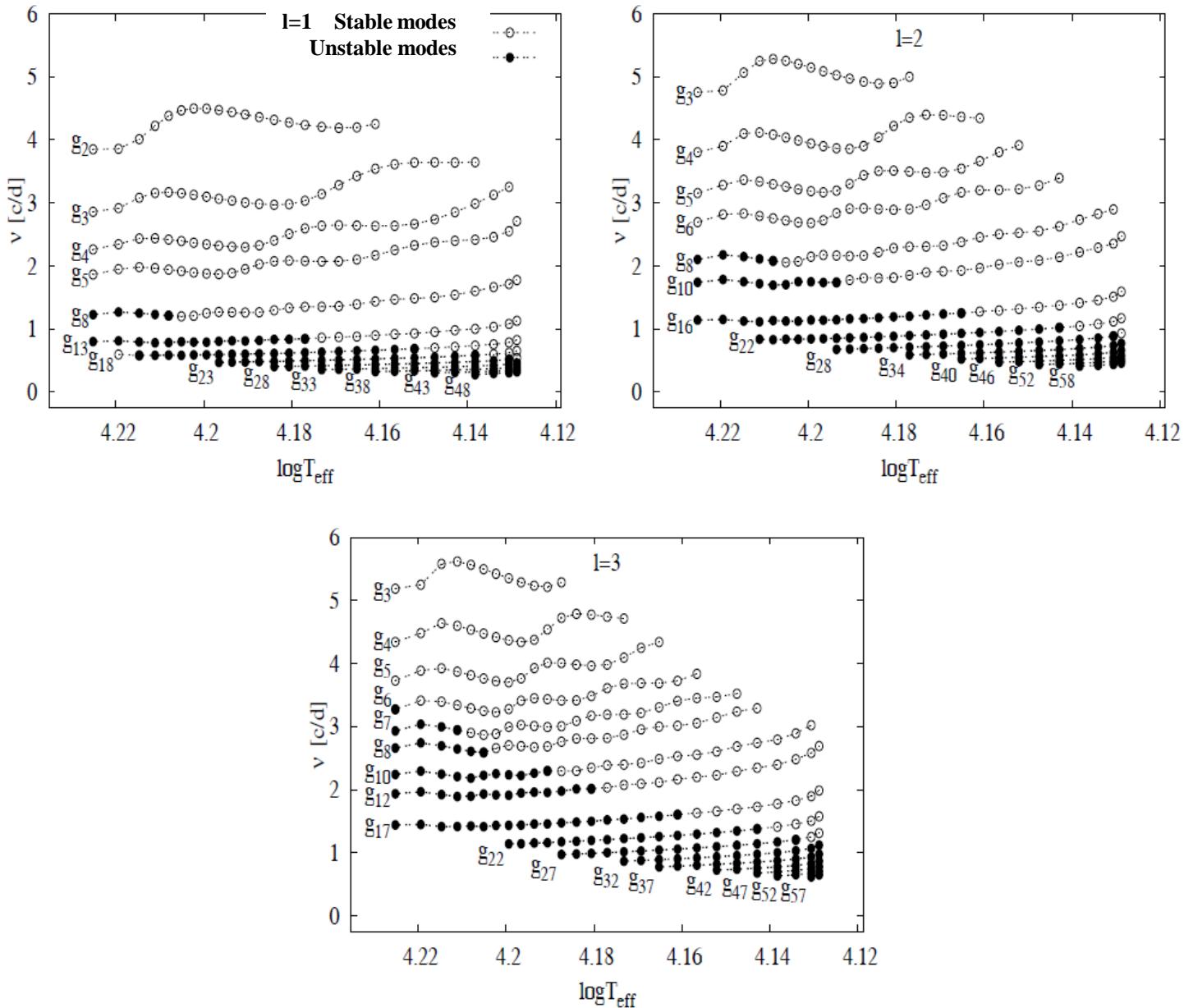


M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
10	4.3699	3.928	3.94	5.6

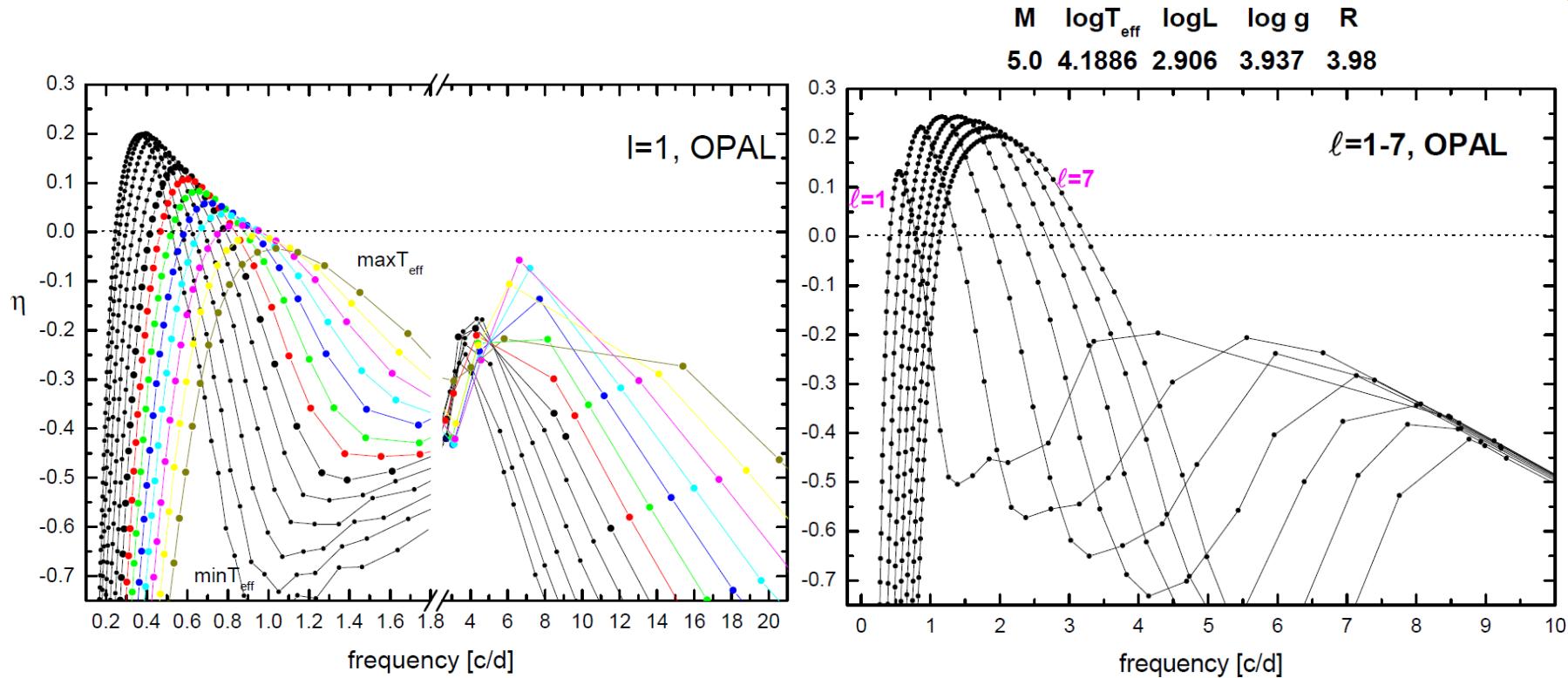
$I=6, g_{15}$   $P=0.513 \text{ [d]}$   $v=1.948 \text{ [c/d]}$



# $5 M_{\odot}$ OP, Z=0.02

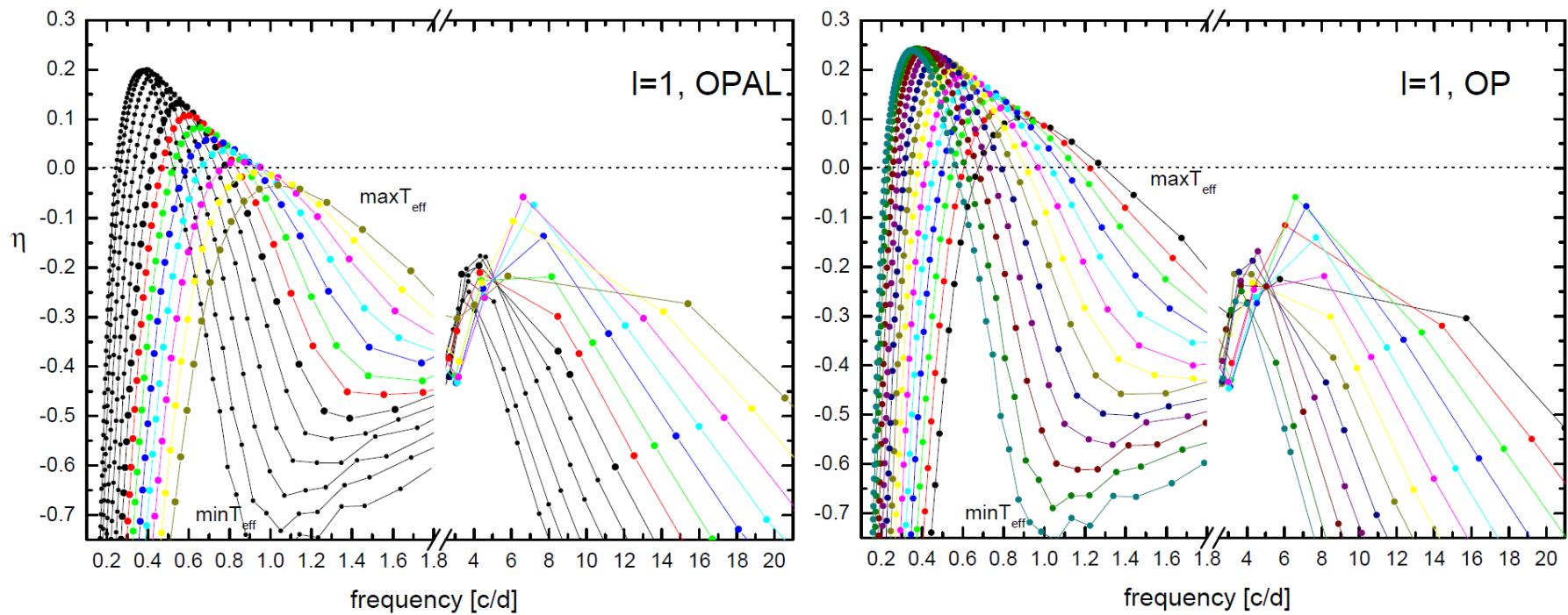


# 5 M<sub>☉</sub> OPAL η vs. frequency



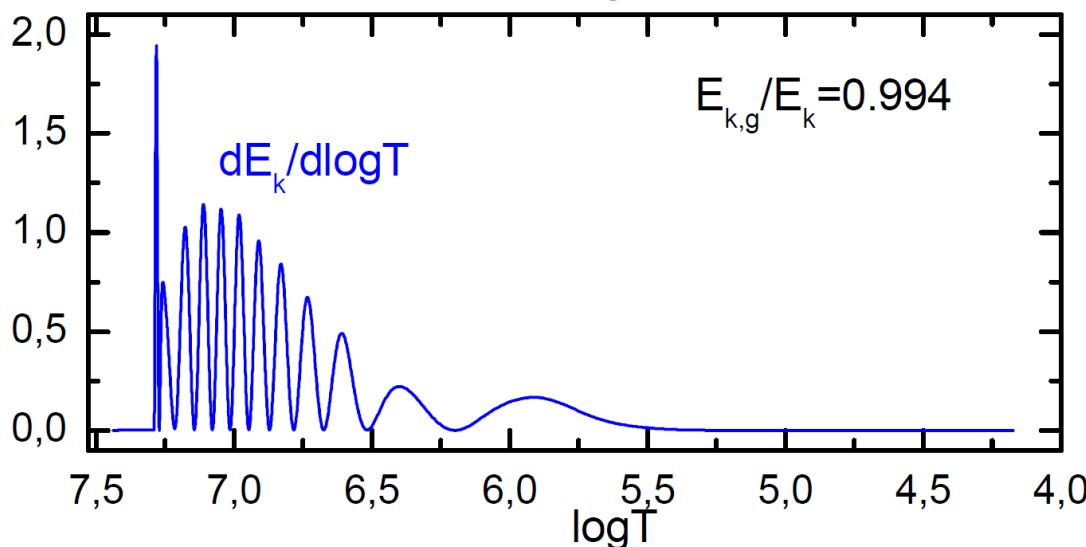
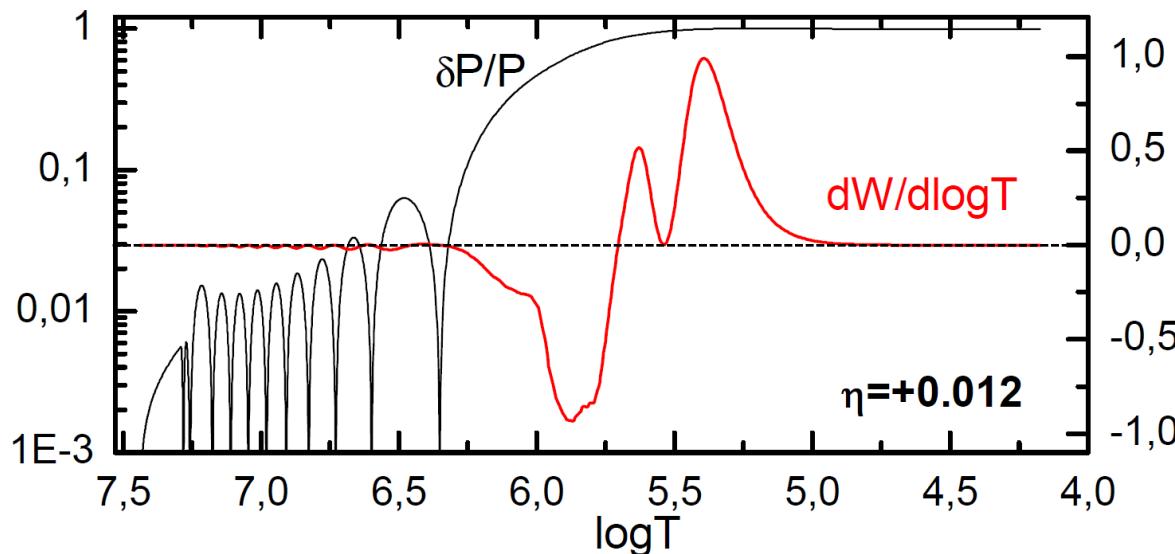
Instability up to  $\ell \approx 17$

# 5 M<sub>⦿</sub> OPAL vs. OP η vs. frequency



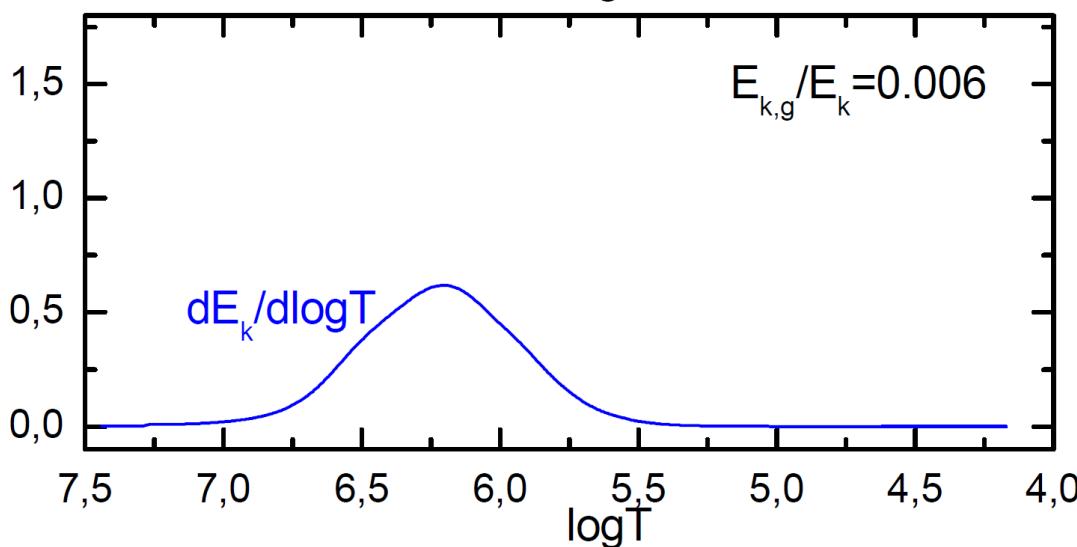
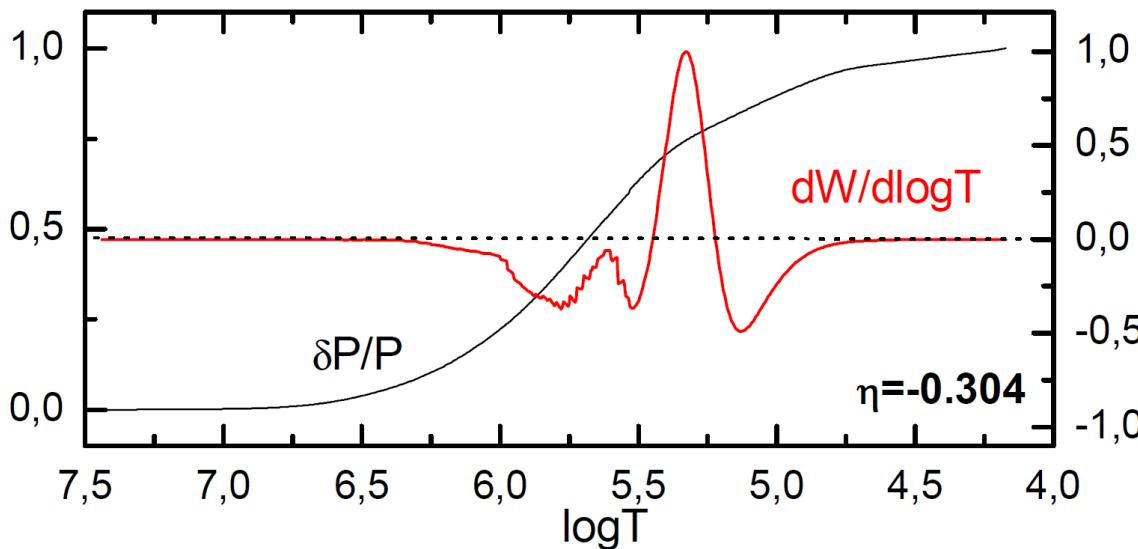
M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
5.0	4.2214	2.806	4.168	3.05

$I=1, g_{12}$   $P=1.145 \text{ [d]}$   $v=0.873 \text{ [c/d]}$



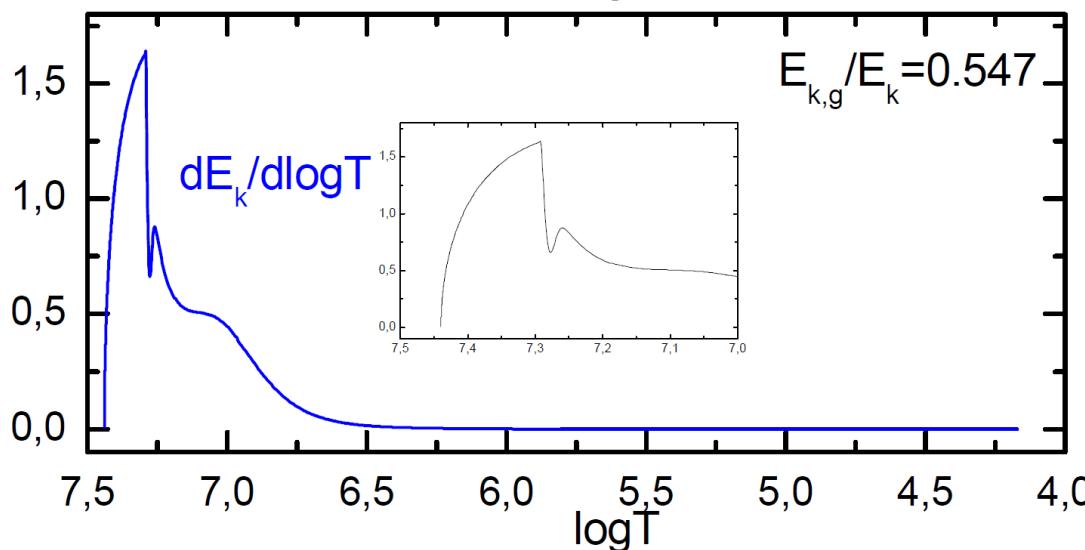
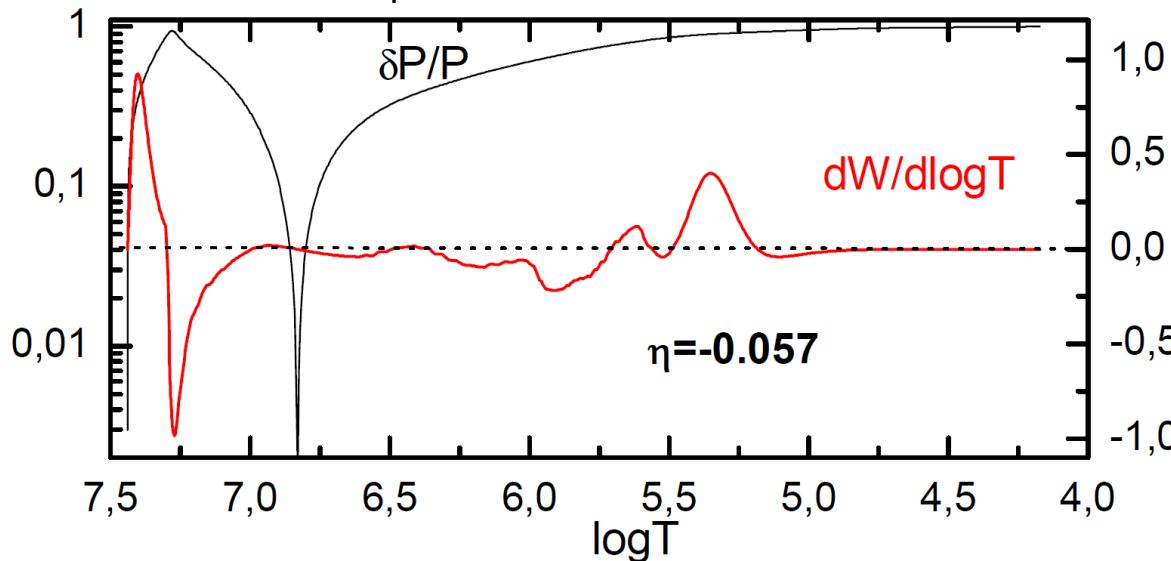
M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
5.0	4.2214	2.806	4.17	3.05

$I=1, p_1 \quad P=0.077 \text{ [d]} \quad \nu=13.030 \text{ [c/d]}$

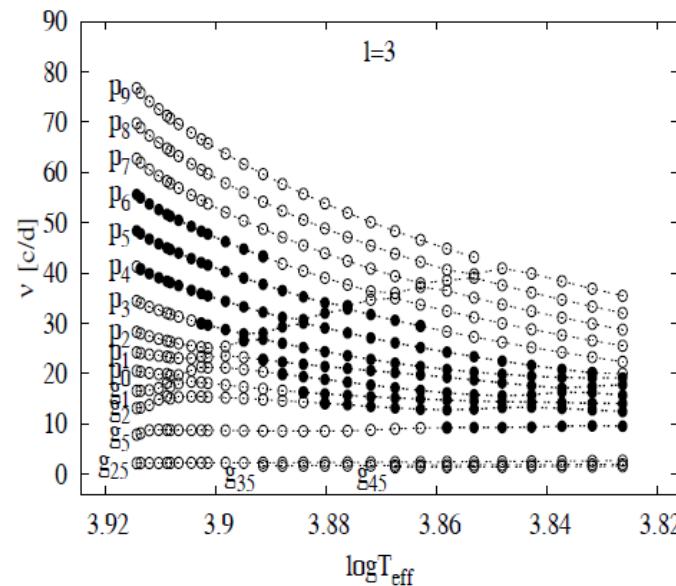
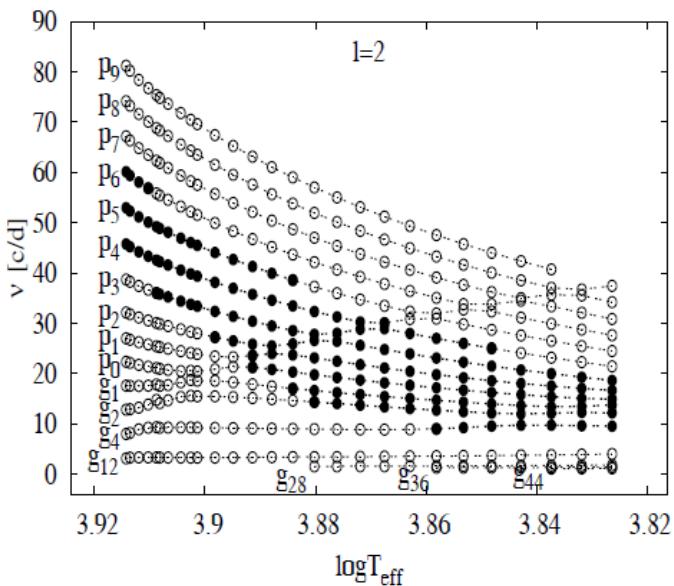
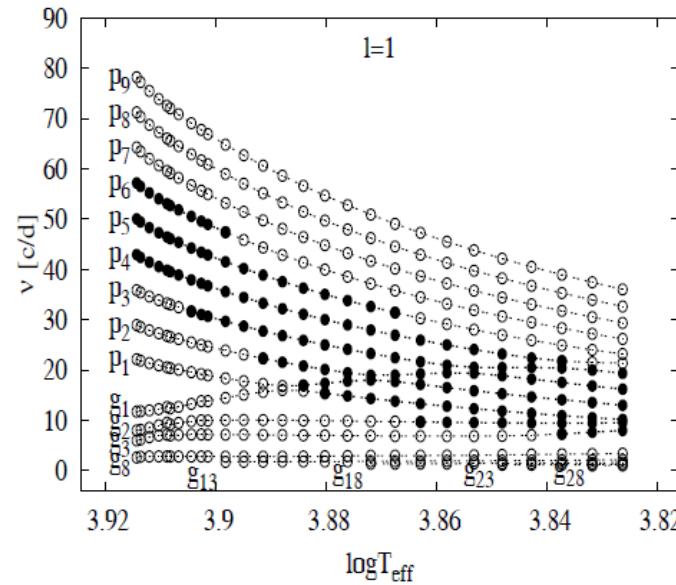
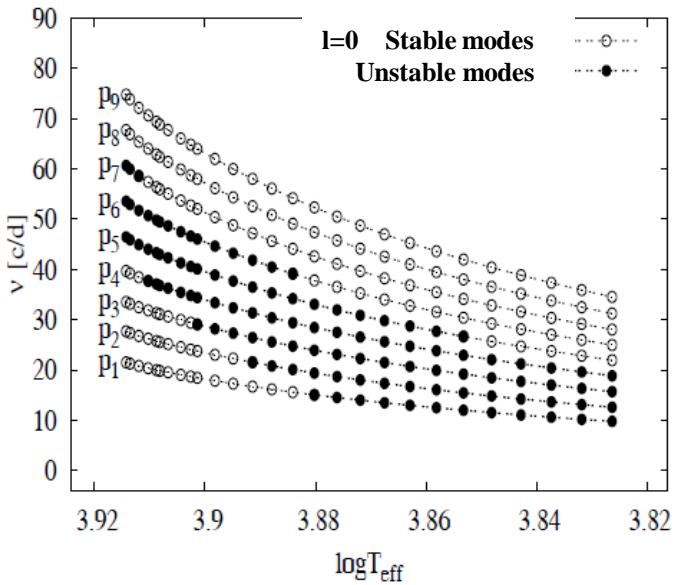


M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
5.0	4.2214	2.806	4.17	3.05

$I=1, g_1$ ,  $P=0.151$  [d],  $v=6.625$  [c/d]

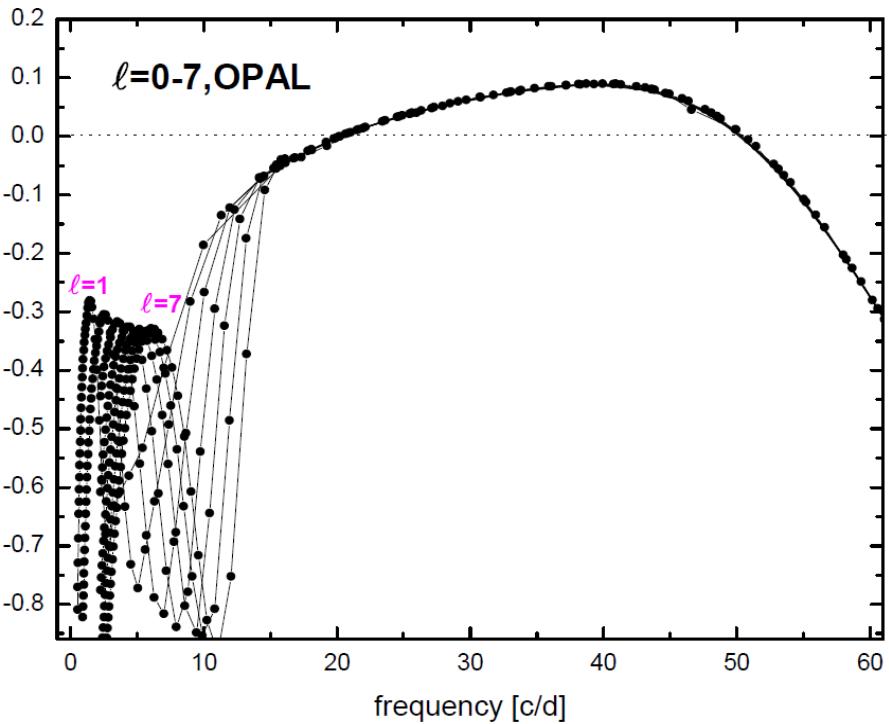
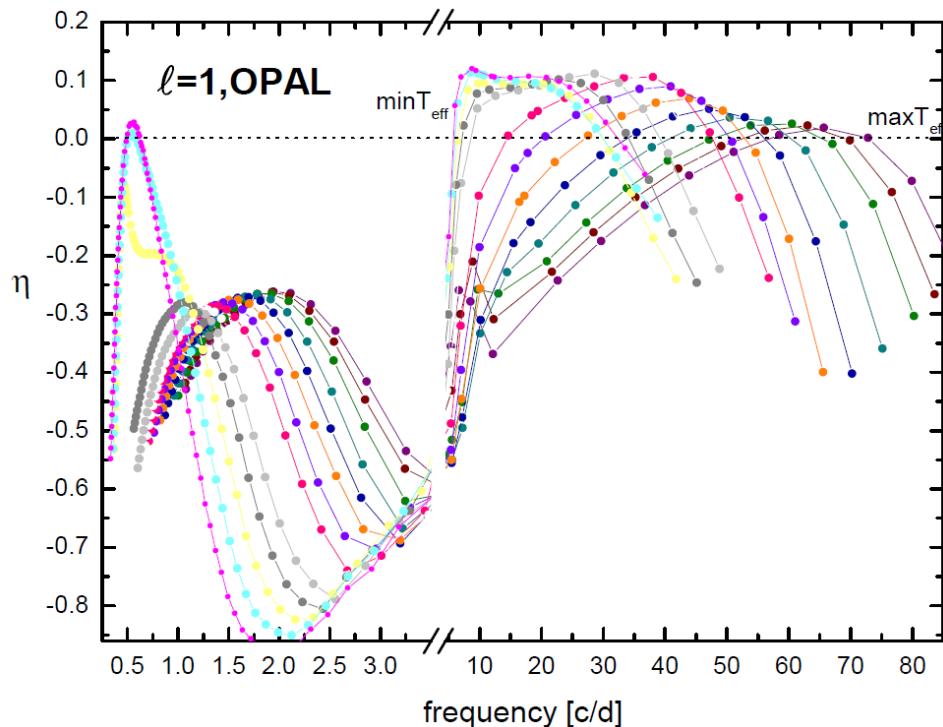


# 1.8 $M_{\odot}$ OP, Z=0.02



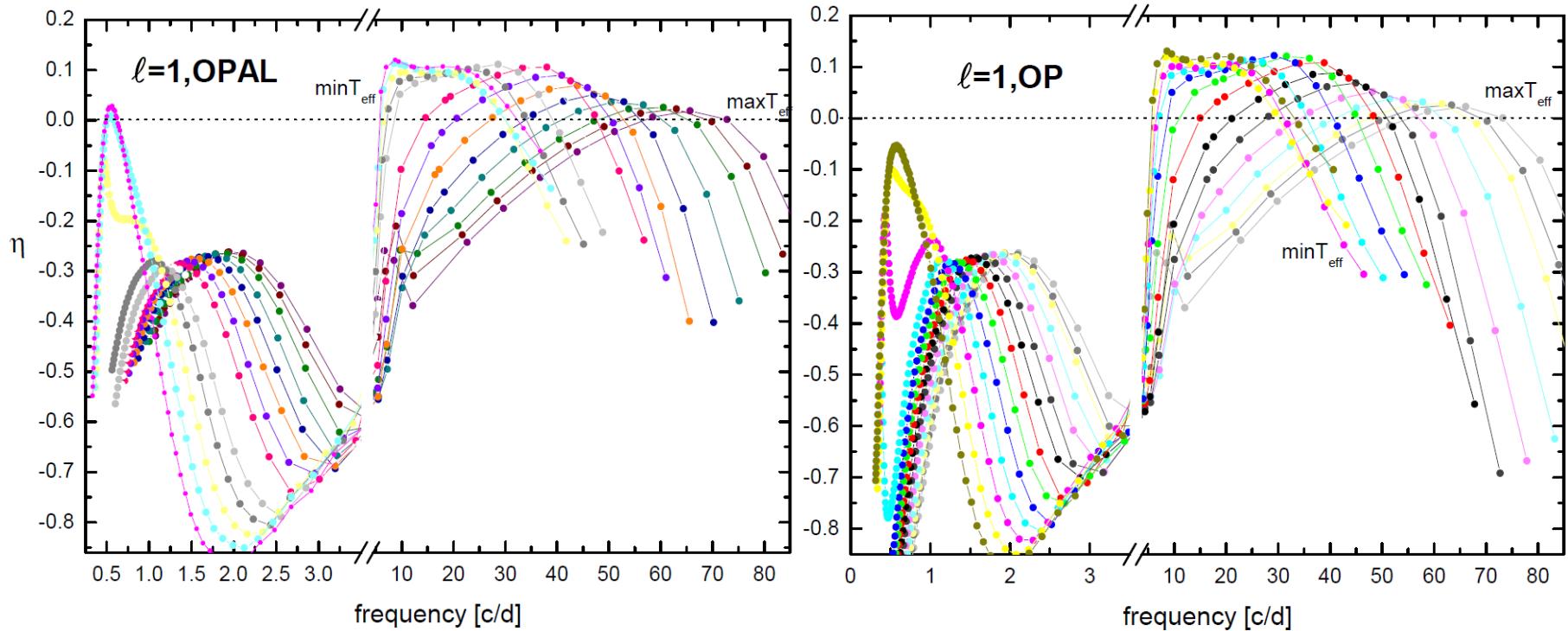
# 1.8 M<sub>⊙</sub> OPAL η vs. frequency

M logT<sub>eff</sub> logL log g R  
1.8 3.9036 1.137 4.12 1.93



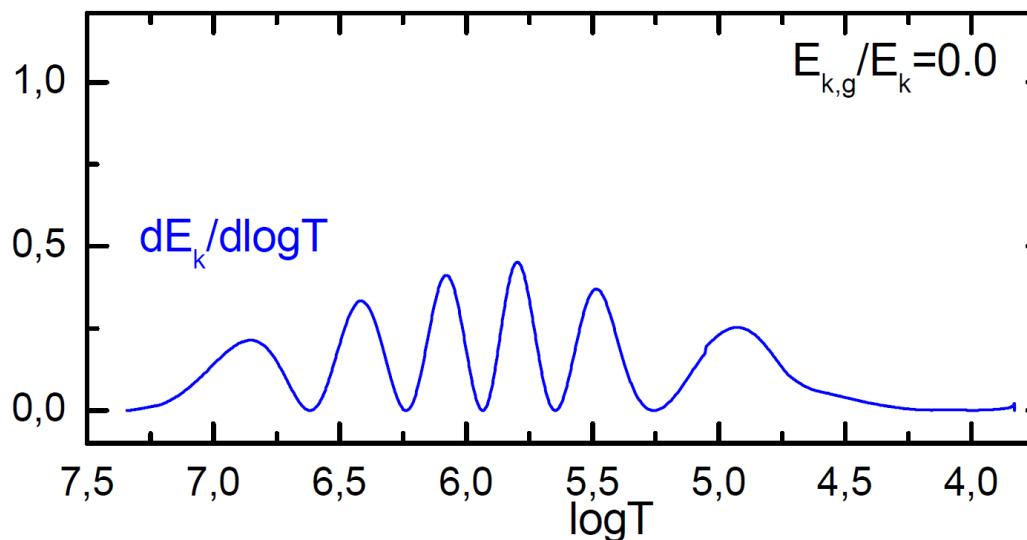
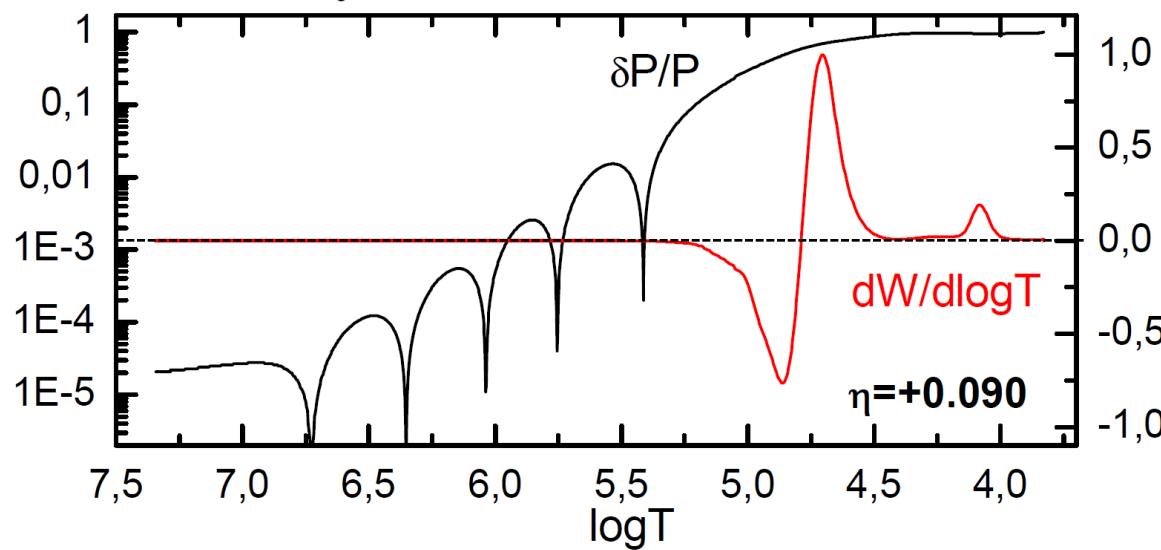
Instability up to  $\ell \approx 60$

# 1.8 M<sub>⊙</sub> OPAL vs. OP η vs. frequency

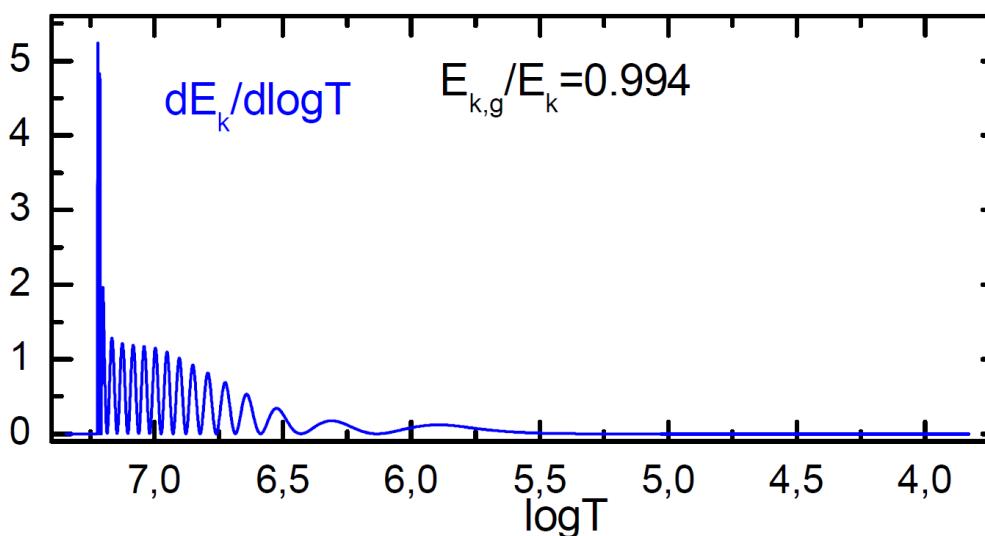
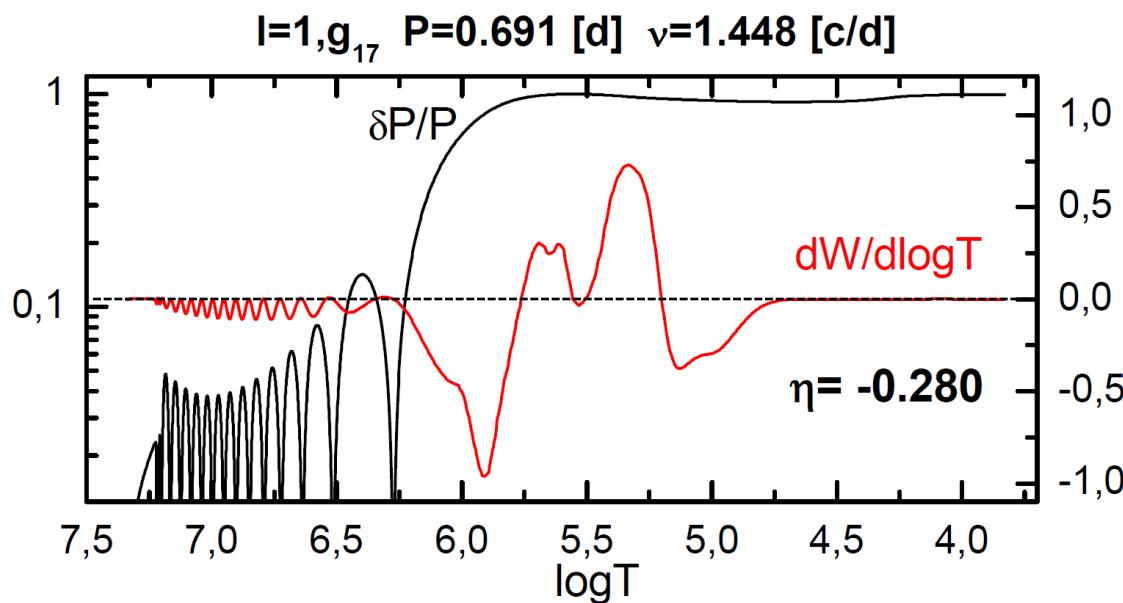


M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
1.8	3.9036	1.137	4.12	1.9

$I=0, p_6 \quad P=0.026 \text{ [d]} \quad \nu=38.716 \text{ [c/d]}$

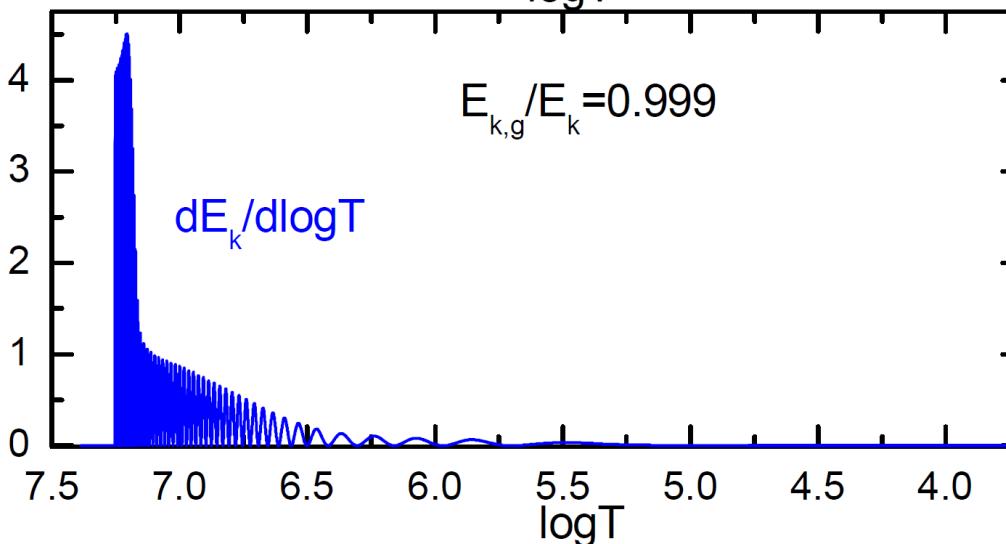
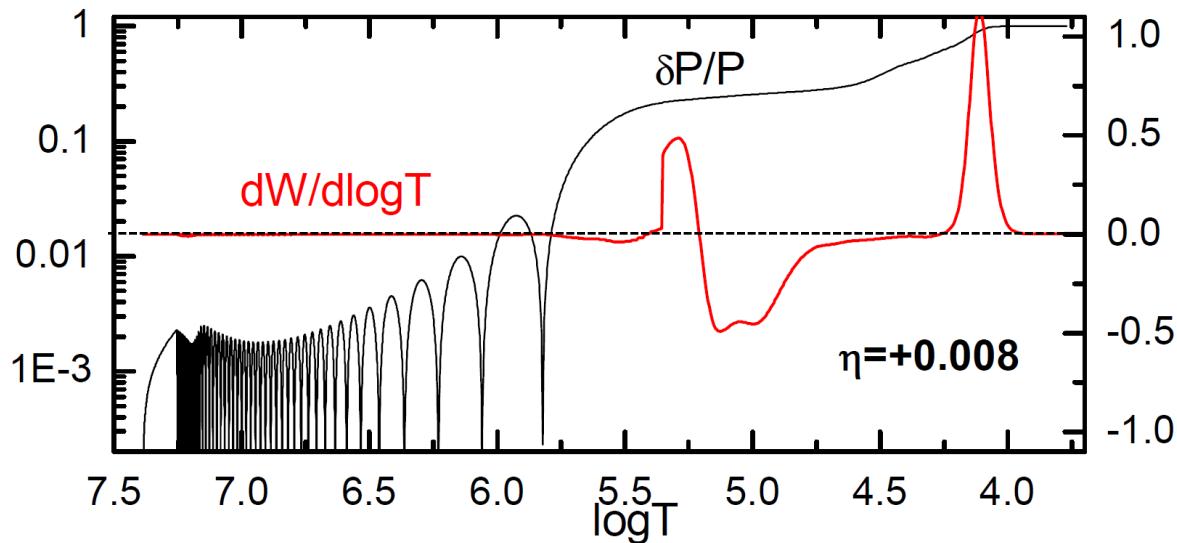


M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
1.8	3.9036	1.137	4.12	1.9



M	$\log T_{\text{eff}}$	$\log L$	$\log g$	R
1.8	3.8459	1.176	3.85	2.63

$I=1, g_{57}$   $P=1.840 \text{ [d]}$   $\nu=0.543 \text{ [c/d]}$



## **Low frequencies in *Kepler* δ Scuti stars**

**L. Balona 2014**

**convective flux blocking**

**(Pesnell 1987, Guzik et al. 2000)**

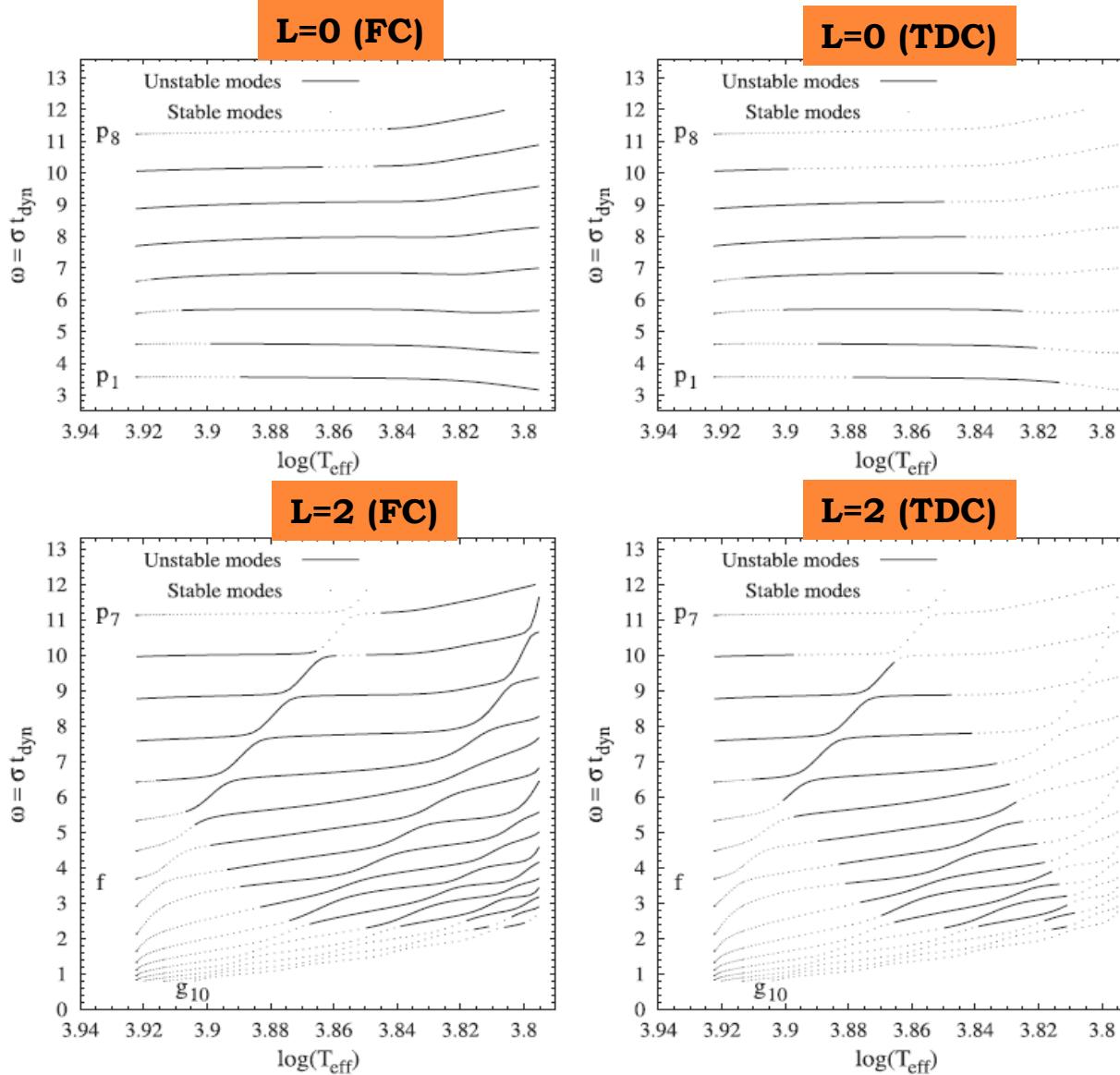
**or**

**Opacity problem**

**or/and**

**stars near TAMS**

# Convection-pulsation coupling



# **OPACITY ...**

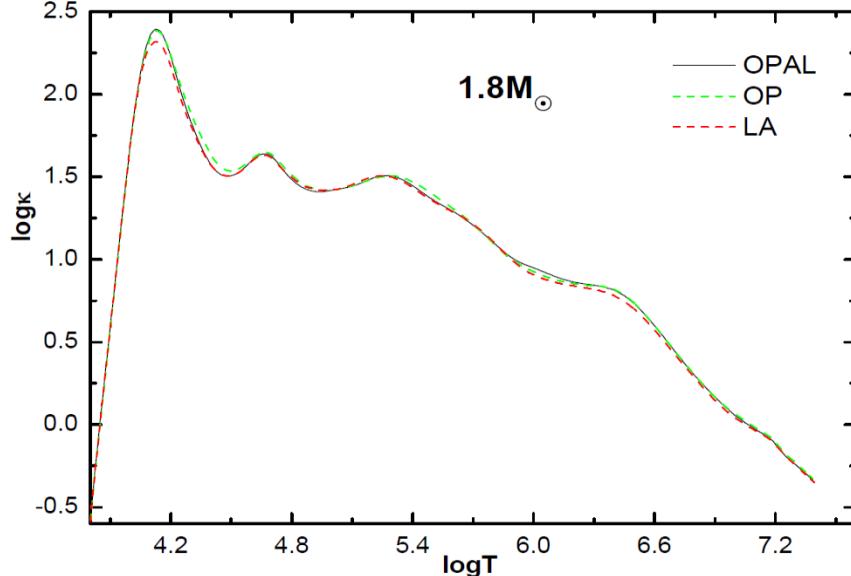
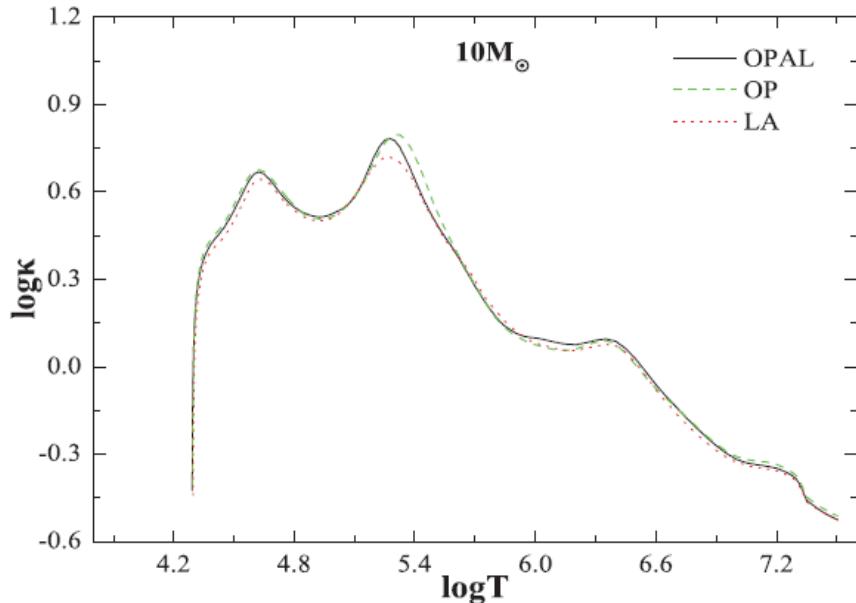
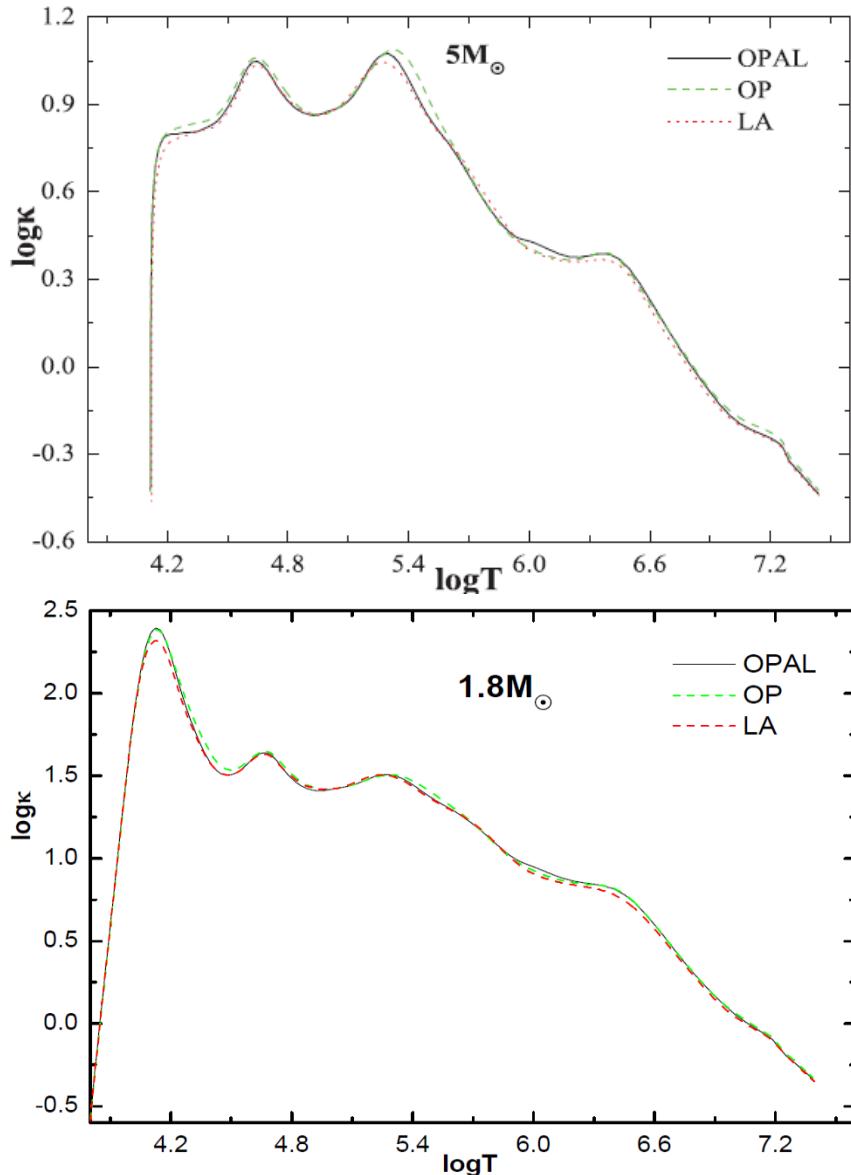
## **the neverending story**

**OPAL  
OP**

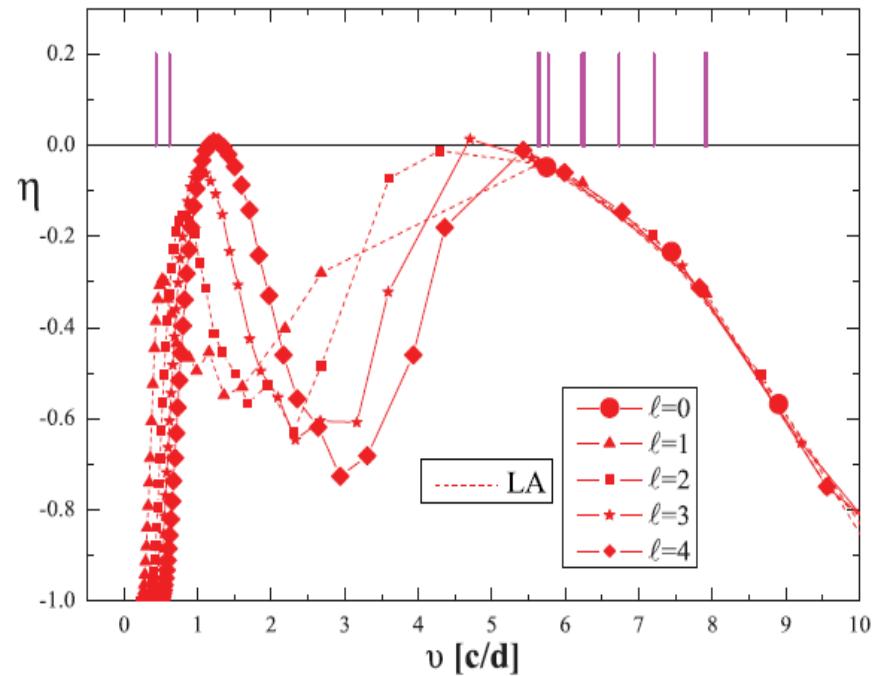
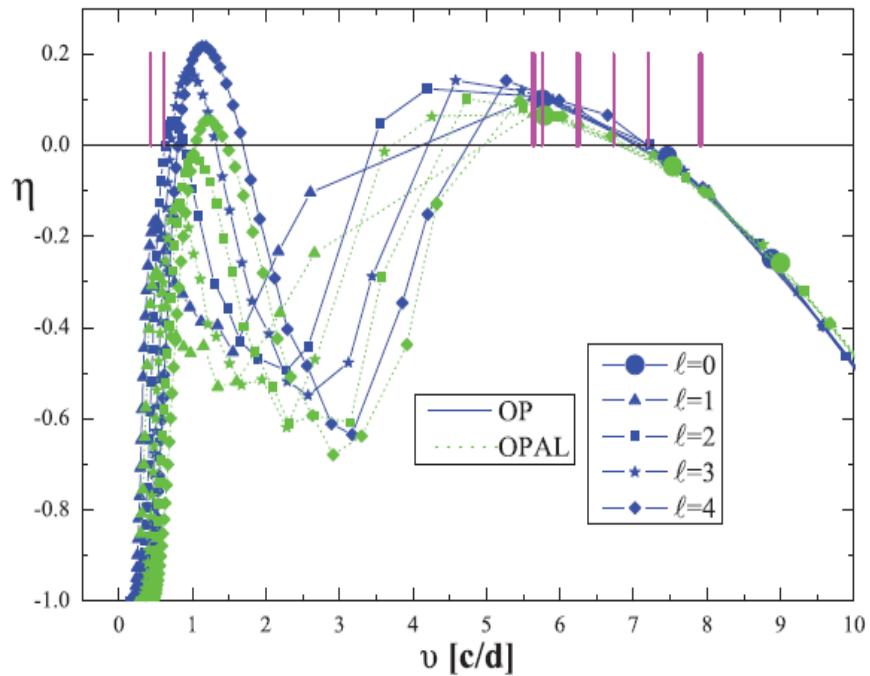
**LA - reactivated**

**Cugier (2012,2014) – OP(OPAL)+Kurucz**

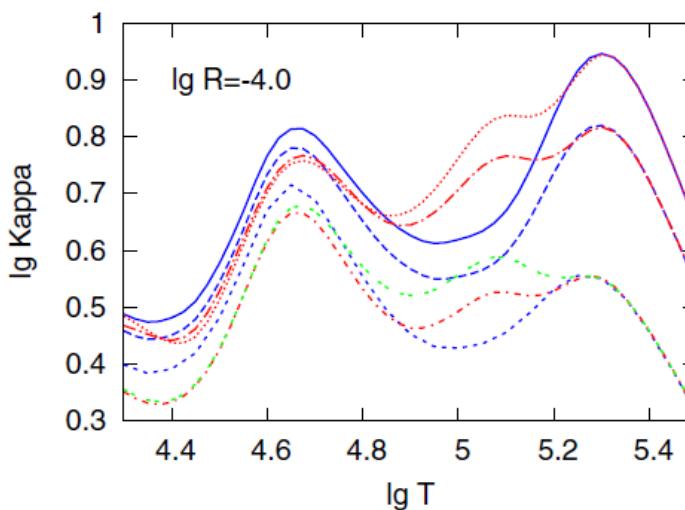
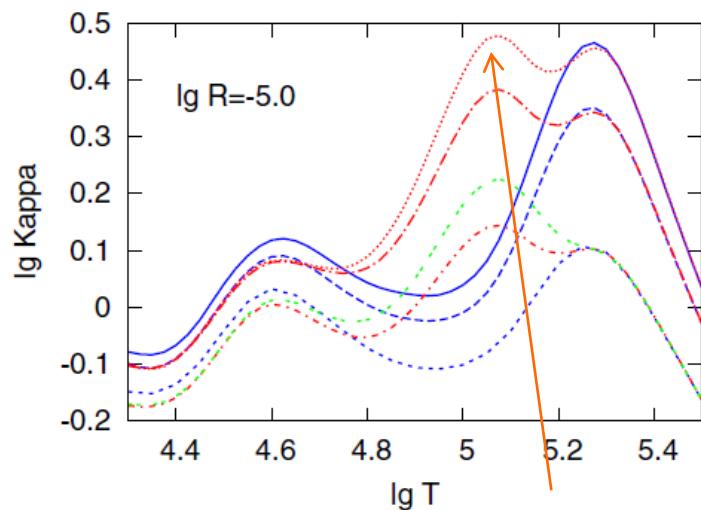
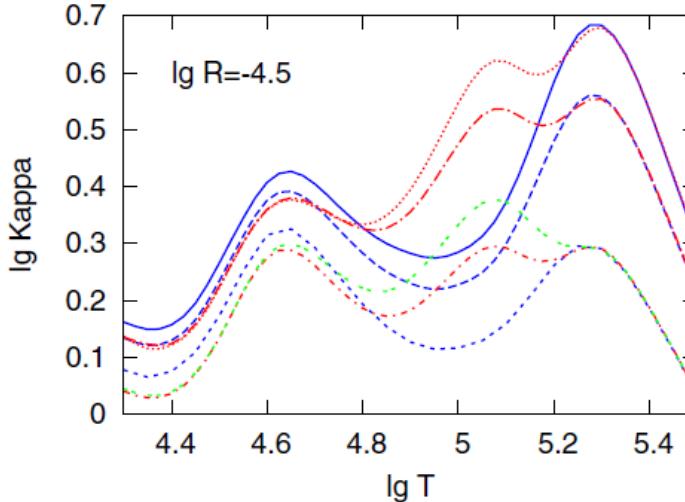
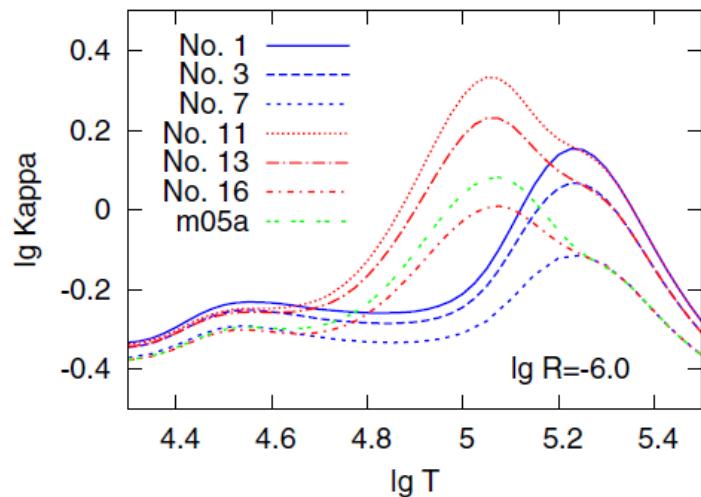
**The Rosseland mean opacity,  $\kappa$ , as a function of  $\log T$  inside the stellar models with masses  $M=1.8, 5, 10 M_{\odot}$  and  $\log \text{Teff} \cong 4.196, 4.373, 3.850$ , respectively.**



**Instability parameter,  $\eta$ , as a function of the frequency for the three seismic models of  $\textcolor{blue}{\nu}$  Eri calculated with the OP, OPAL and LA opacity data.**



**Rosseland-mean opacities  $\kappa[\text{cm}^2 \text{ g}^{-1}]$  vs.  $\log T$  for  $\lg R = -6.0$ ,  $-5.0$ ,  $-4.5$ , and  $-4.0$ . The OPAL (blue lines) and K-OPAL (red lines) data were plotted for  $Z = 0.0266$ ,  $0.0168$  and  $0.0054$ .**



The new bump at  $\log T = 5.06$

H. Cugier 2014

# **PULSATING WHITE DWARFS**

**Recent reviews:**

**G. Fontaine & P. Brassard 2008**

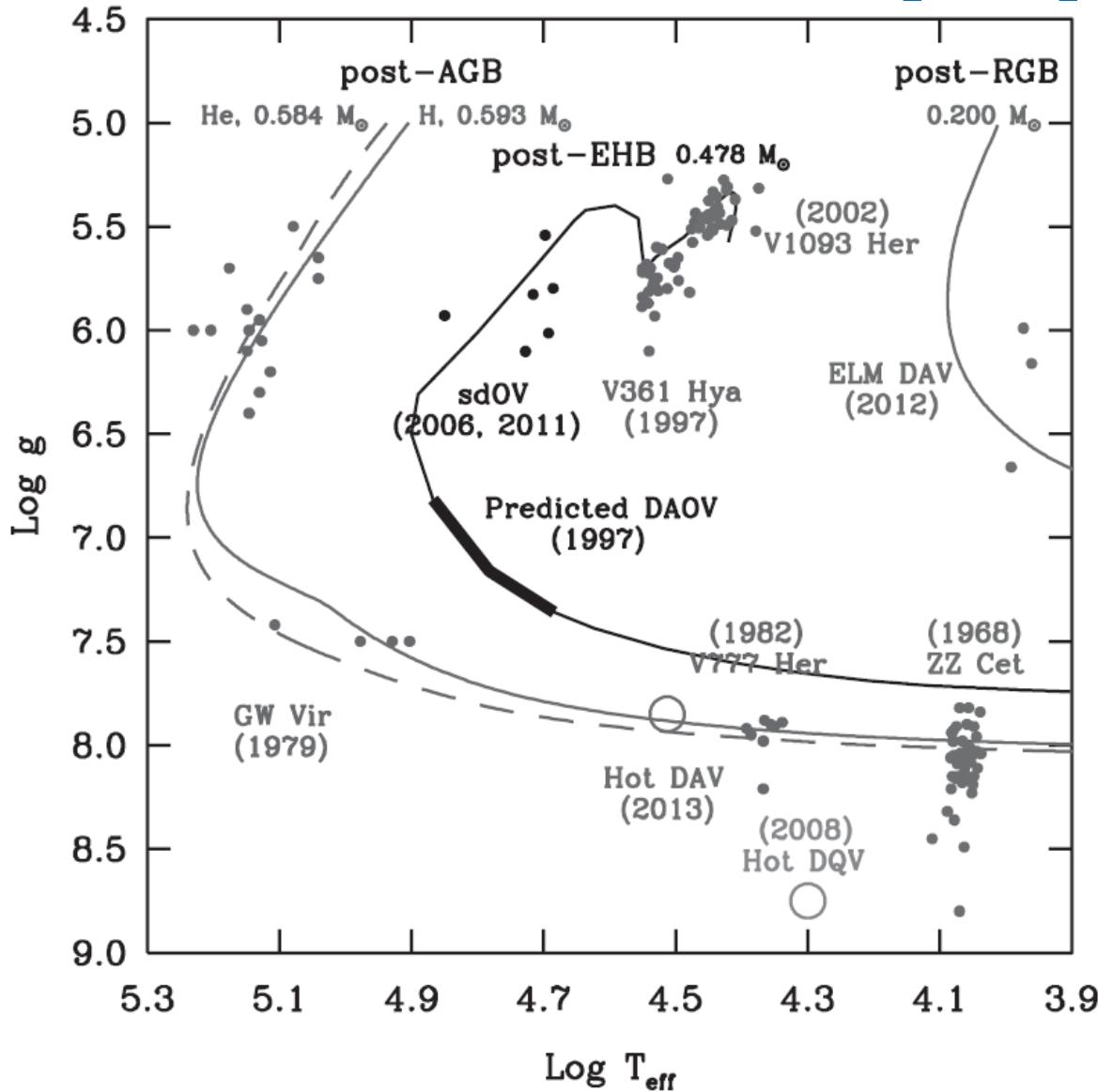
**D. E. Winget & S. O. Kepler, 2008**

**H. Saio 2013**

**G. Fontaine et al. 2014**

**Most stars (~97%) will end up  
their evolution as white dwarfs**

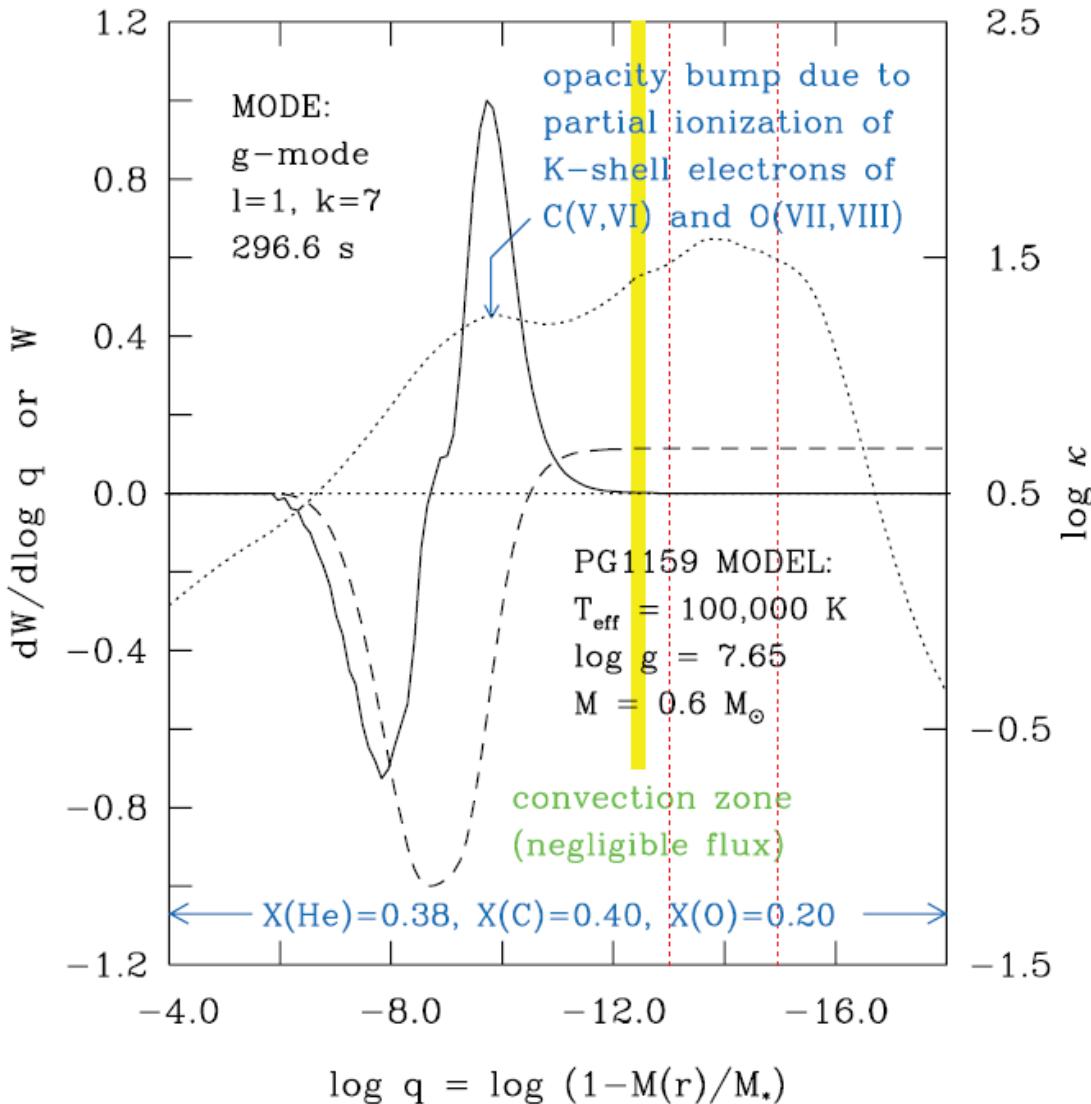
# the locus of various classes of compact pulsators



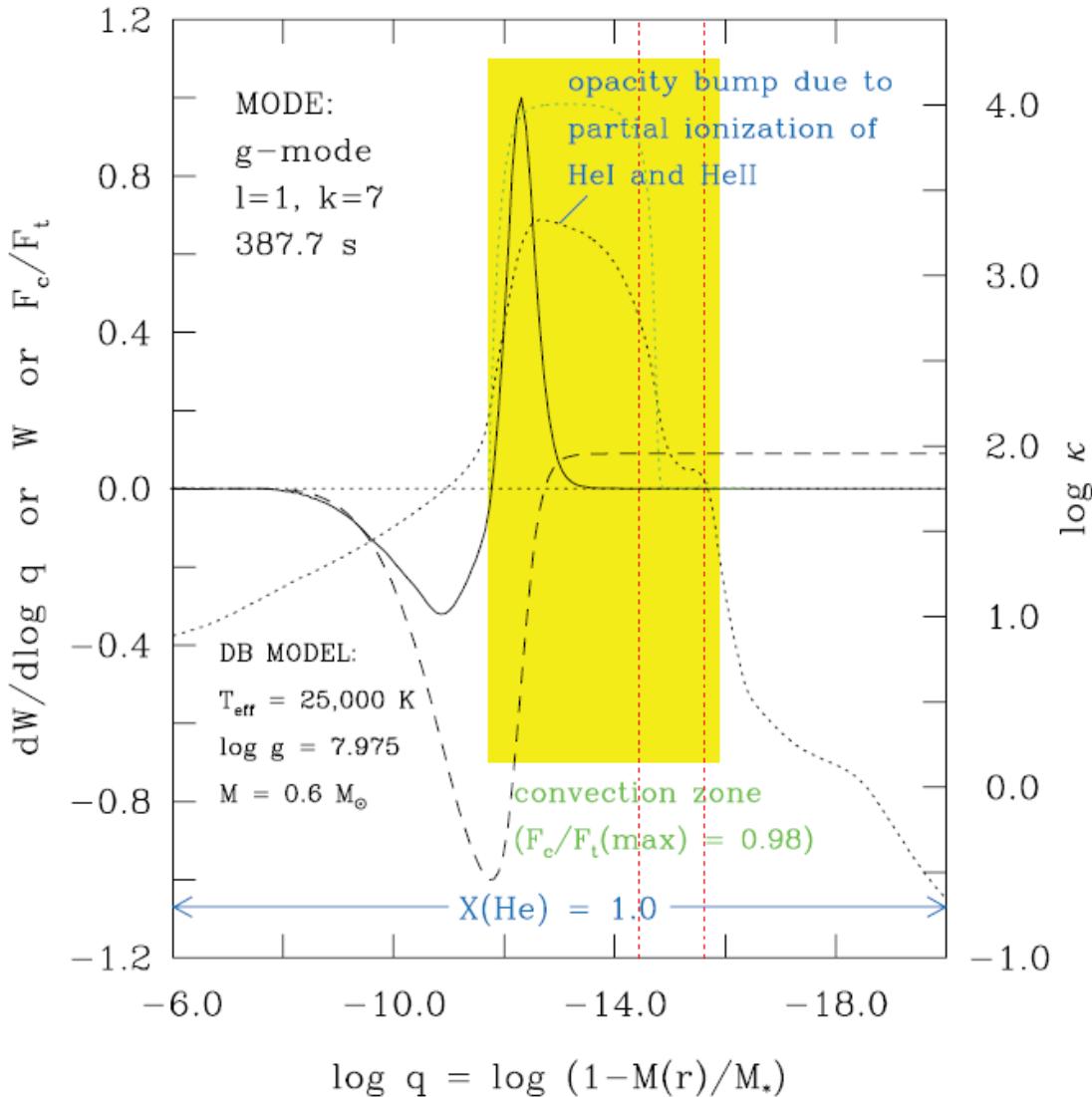
- 1 GW Vir (PNNV + DOV) – He/C/O-atmospheres,  $T_{\text{eff}} \approx 120\,000$  K**
- 2 V777Her (DBV) – He-atmospheres,  $T_{\text{eff}} \approx 25\,000$  K**
- 3 ZZ Cet (DAV) – H-atmospheres,  $T_{\text{eff}} \approx 12\,000$  K**
- 4 Hot DQV – C-atmospheres,  $T_{\text{eff}} \approx 20\,000$  K  
Dufour et al. 2007**
- 5 ELM DAV – thick H-envelope,  $T_{\text{eff}} < 10\,000$  K, p-modes ?  
Hermes et al. 2013**
- 6 Hot DAV – thin H-envelope, ,  $T_{\text{eff}} \approx 30\,000$  K  
Kurtz et al. 2013**

**g-modes with  $P=100 - 2500$ s**

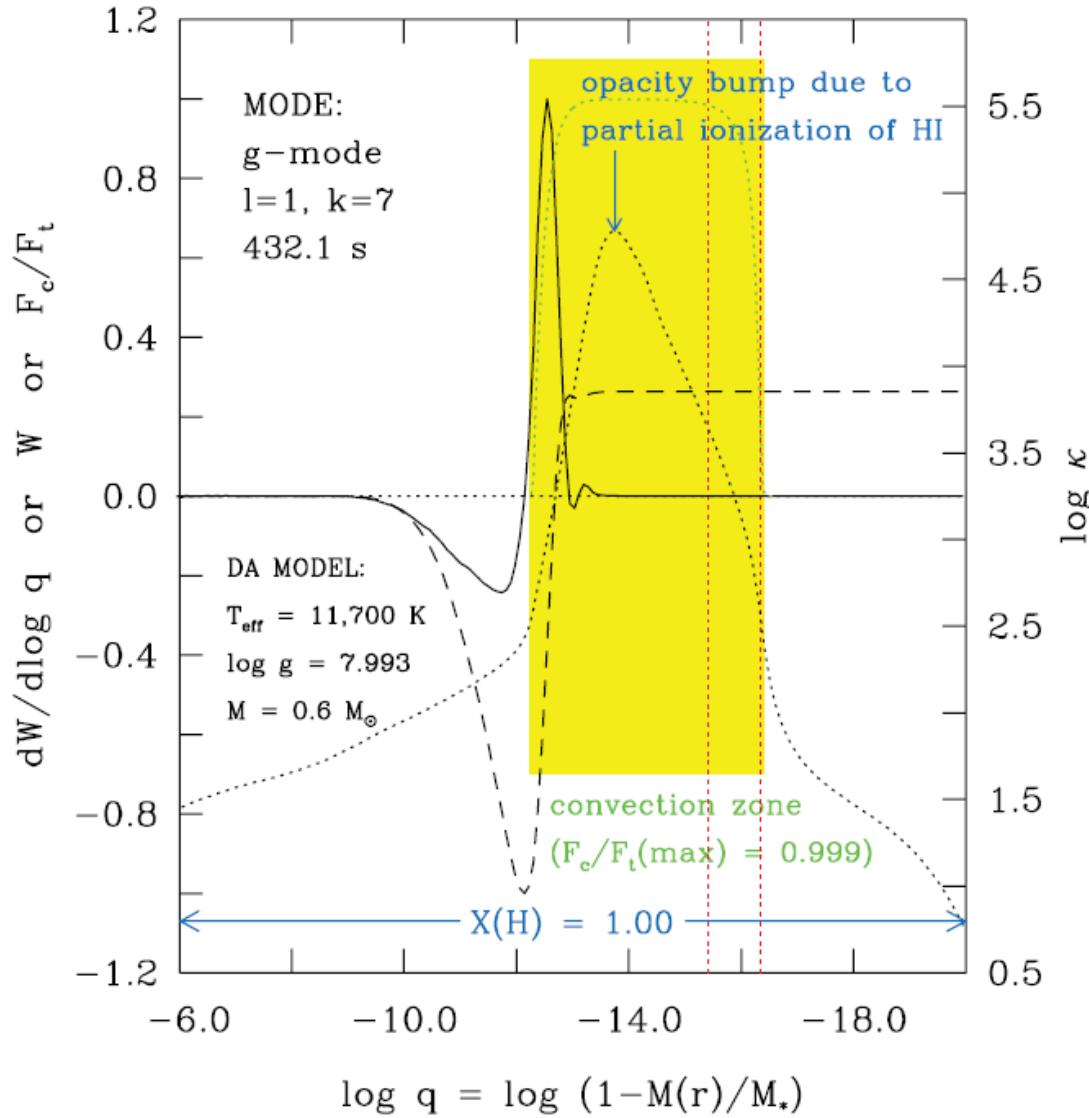
# Energetic properties of a typical g-mode excited in a GW Vir star model



# Energetic properties of a typical g-mode excited in a V777 Her star model



# Energetic properties of a typical g-mode excited in a ZZ Cet star model

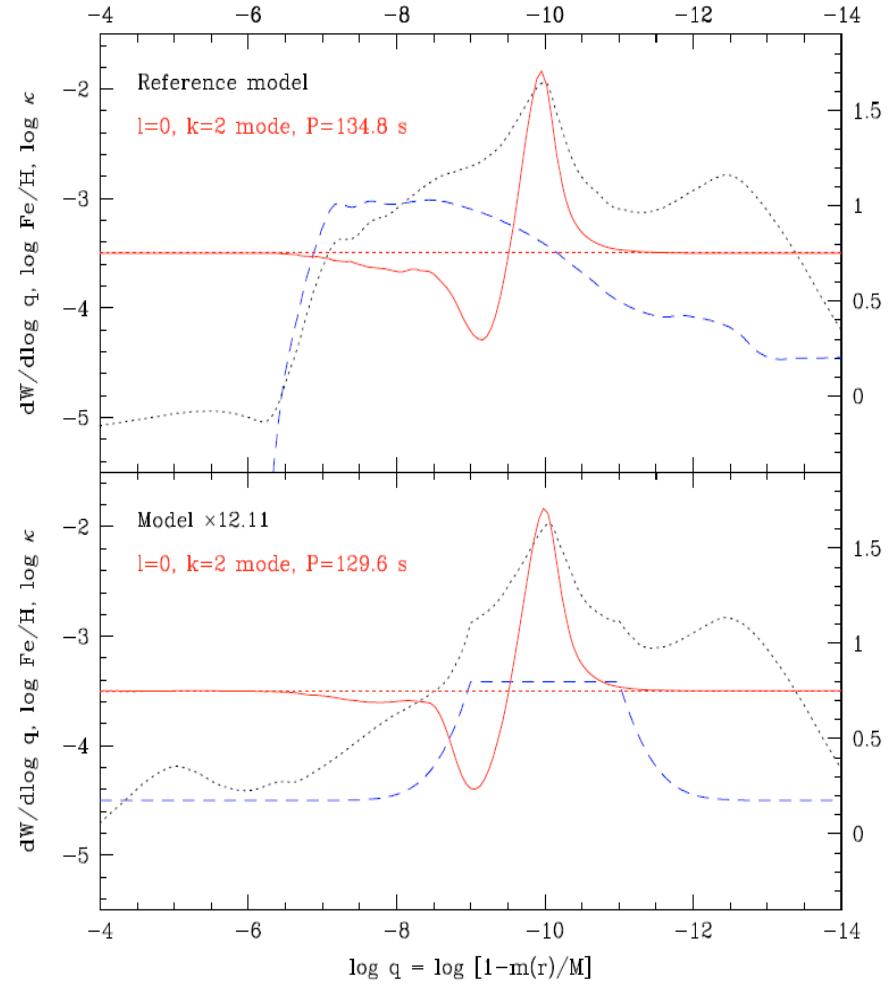
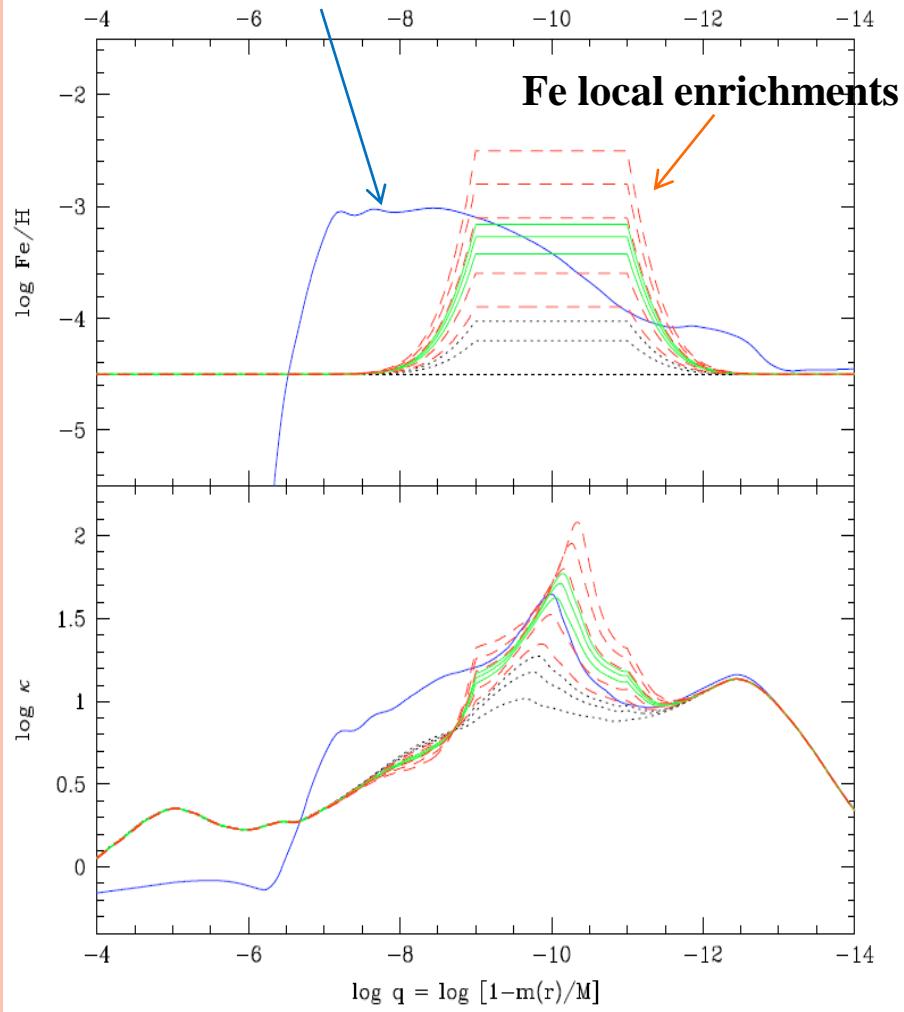


# **hot subdwarf stars**

## **sdB, sdO**

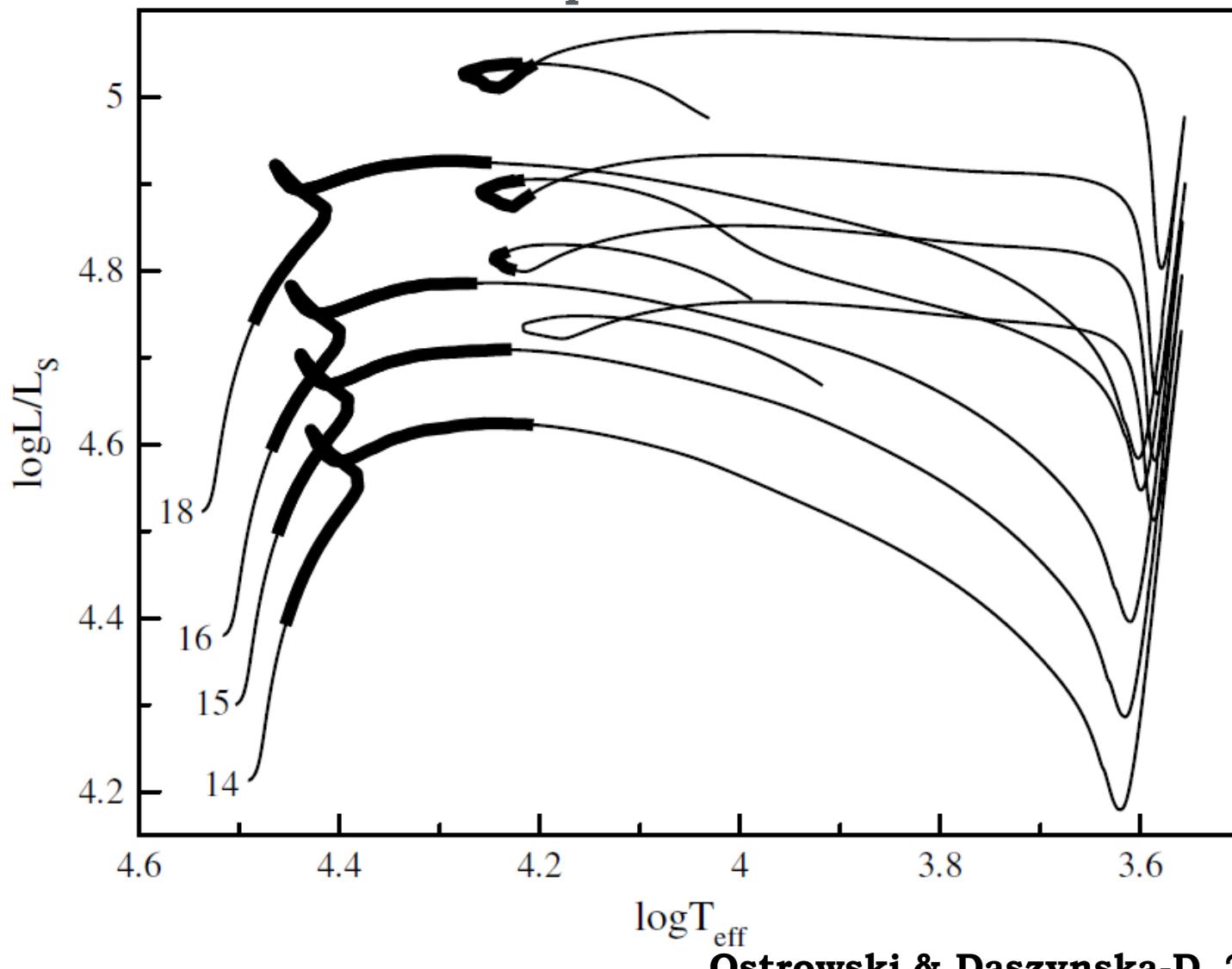
**Recent review:**  
**S. Randall et al. 2014**  
**Six distinct types**

the Fe profile obtained from  
equilibrium between gravitational  
settling and radiative levitation



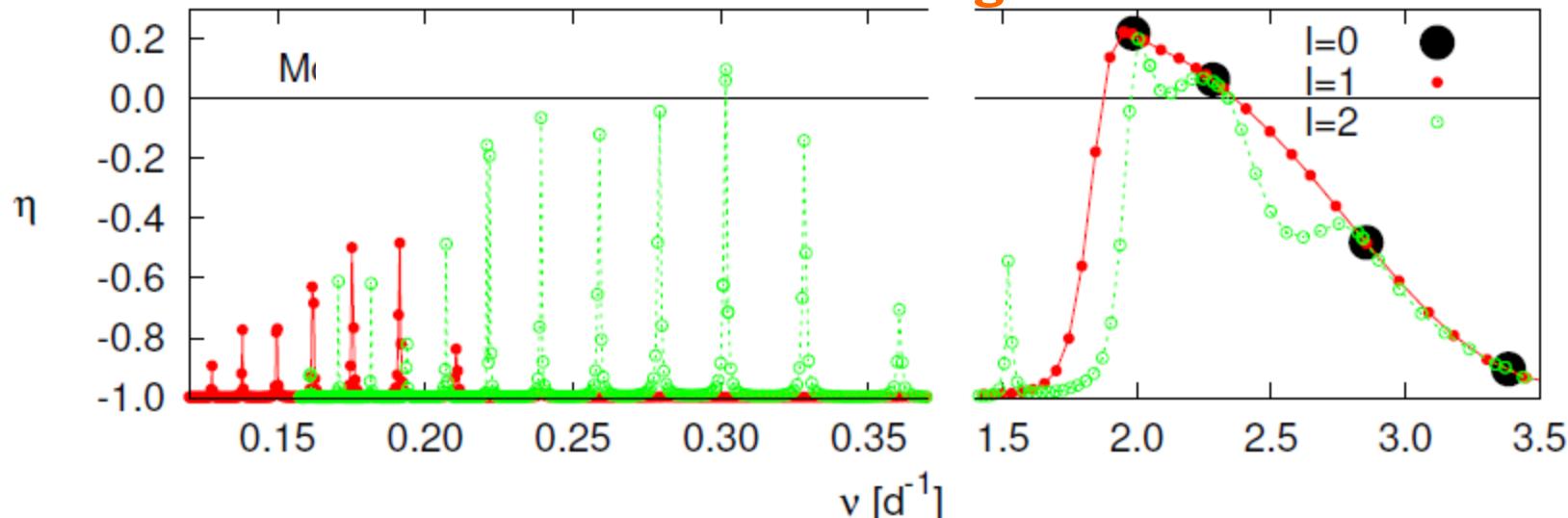
# B-type supergiants

**Instability domains for the modes of the degree  $l=0,1,2$   
excited in the OPAL models with masses of  $14 - 18M_{\odot}$ .  
MESA code + nonadiabatic pulsational code of Dziembowski**

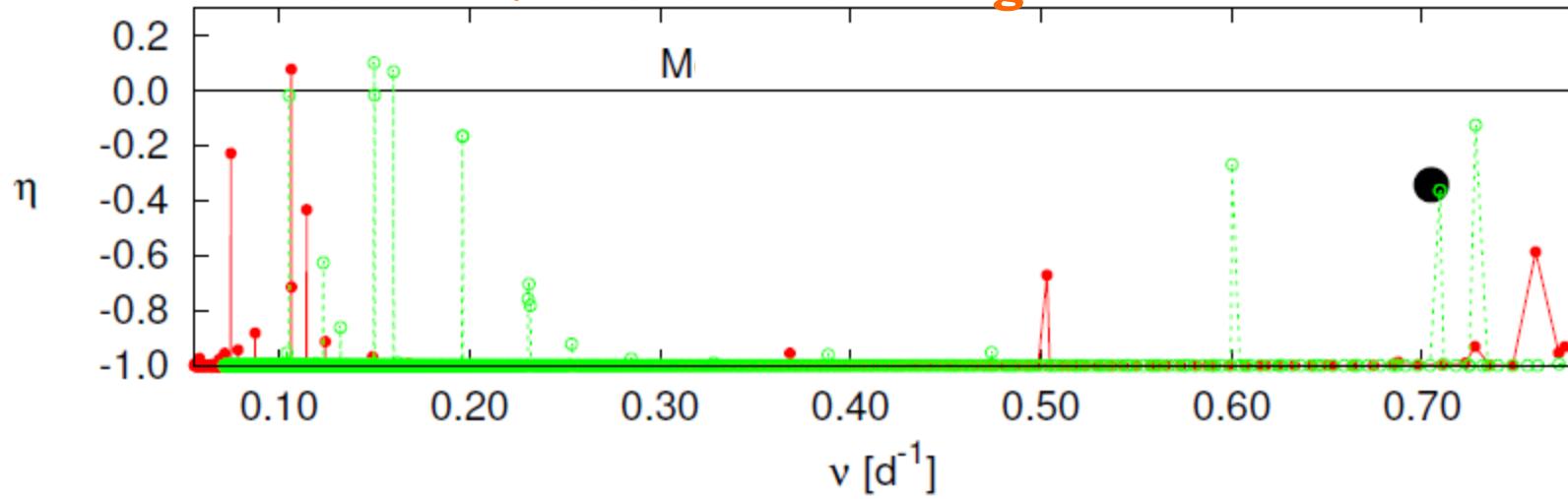


Ostrowski & Daszynska-D, 2014

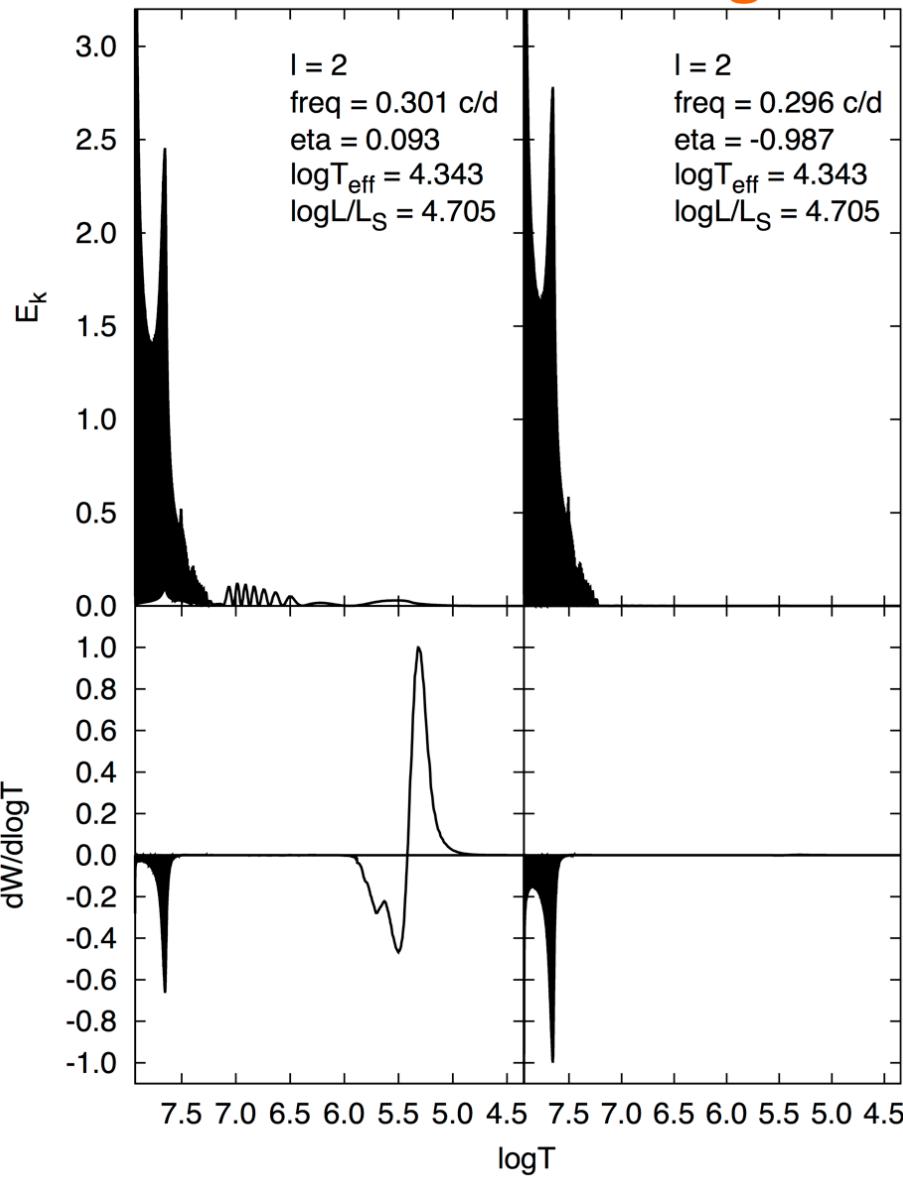
## MODEL 1 - H shell burning



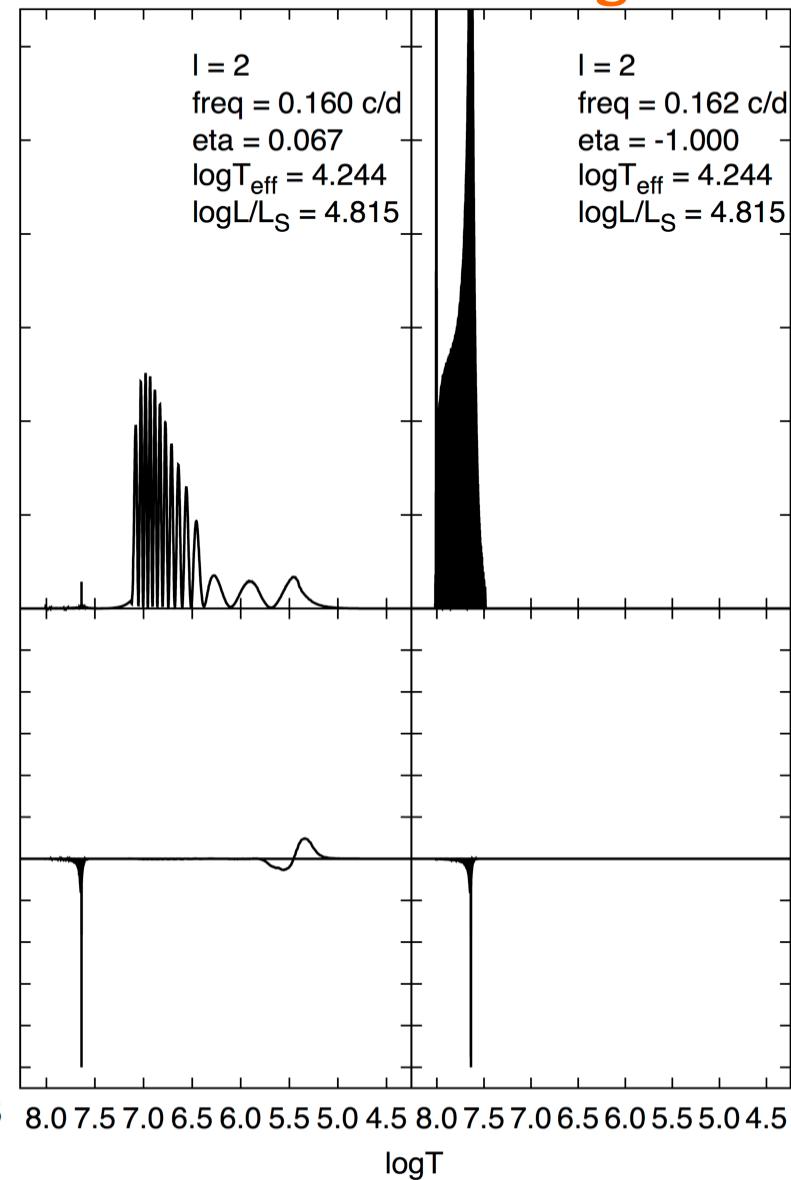
## MODEL 2 - Core He burning



## MODEL 1 H shell burning



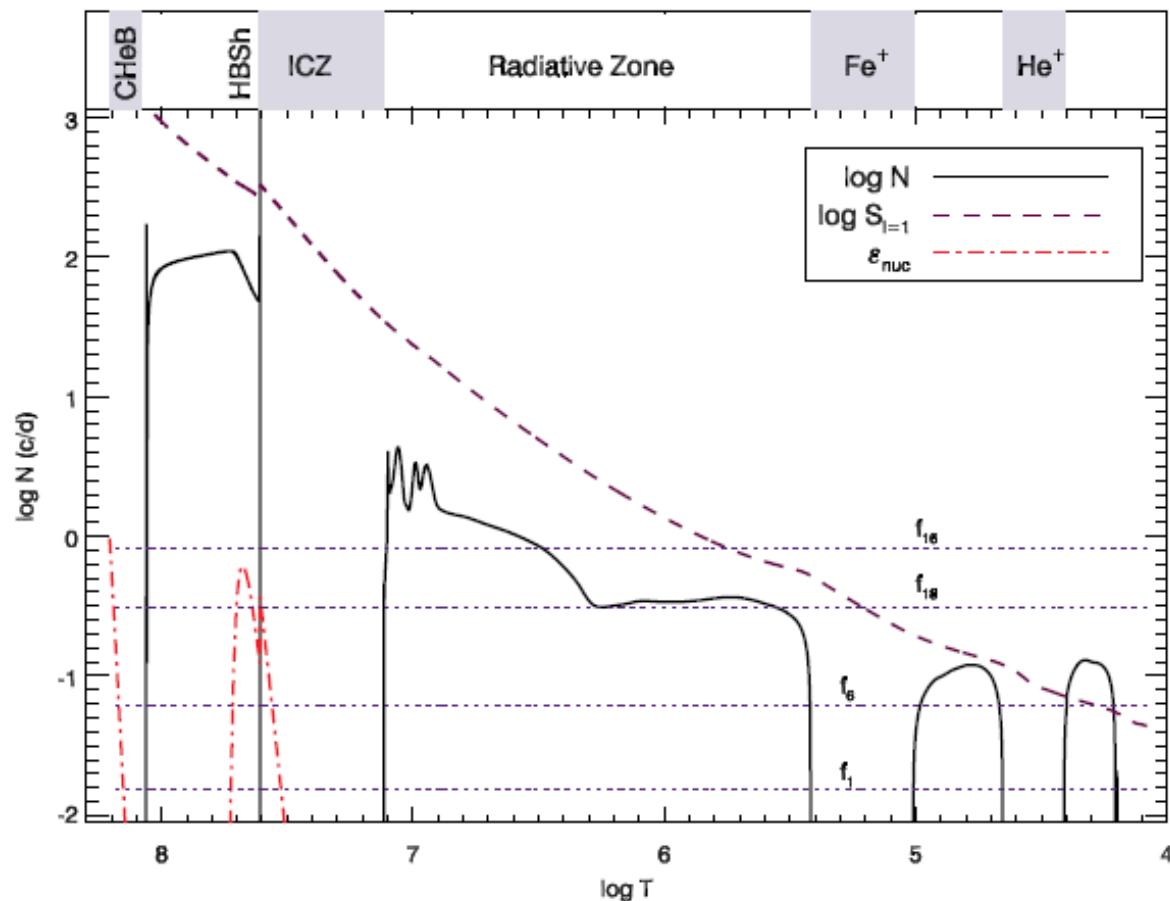
## MODEL 2 Core He burning



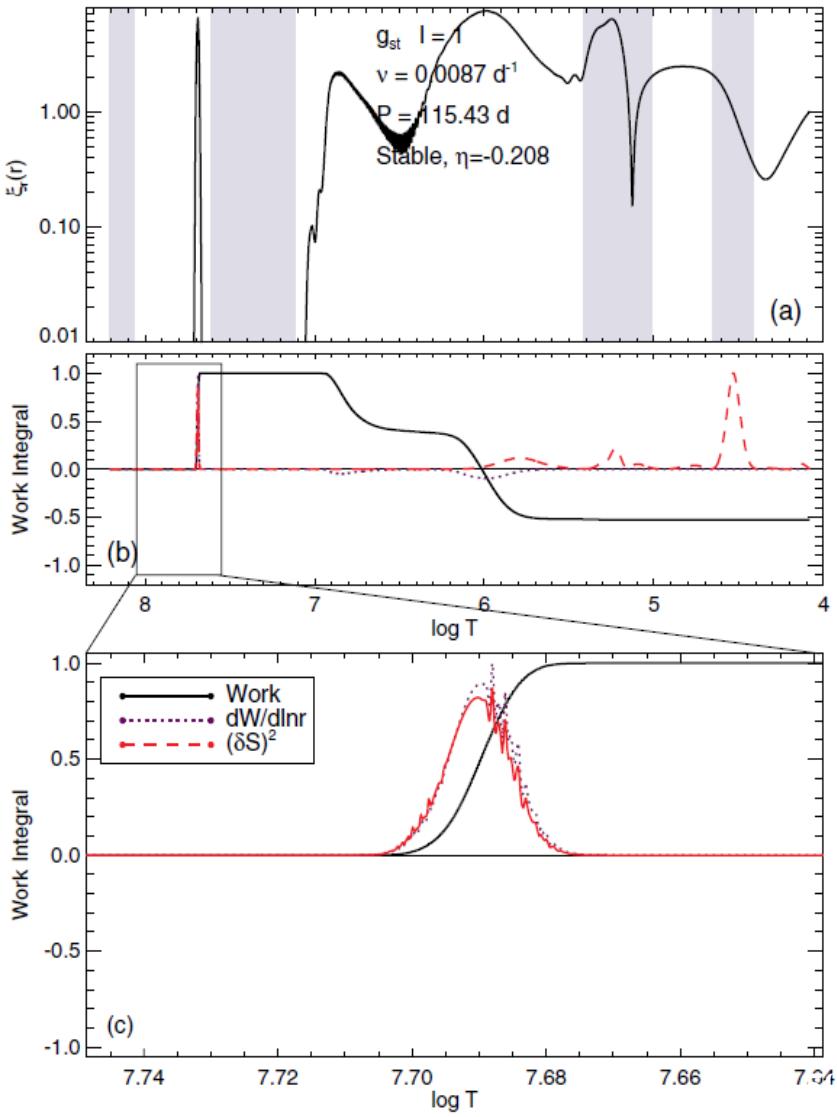
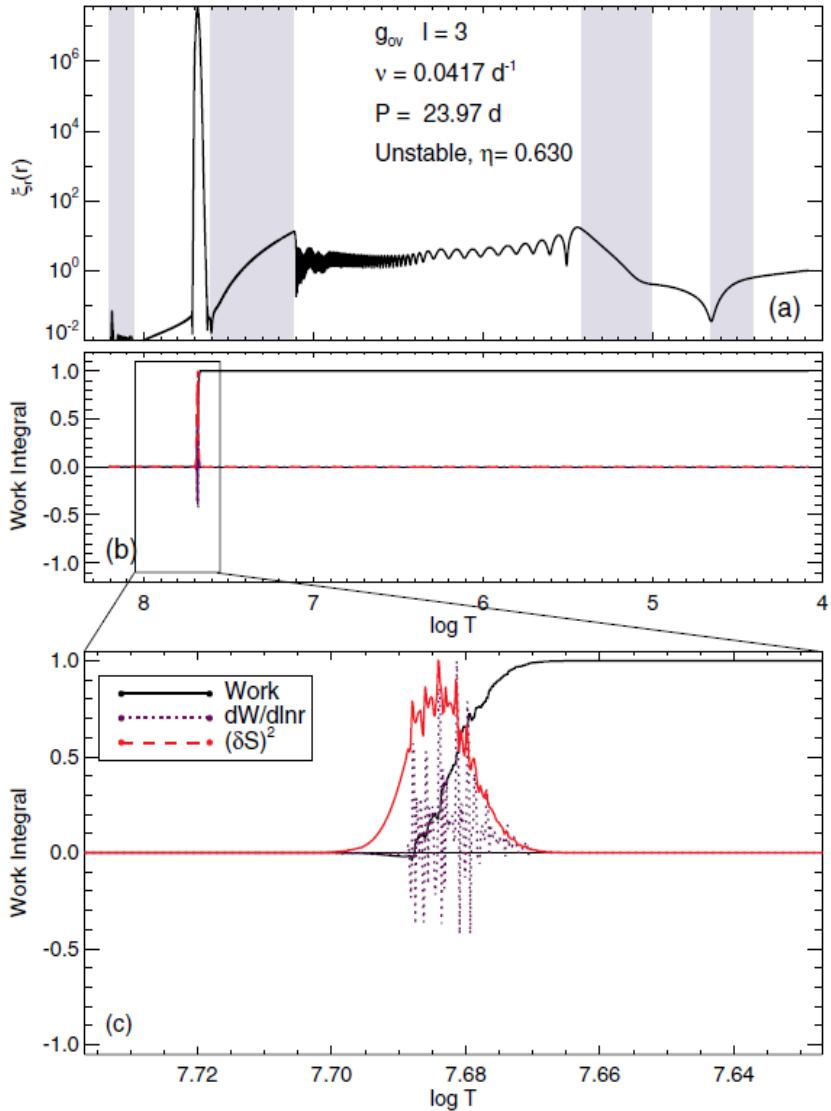
**$\varepsilon$ -mechanism**

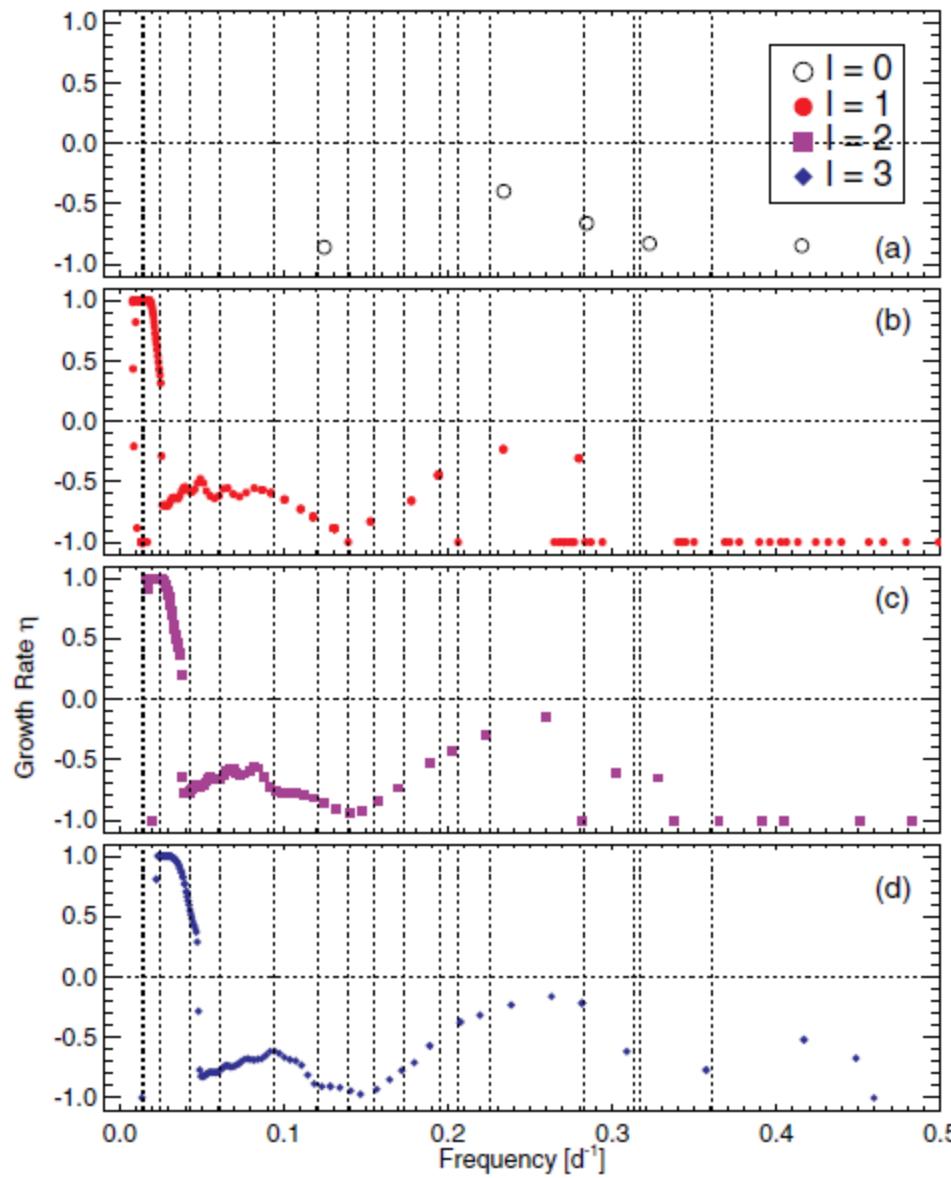
**Ledoux (1941)**

# Propagation diagram for the model of Rigel ( $\beta$ Ori, B8 Ia)



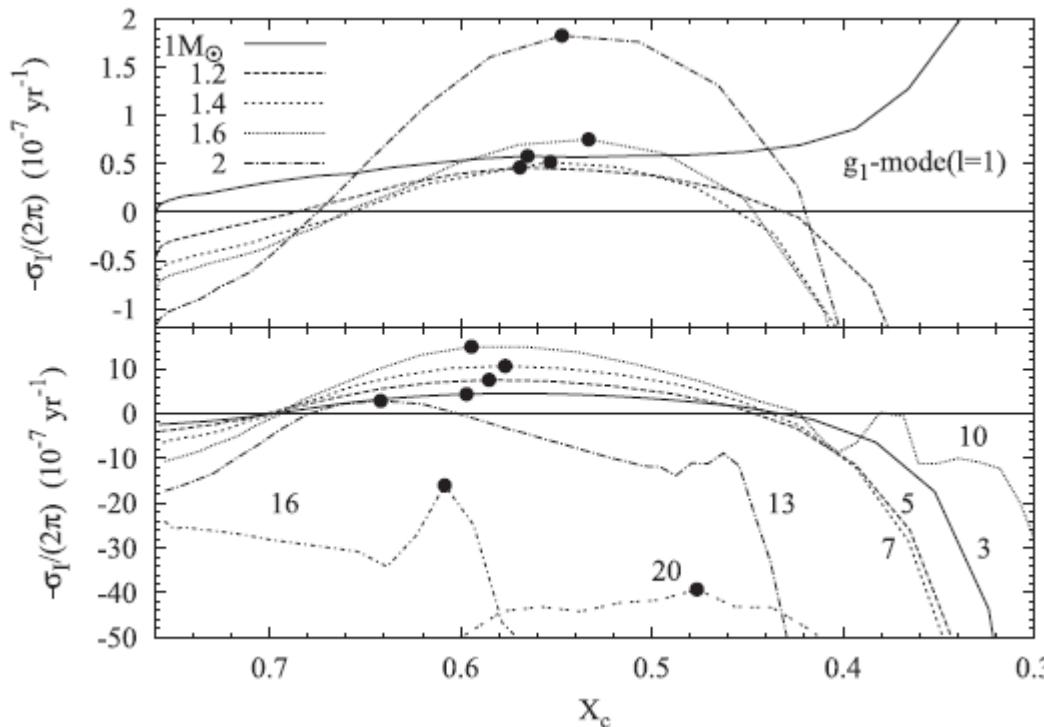
# Example of unstable and stable g-mode in the Rigel model



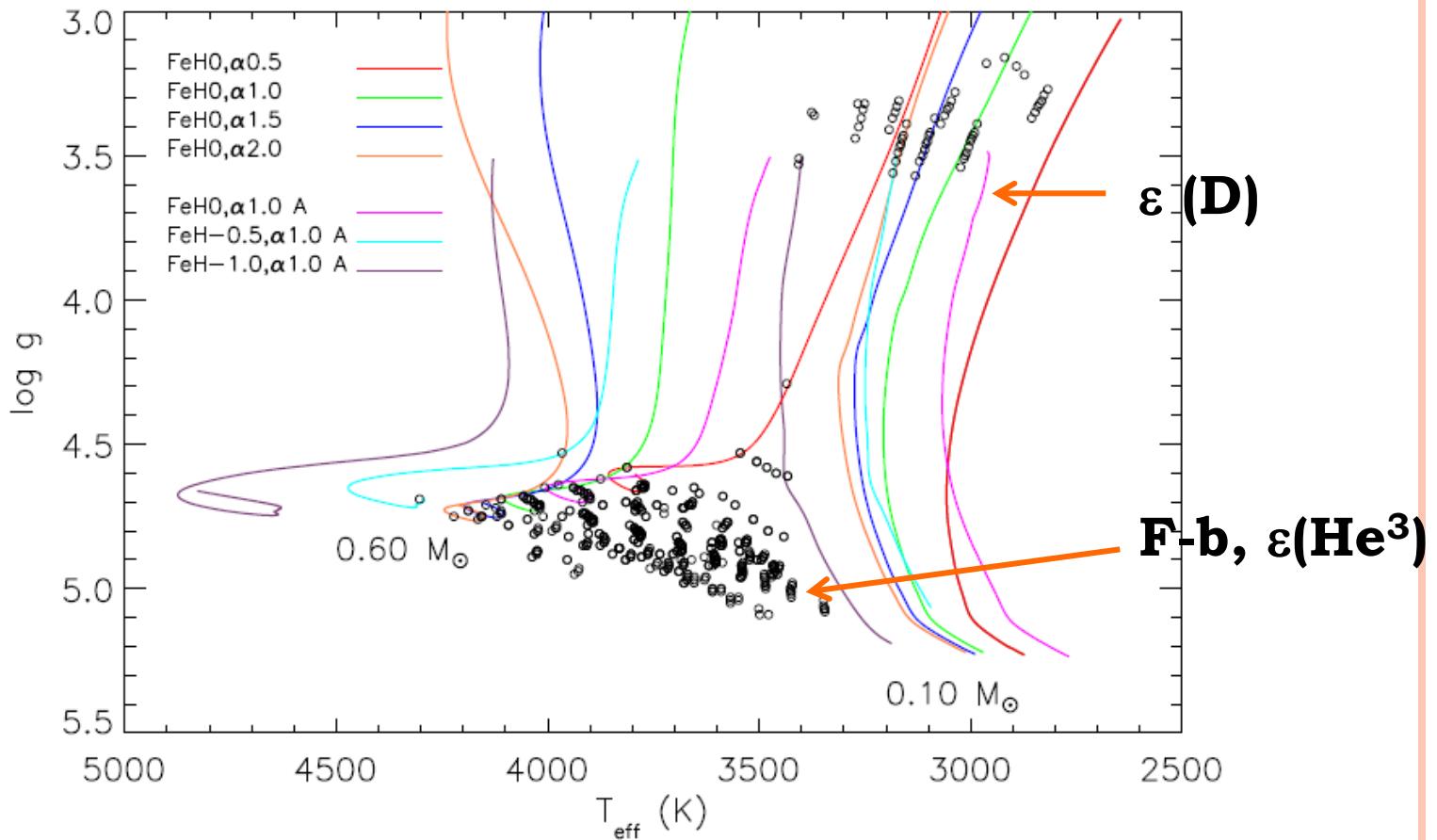


# $\varepsilon$ -Mechanism – pulsational instability of Population III Stars

„stars with  $M < 13M_{\odot}$  become unstable against the dipole  $g_1$ - and  $g_2$ -modes during the early evolutionary phase at which the pp-chain is still the dominant nuclear energy source”

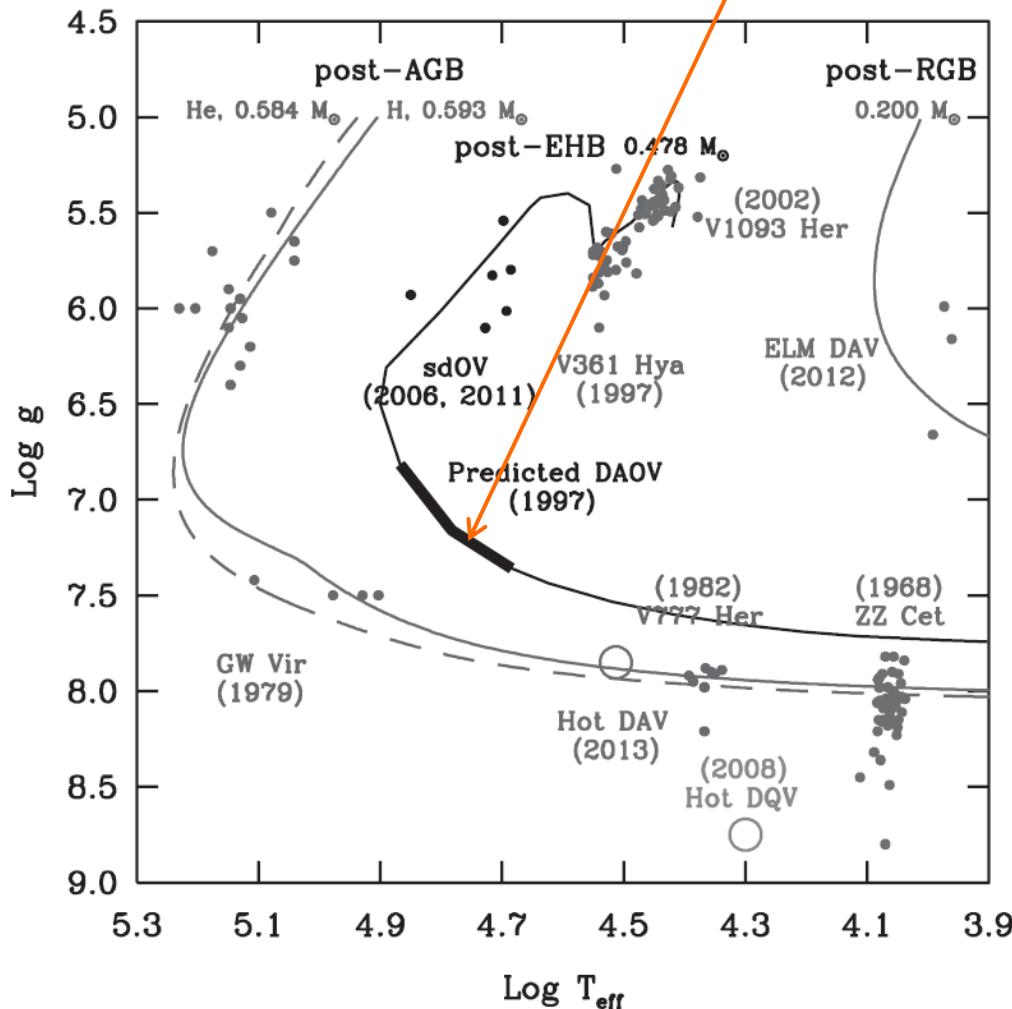


## instability strip for M dwarf stars



# post-EHB DAO white dwarfs

## g-mode pulsations driven by the $\varepsilon$ -mechanism



# Effects of rotation on pulsational instability and mode properties

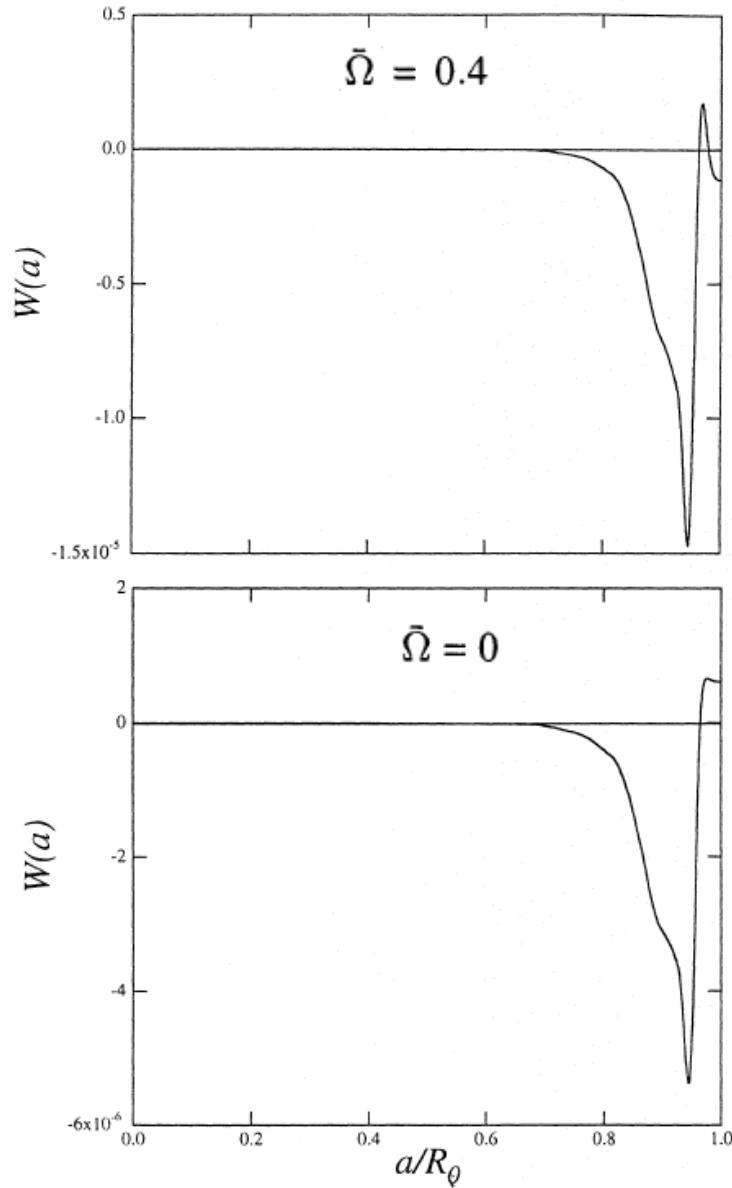
Osaki, Lee, Baraffe, Townsed, Saio, Reese, Ballot,  
Goupil, Savonije, Lignières, Suarez, Mathis,  
Neiner, ...

- $\Omega \sim 0.5\Omega_{\text{crit}}$
- $\omega \sim \Omega$

$$\eta(\omega, \lambda/\omega^2, \Omega)$$

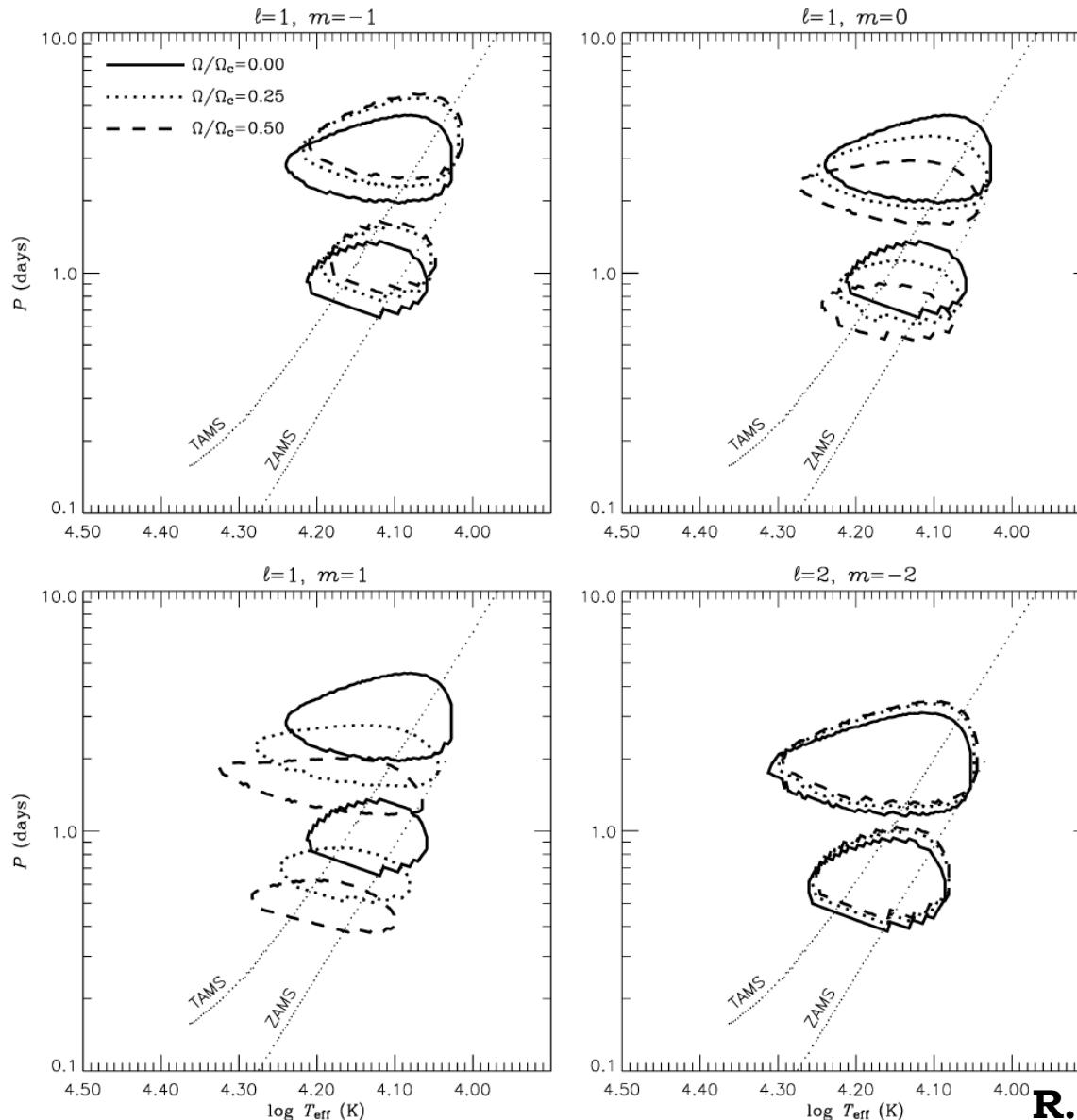
$$\Omega \approx 0 \rightarrow \lambda = \ell (\ell + 1)$$

## Cumulative W for the l=2 m=-2 p-mode of the $10 M_{\odot}$ star (MS)

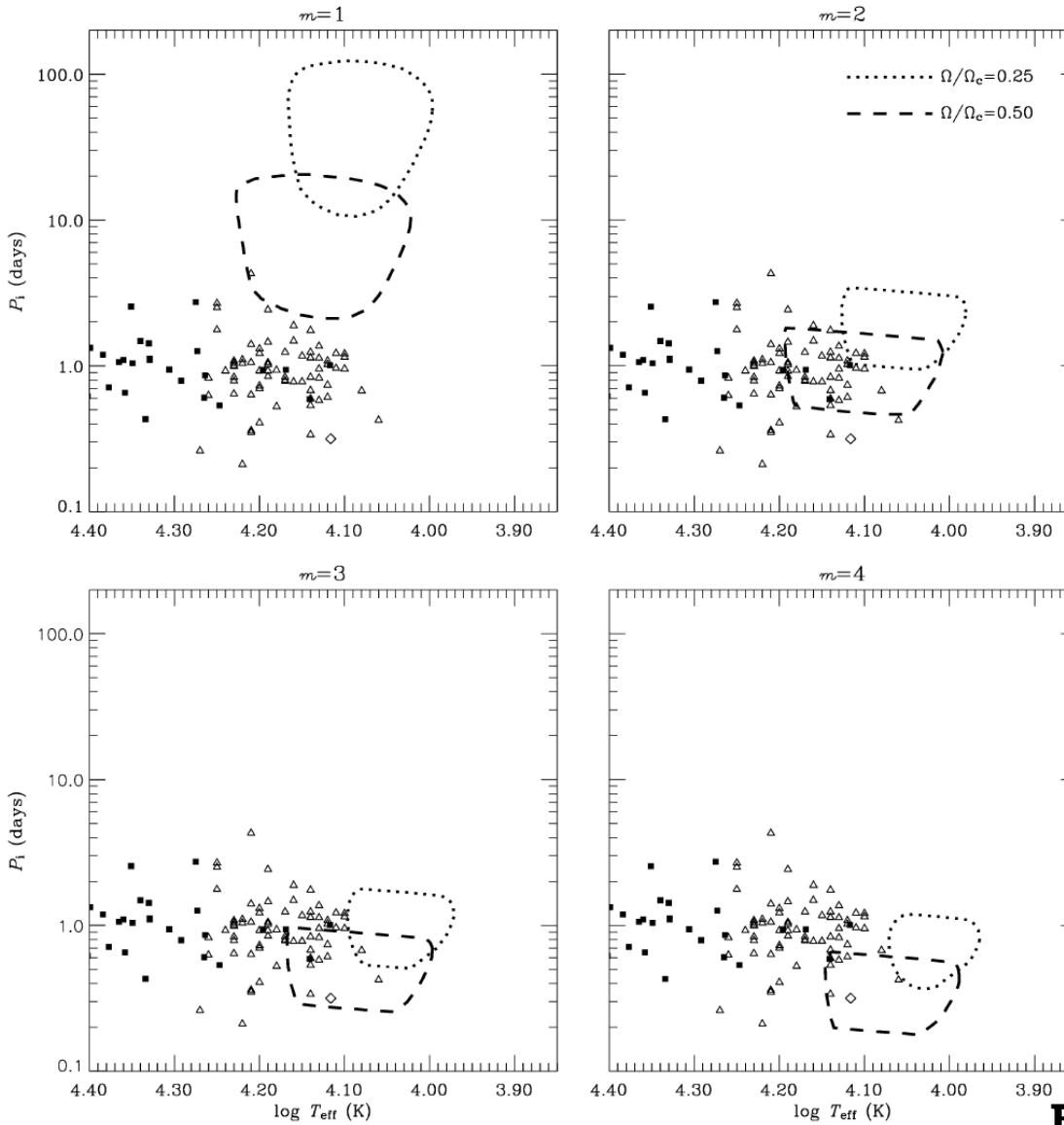


U. Lee & I. Baraffe 1995

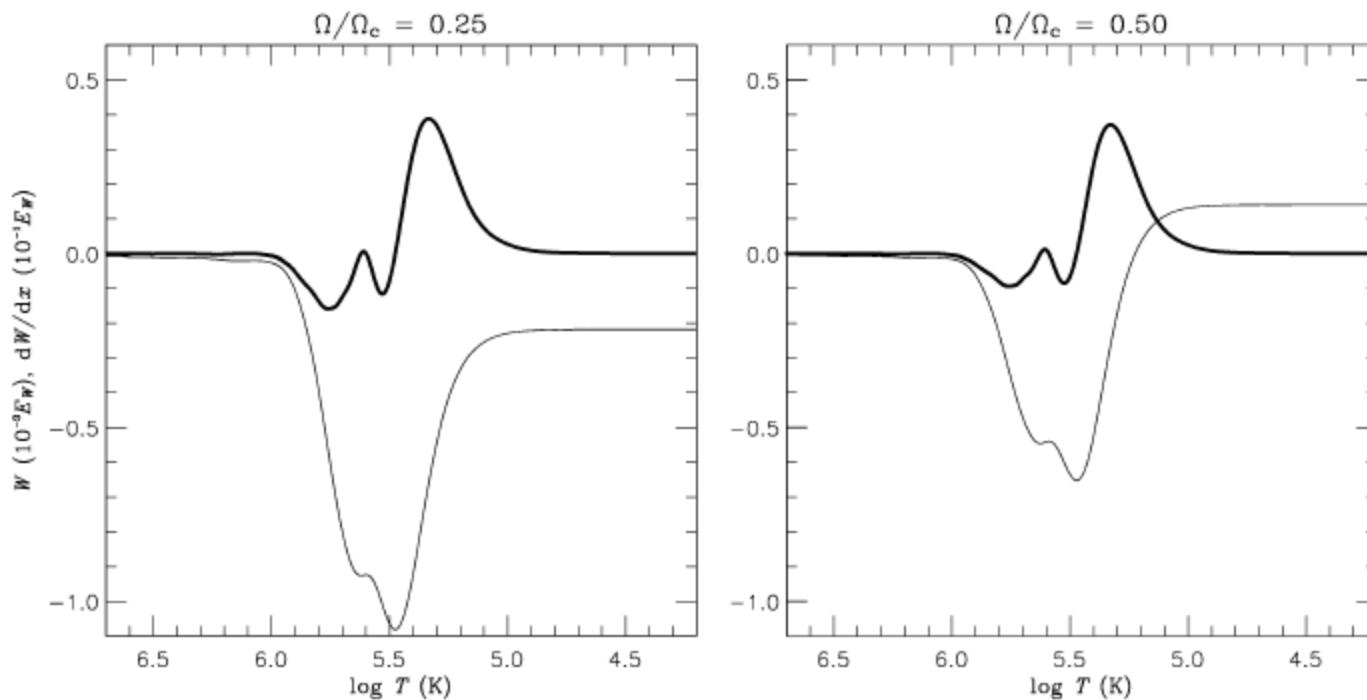
# Effect of the Coriolis force on the instability domains of SPB stars



# Instability strip of retrograde mixed modes in B-type stars



## The cumulative and differential work integral for the m=1, n=20 mixed mode of the 53 Per model



# **Solar-like oscillations**

**Appourchaux, Basu, Baudin, Bedding, Belkacem,  
Chaplin, Christensen-Dalsgaard, De Ridder, Di  
Mauro, Dupret, Duvall, Dziembowski, Elsworth,  
Garcia, Goode, Gough, Goupil, Hekker, Houdek,  
Kjeldsen, Kosovichev, Mosser, Roxburgh,  
Shibahashi, Samadi, Vorontsov**

...

**Chaplin & Miglio, 2013, ARA&A**

# Solar-like oscillations

the observed amplitude of a pulsational mode changes with time

the energy distribution

$$p(E)dE = \langle E \rangle^{-1} \exp(-E/\langle E \rangle) dE$$

the height of a single peak

$$H = \frac{2E}{\eta I} = \frac{P}{\eta^2 I}$$

e.g. Chaplin et al. 2005, Houdek 2006

# Scaling relations

Baudin et al. 2005, Chaplin et al. 2008

Belkacem et al. 2011, Kjeldsen & Bedding 2011

Mosser et al. (2011, 2012), Samadi et al. 2012,

Belkacem et al. 2013 ...

the frequency of  $H_{\max}$

$$\nu_{\max} \propto \nu_{\text{ac}} \propto \frac{MT_{\text{eff}}^{3.5}}{L}$$

the height

$$H = \frac{P}{2\eta^2 \mathcal{M}}$$

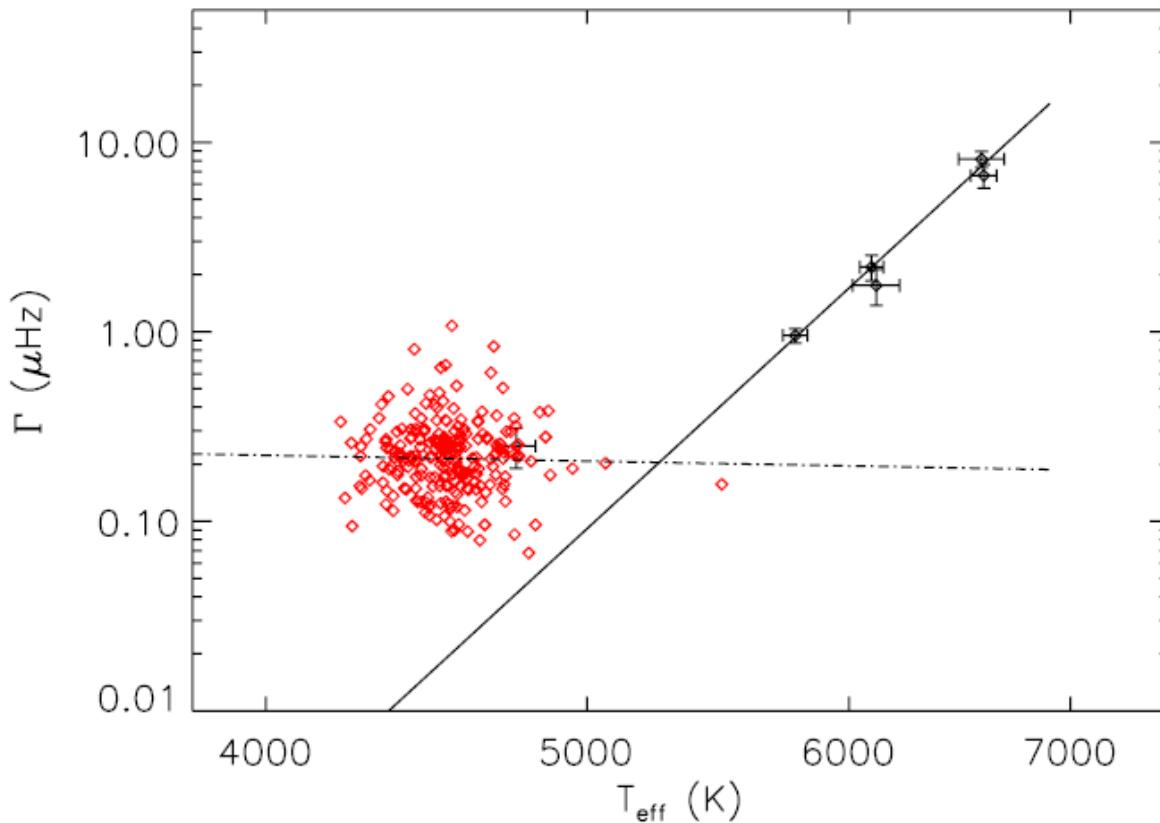
the damping rate

$$\eta \propto T_{\text{eff}}^{10.8} g^{-0.3}$$

the mode linewidth

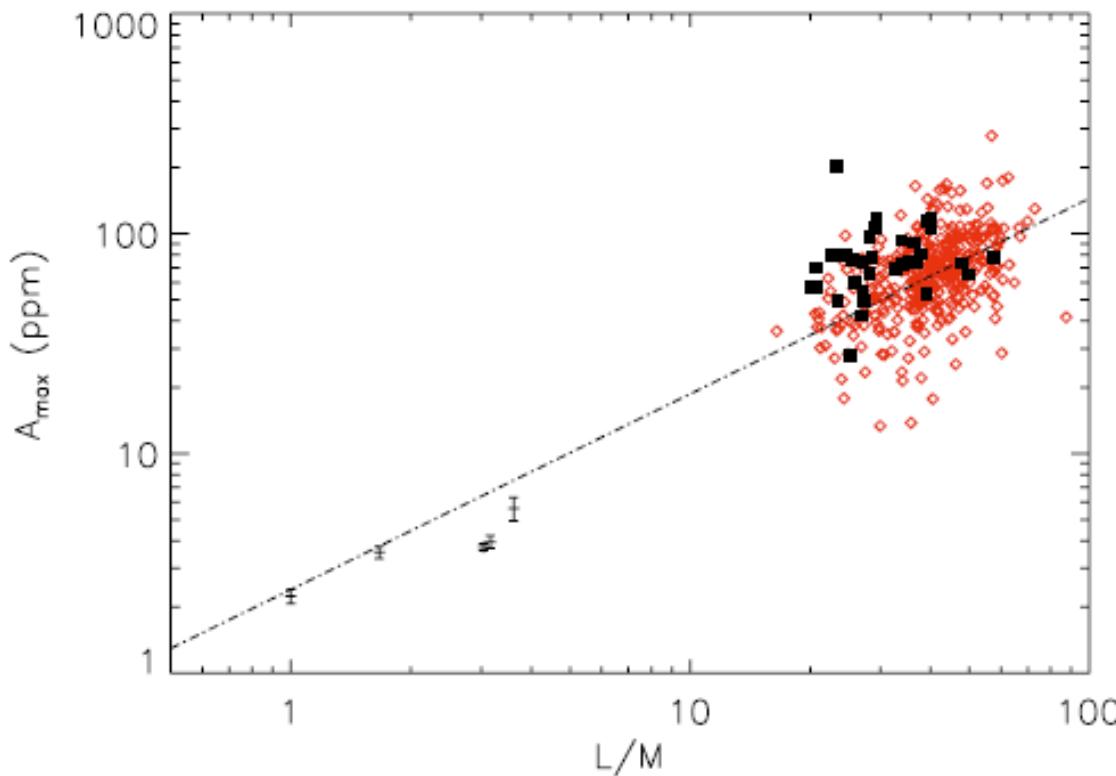
$$\Gamma \propto T_{\text{eff}}^{8.75-4\beta} g^\beta$$

mode linewidths for the red giants (Teff < 5000K)  
and for the main-sequence stars (Teff > 5000 K).



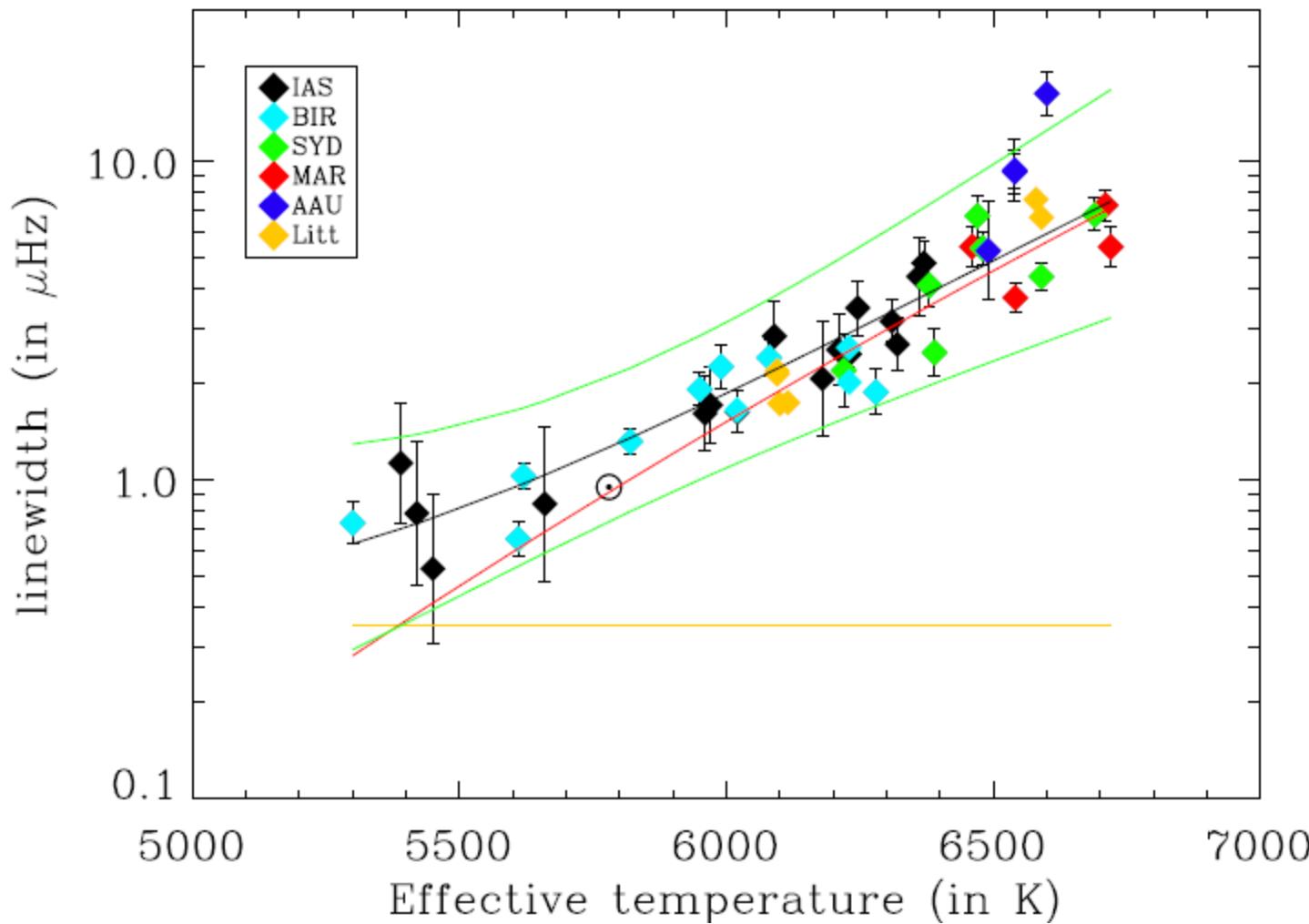
$$\Gamma \propto T_{\text{eff}}^s \quad \text{with} \quad \begin{cases} s \approx -0.3 \pm 0.9 & \text{for RG} \\ s \approx 16 \pm 2 & \text{for MS} \end{cases}$$

## Mode amplitudes ( $\propto \sqrt{H\Gamma}$ ) for the red giants (L/M > 5) and for the MS stars (L/M < 5)



$$A_{\max}^I \propto \left(\frac{L}{M}\right)^s \text{ with } \begin{cases} s = 0.92 \pm 0.14 & \text{for RG} \\ s = 0.42 \pm 0.14 & \text{for MS} \end{cases}$$

## mode linewidths of MS and subgiant stars



# TIDAL EFFECTS

- Perturbation of free oscillations
- Tidally forced oscillations

# **Tidal perturbation of free oscillations**

**Fitch (1967, 1969) – some  $\delta$  Sct and  $\beta$  Cep stars**

**XX Pyx – T. Arentoft et al. (2001), C. Aerts et al. 2002**

**DG Leo – P. Lampens et al. 2005**

**C. Aerts 2007**

# **Tidally forced oscillations**

**Cowling 1941**

**Kato, Zahn, Savonije, Papaloizou, Witte, Lee,  
Shibahashi, Kurtz**

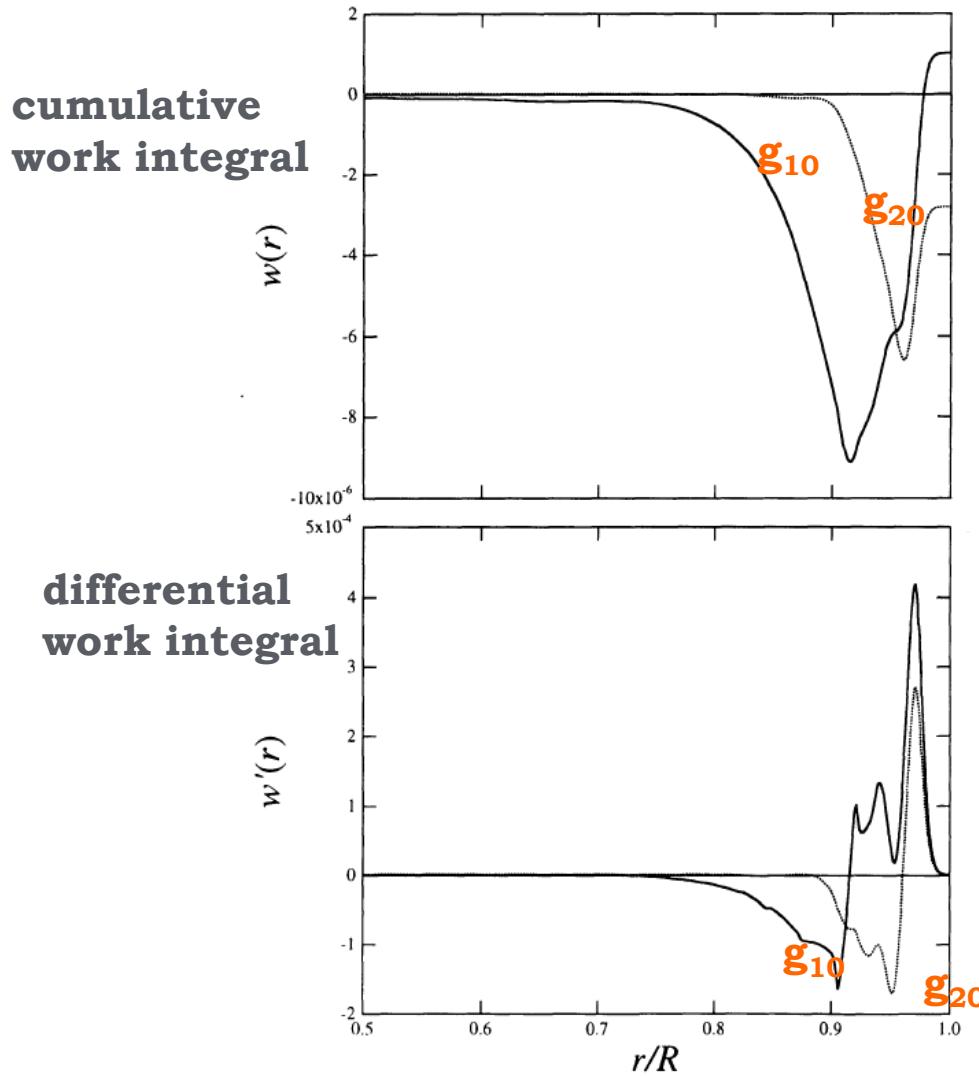
**$\delta$  Sct/γ Dor star HD209295 - Handler et al. 2002**

**the SPB star HD177863 -De Cat et al. 2000,  
Willems & Aerts 2002**

**the Kepler SPB star - Papics et al. 2013**

**the Kepler  $\delta$  Sct stars - K. M. Hambleton et al. 2013,  
C. Maceroni et al. 2014**

# Tidally forced oscillations in resonance with $g_{10}$ and $g_{20}$ quadruple mode of the $5 M_\odot$ model



**Silvotti et al. 2014**

$$v_{\text{puls}} = 3v_{\text{orb}}^P$$

**g-mode pulsations tidally excited by a  
planetary companion**

# CONCLUSIONS

Diversity of stellar pulsations

Seismic model → frequency + mode properties  
(nonadiabatic effects, convection, rotation etc.)

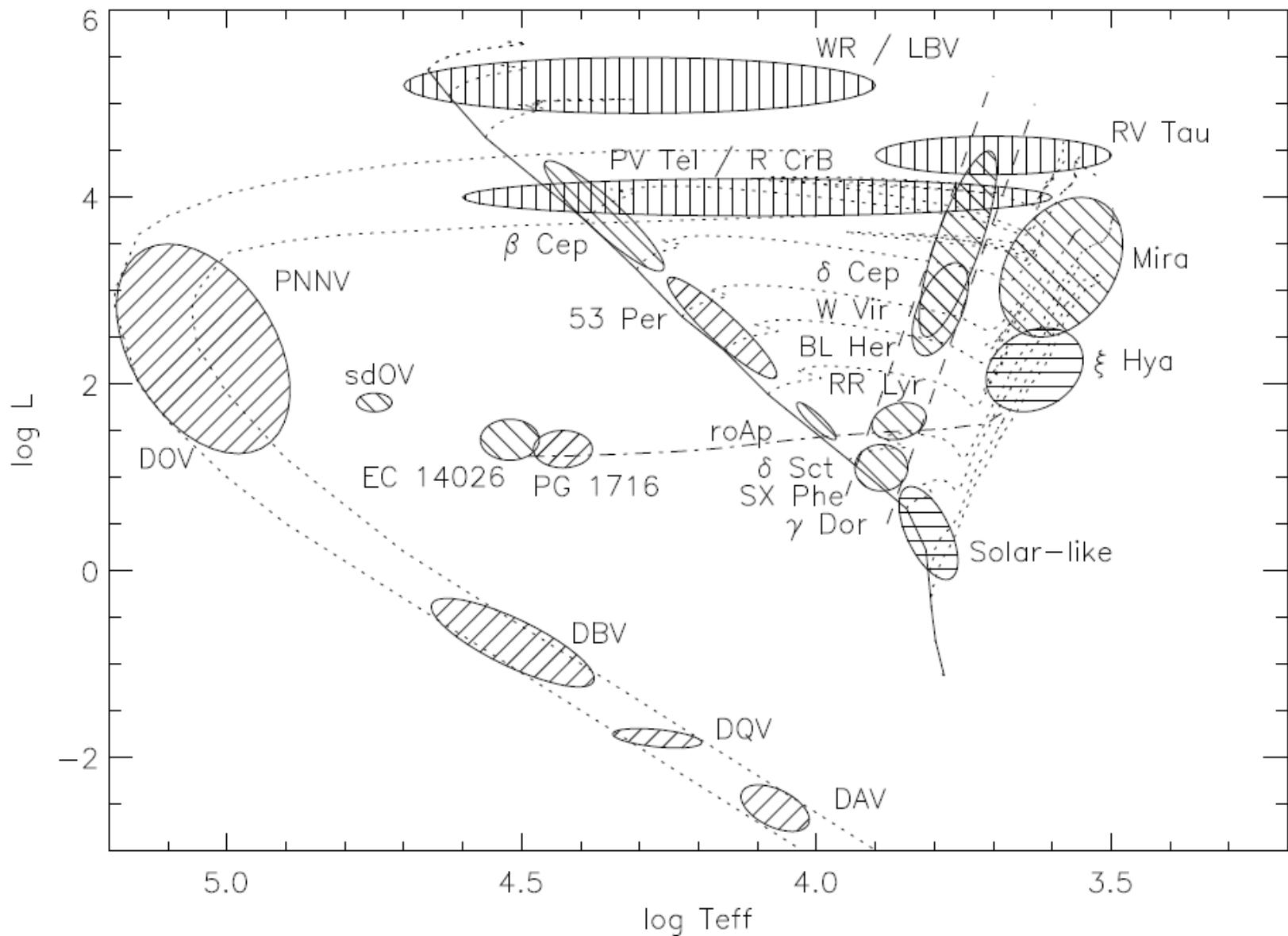
More data

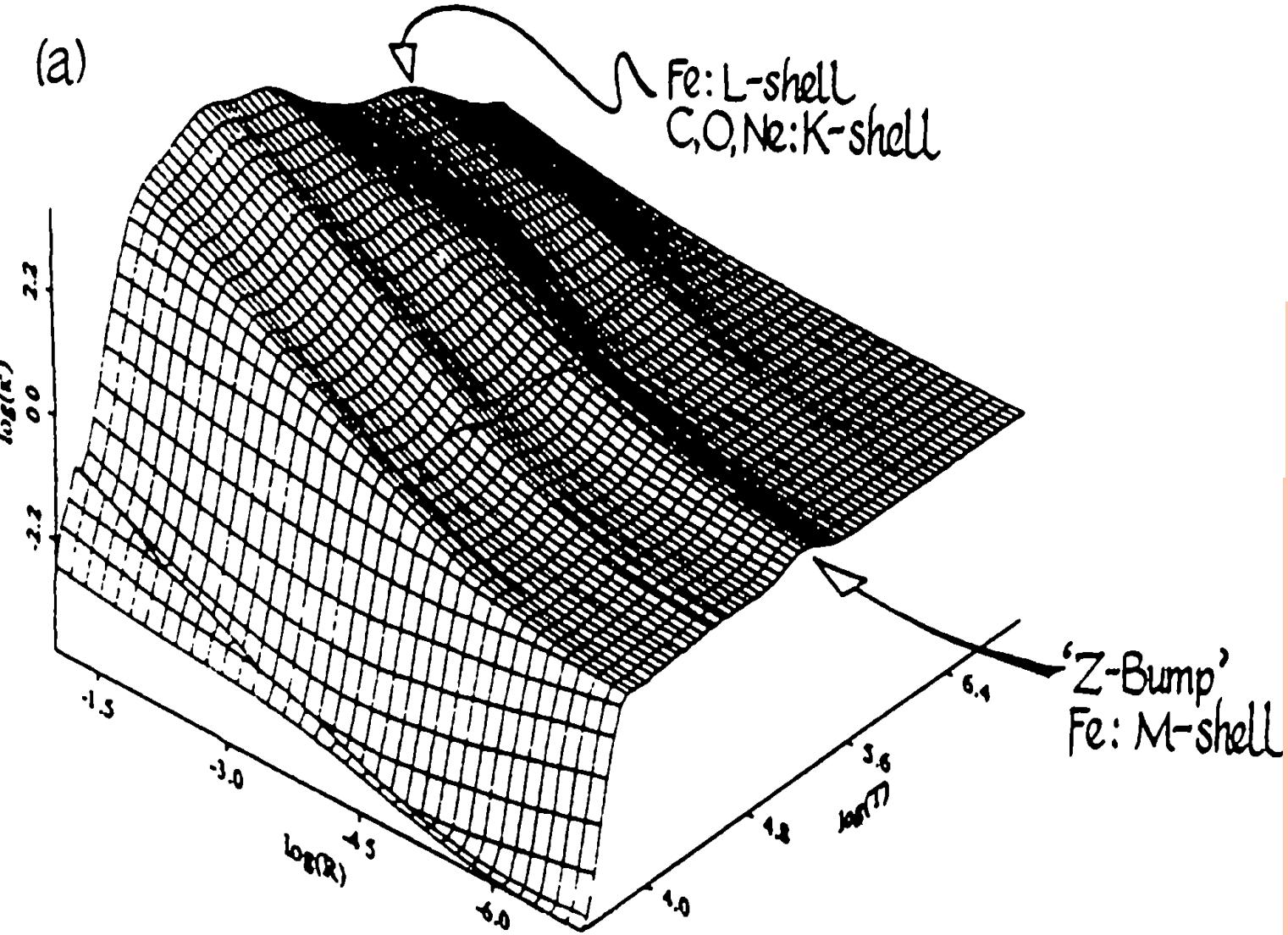
better understanding  
new solutions

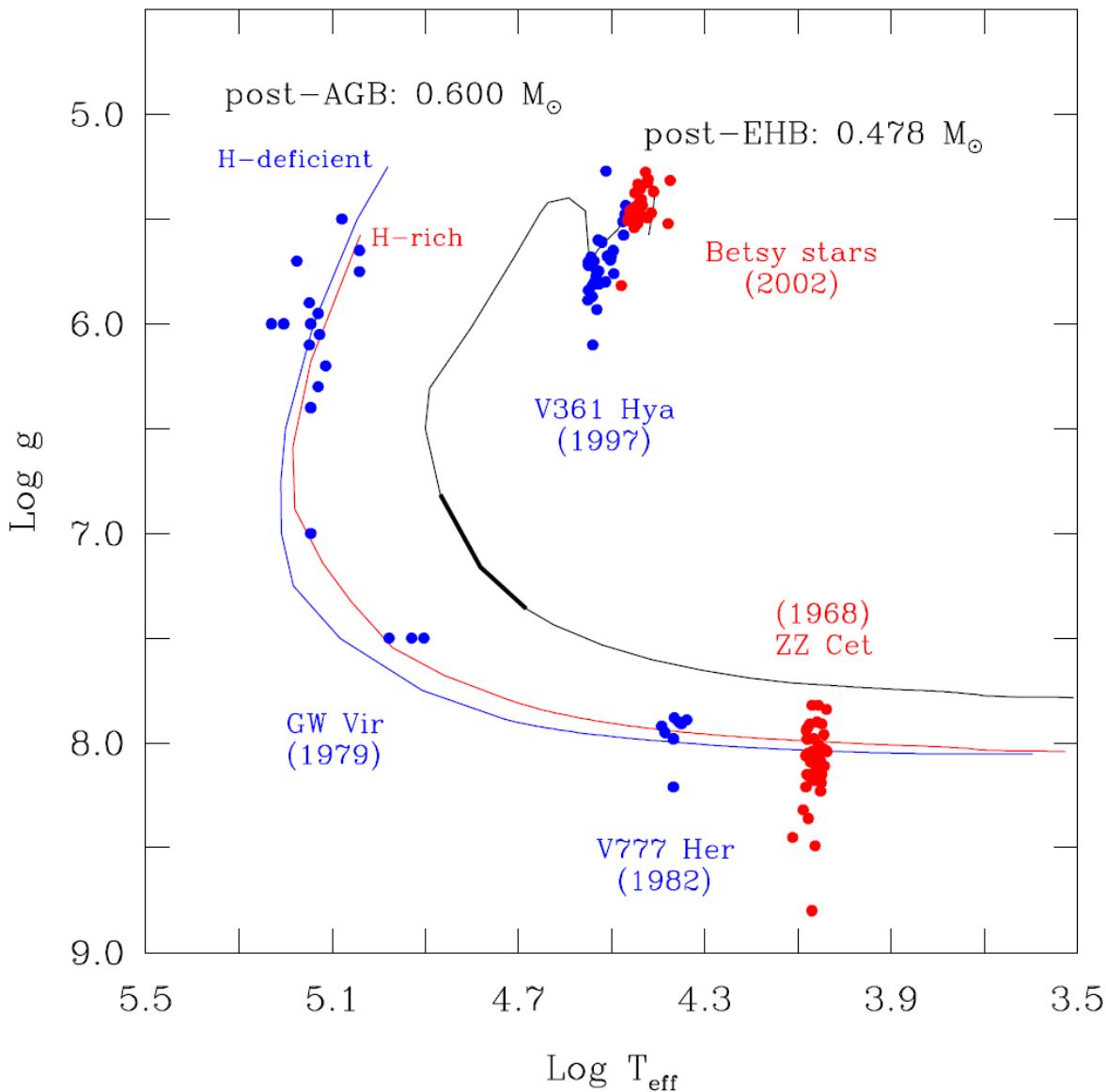
new challenges  
new problems

Precision asteroseismology







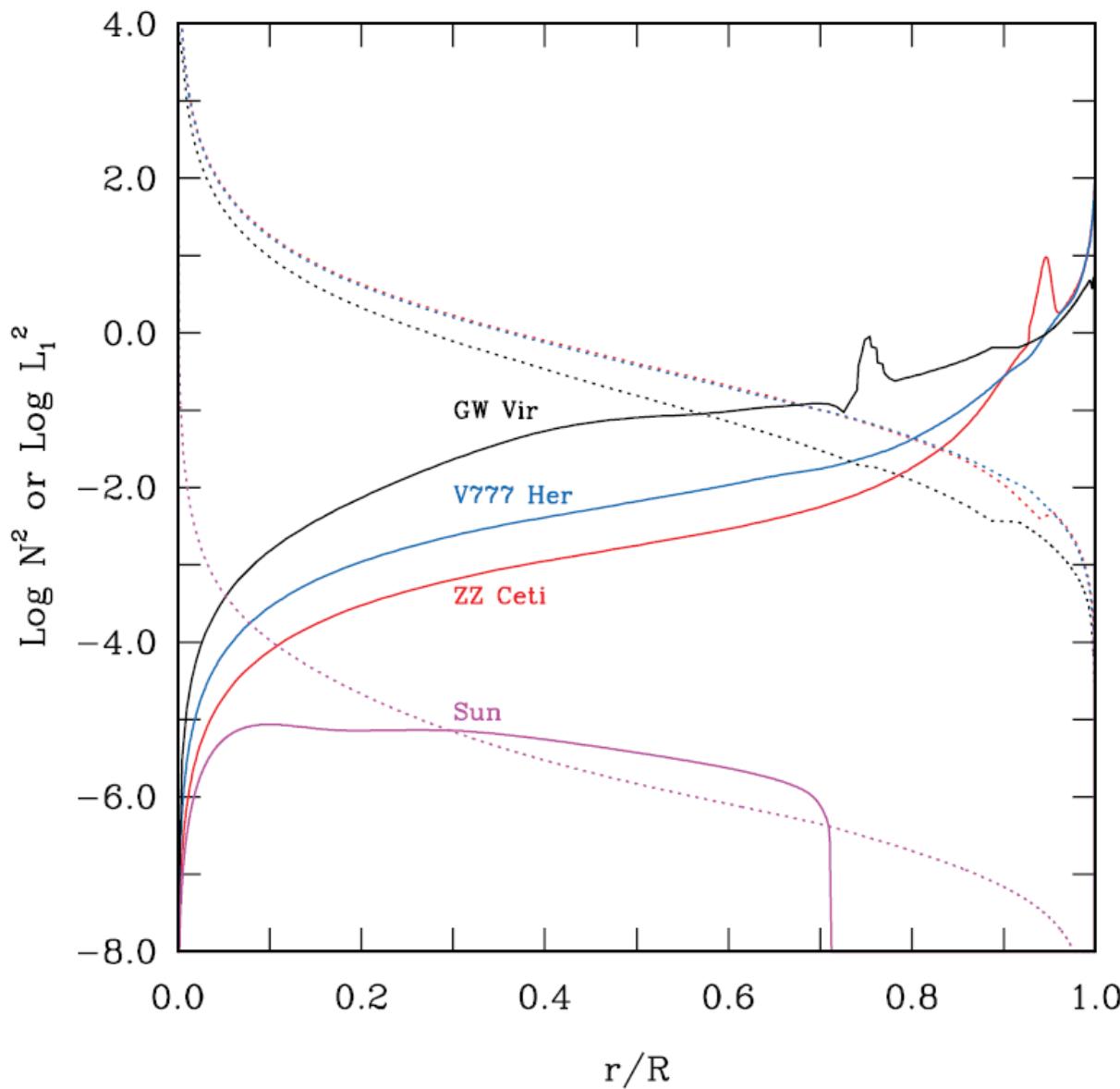


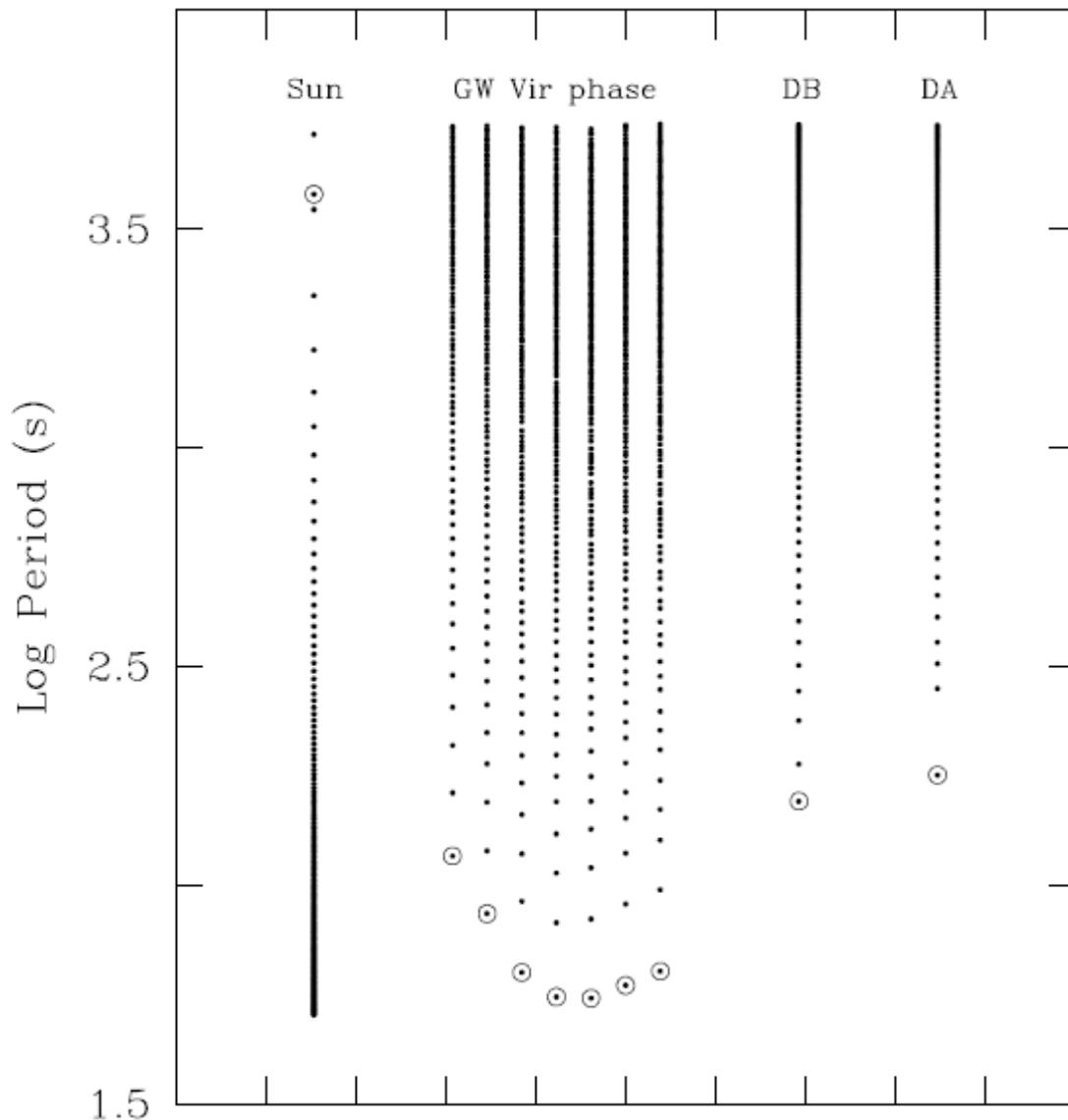
Mass (M <sub>⊕</sub> )	P	Excit. mechan.	Comments
0.10 - 0.40	4 - 11 h	ε (D)	age $\lesssim$ 2 Myr
0.40 - 0.60	1 - 2 h	F-b	age $\leqslant$ 50 Myr
	1 - 3 h	F-b, ε (He <sup>3</sup> )	g-modes
0.20 - 0.30	20 - 30 min	ε (He <sup>3</sup> )	
0.35 - 0.60	20 - 60 min	F-b	

**75-80 % - post-AGB with thin hydrogen envelope**

**~20% - post-AGB without H (born-again scenario)**

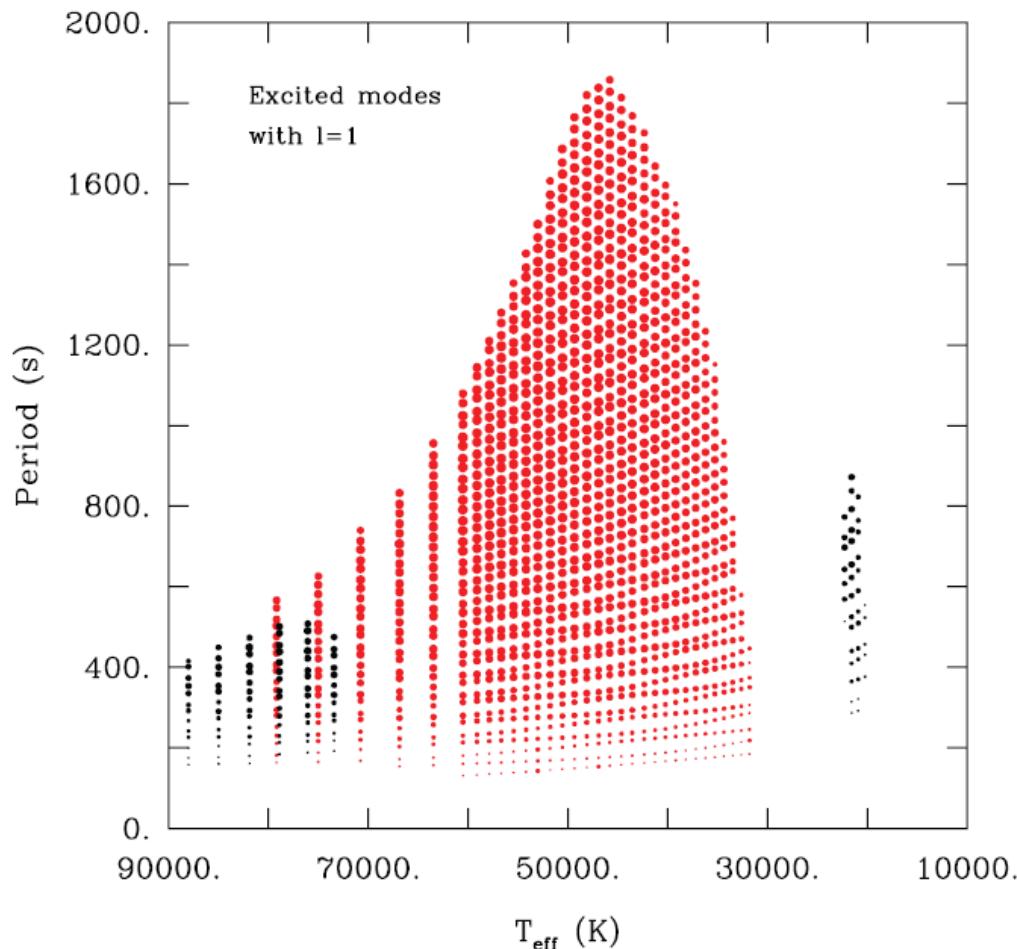
**2% - post-EHB, sdB**



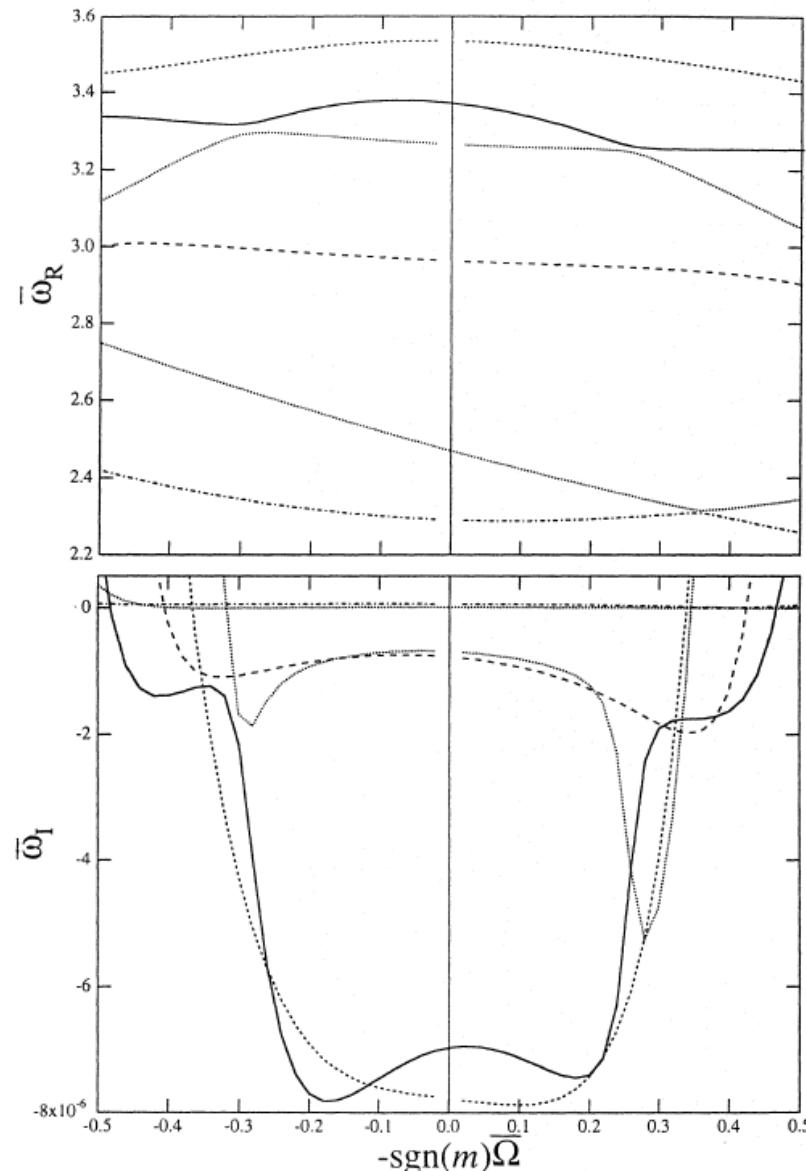


# Periods of unstable dipole modes in two post-PG 1159 evolutionary models:

**red** –  $M=0.6 M_{\odot}$ , envelope  $X(He)=0.38$ ,  $X(C)=0.40$ ,  $X(O)=0.20$ ,  $Z=0.02$   
**black**– diffusion and mass loss included

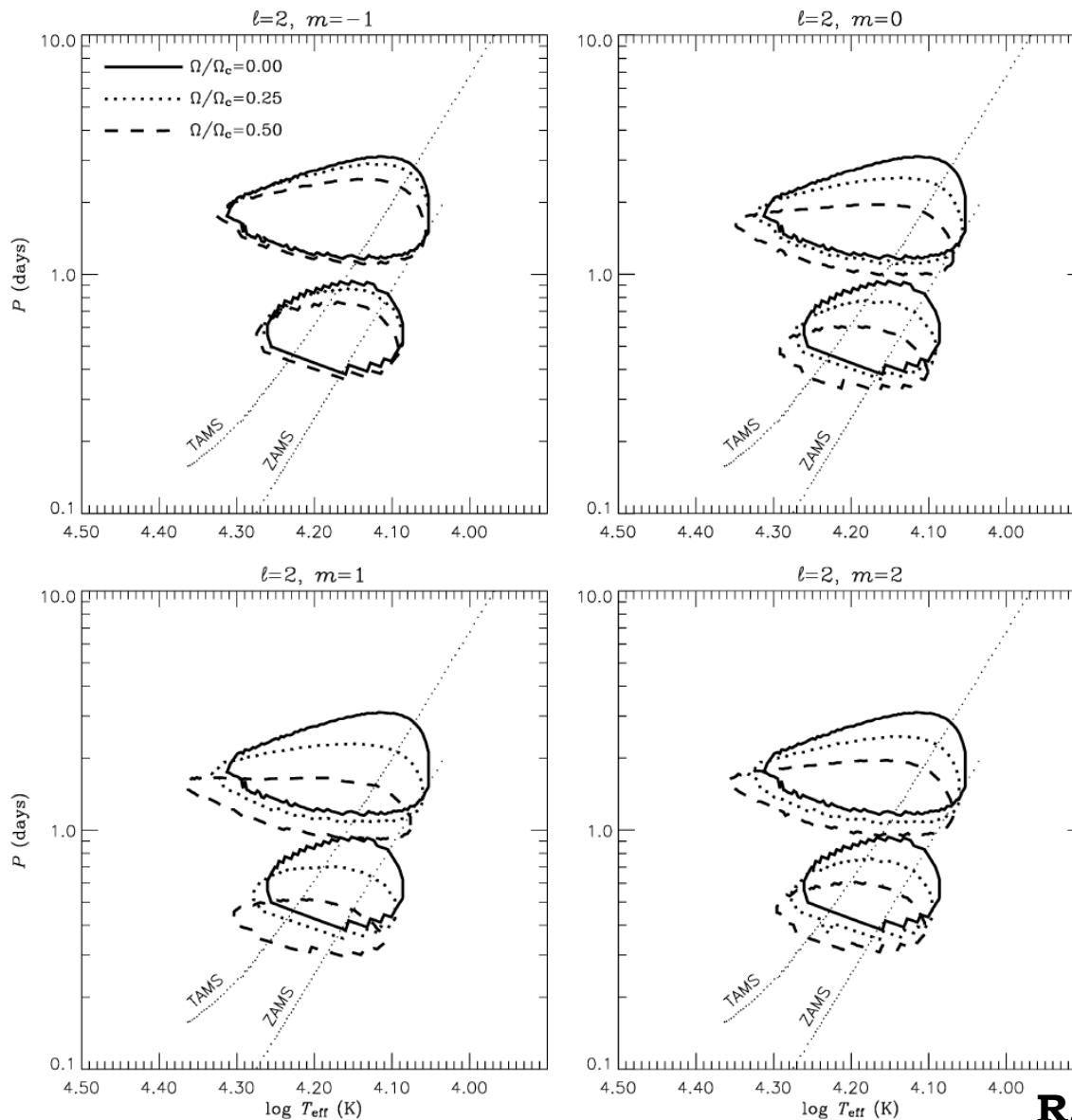


## Effects of rotation on frequencies of p-modes with $|m|=1$ for the $10 M_{\odot}$ star (MS)



U. Lee & I. Baraffe 1995

# Effect of the Coriolis force on the instability domains of SPB stars



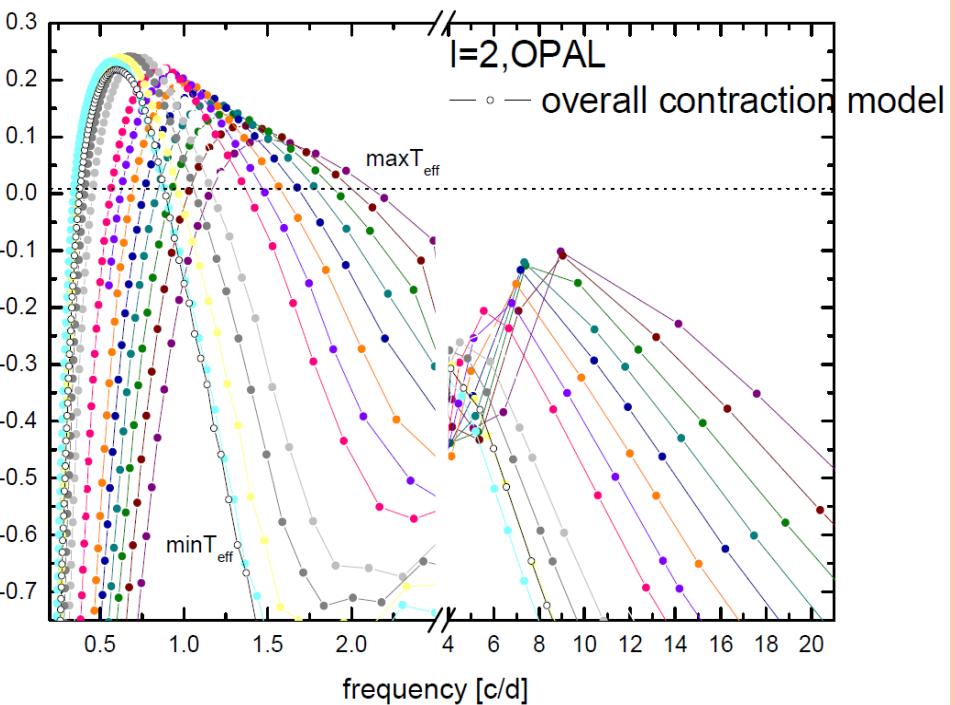
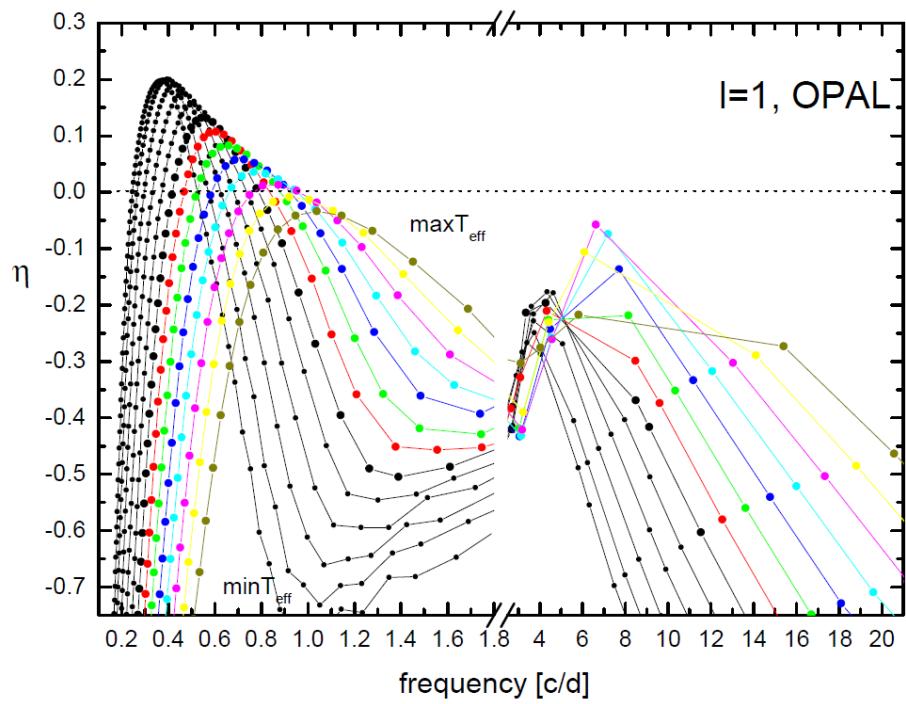
**Tempo zmian okresów jest bezpośrednim pomiarem czasu chłodzenia, który zależy od składu chemicznego zdegenerowanego jądra. Jest to bezpośredni test przewidywań teorii ewolucji.**

**Wyznaczenie wieku białych karłów wzdłuż ciągu chłodzenia jest metodą pomiaru wieku dysku galaktycznego w okolicach Słońca.**

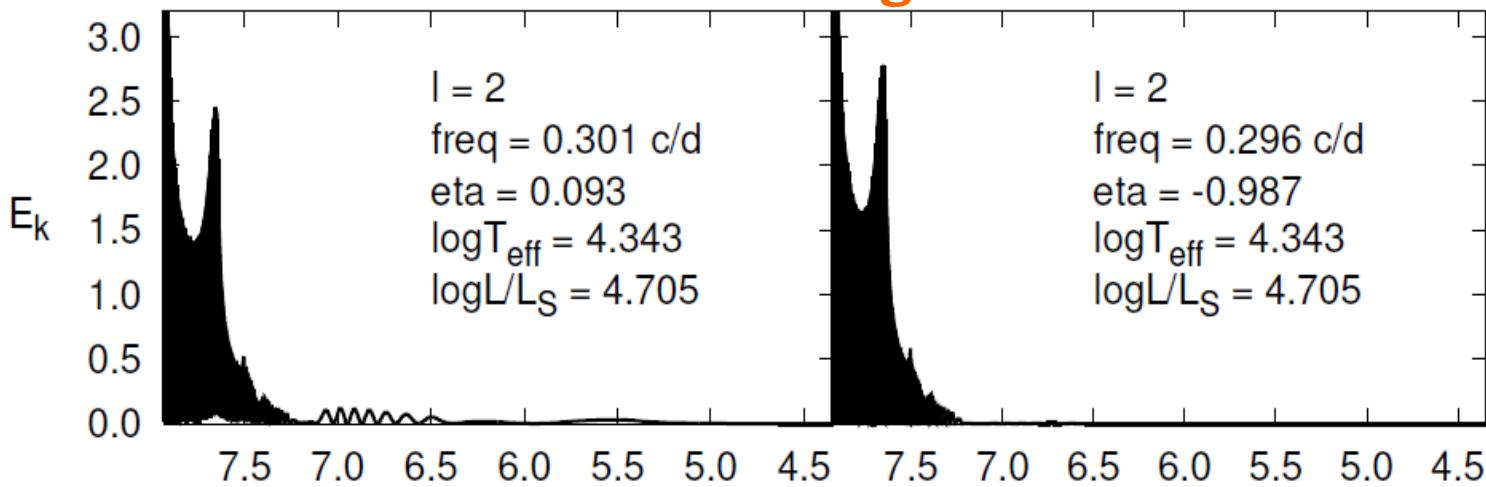
## *Co możemy otrzymać z asterosejsmologii białych karłów ?*

- ◆ *Całkowita masa z odstępów w okresach (reżim asymp.)*
- ◆ *Masa warstw zewnętrznych z odchyłek rozkładu okresów od regularnego, diagram  $\Delta P$  vs.  $P$*
- ◆ *Jasność gwiazdowa*
- ◆ *Okres rotacji z rozszczepienia modów*
- ◆ *Pole magnetyczne z rozszczepienia magnetycznego  
Pole magnetyczne rozszczepia mody na  $l+1$  składowych, a przesunięcie w częstotliwościach jest proporcjonalne do  $m^2$*
- ◆ *Ewolucyjną skalę czasową  
tempo zmian okresów → czas chłodzenia*

# $5 M_{\odot}$ OPAL



## MODEL 1 - H shell burning



## MODEL 2 - Core He burning

