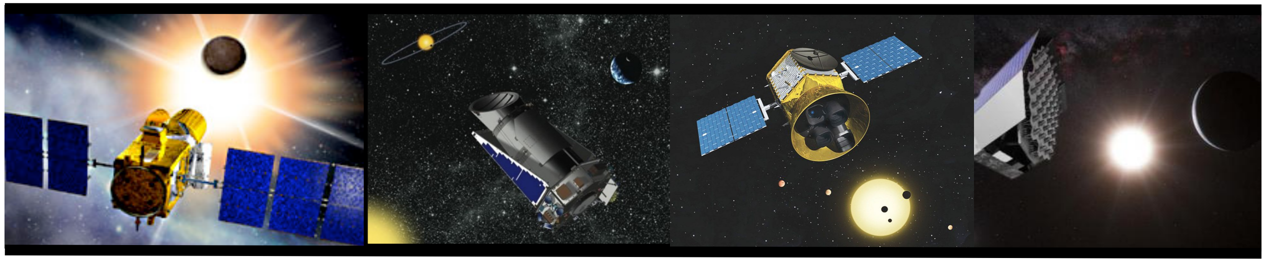
Tidal dissipation in stars and planetary fluid layers

S. Mathis, P. Auclair-Desrotour,
M. Guenel, C. Le Poncin-Lafitte

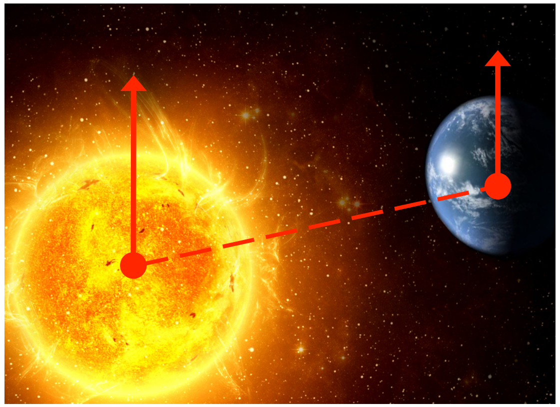
General context

The space photometry revolution: the discovery of new planetary systems and the characterisation of their host stars



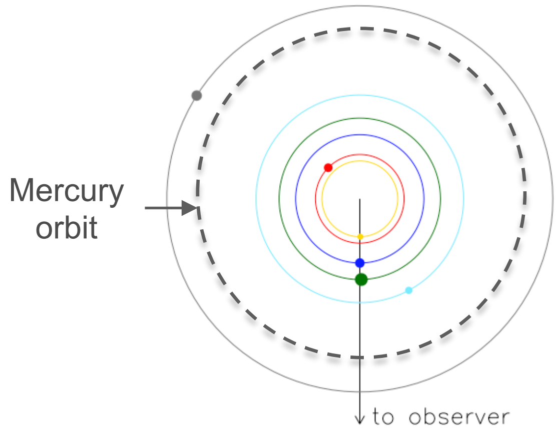
CoRoT *Kepler – K2* *CHEOPS & TESS* *PLATO*

Stellar and planetary rotation history



Albrecht et al. 2012; Gizon et al. 2013;
<u>Talks C. Damiani & T. Mazey</u>

Orbital architecture



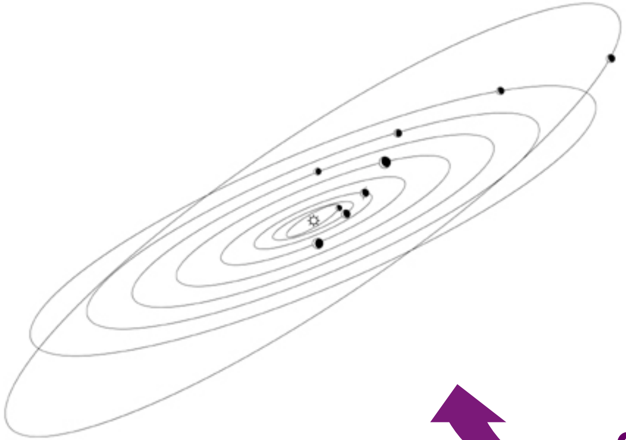
Kepler 11

Lissauer et al. 2011

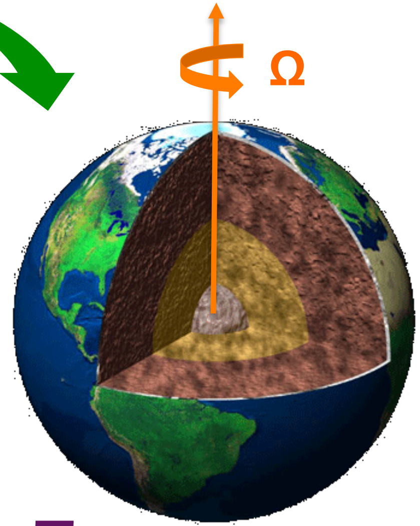
→ Need to understand angular momentum exchanges within star-planet systems → TIDES

Tidal dissipation

Internal velocity field



Tides



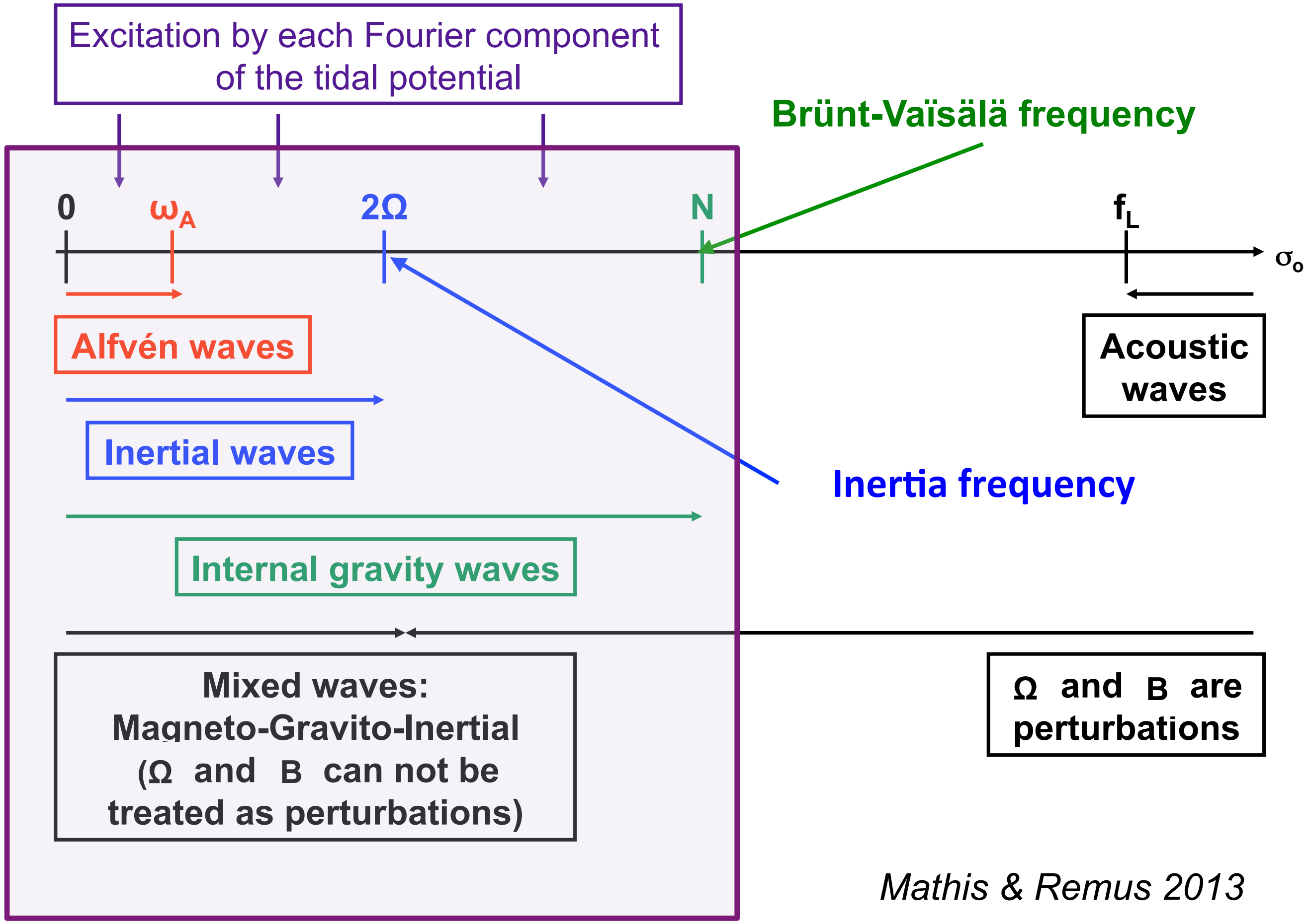
Tidal
quality factor Q
α D^{-1}

$$\frac{da}{dt} = -\frac{3k_2 R_A^5 G n M_B}{M_A a^4} Q^{-1}(\omega)\,\mathrm{sgn}(\omega)$$

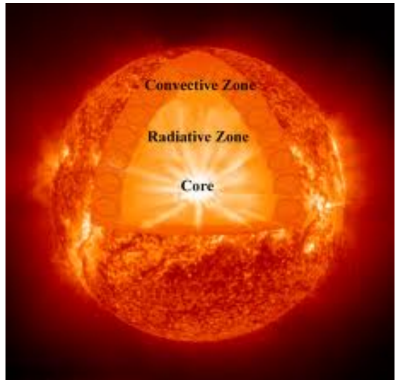
Energy dissipation
→ Dynamics of spin & orbits

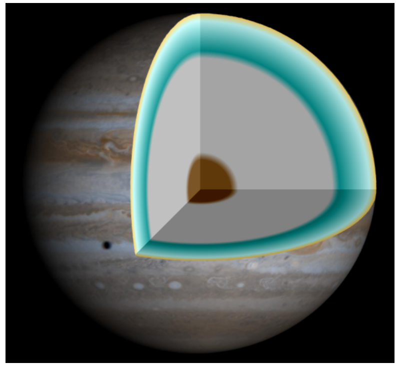
Spin/orbit | Tidal dissipation | Internal structure

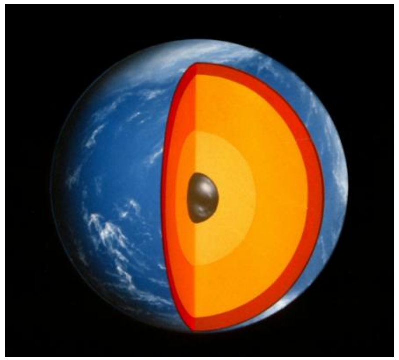
Tidal waves in stars and fluid planetary layers



Mathis & Remus 2013



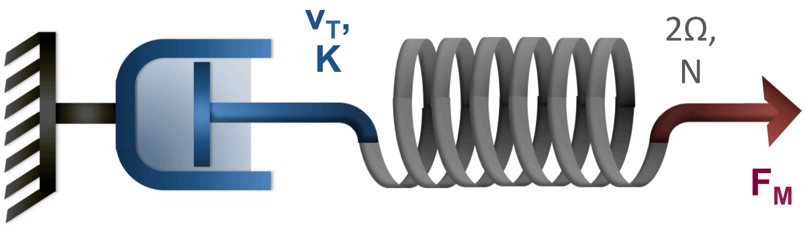


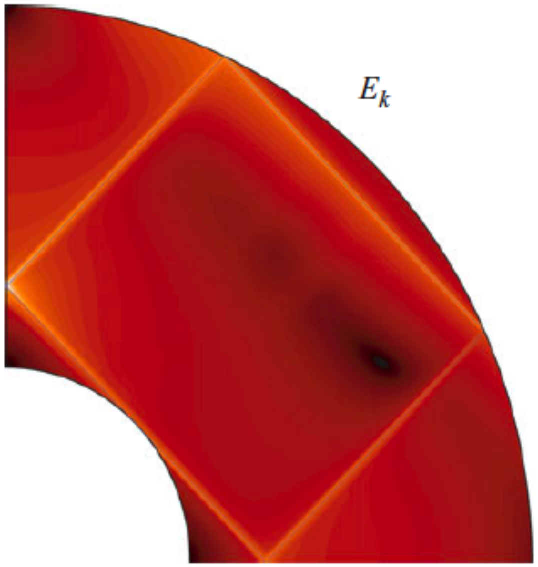


- **Convective layers:** turbulent friction & thermal diffusion
- **Stable layers:** thermal diffusion

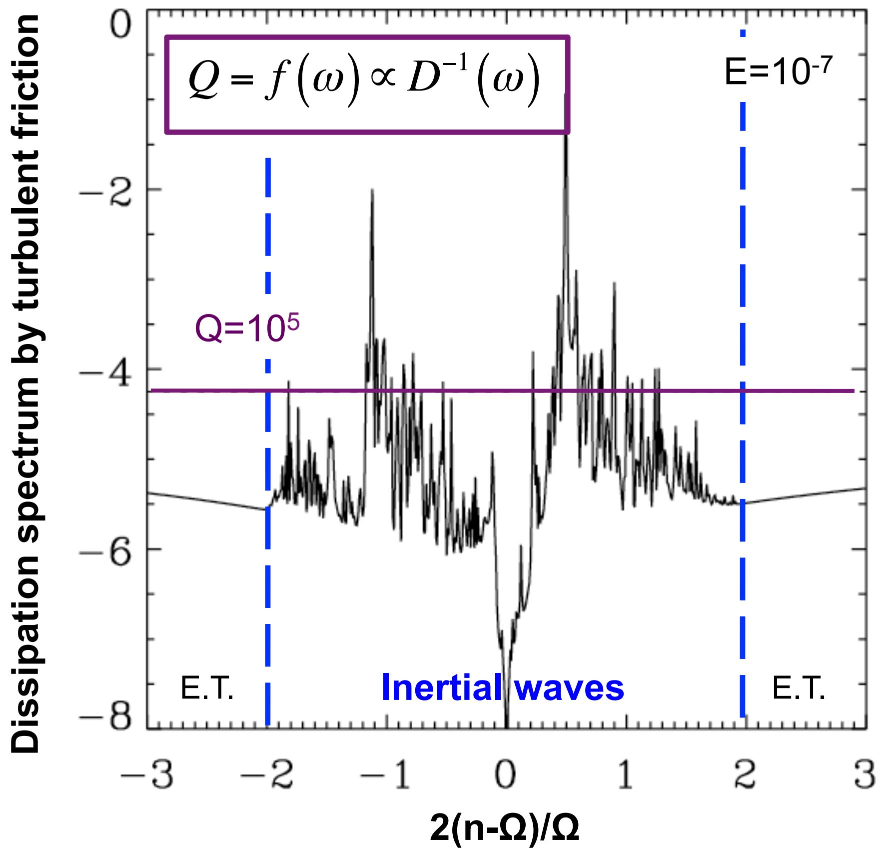
A resonant erratic tidal dissipation spectrum

Forced (gravito-) inertial waves
→ resonant response





$Nr = 400\ L = 1200\ M = 0^+\ E = 2.0 \times 10^{-9}$
$\eta = 0.350\ \ \omega = 0.707\ CL = ff$



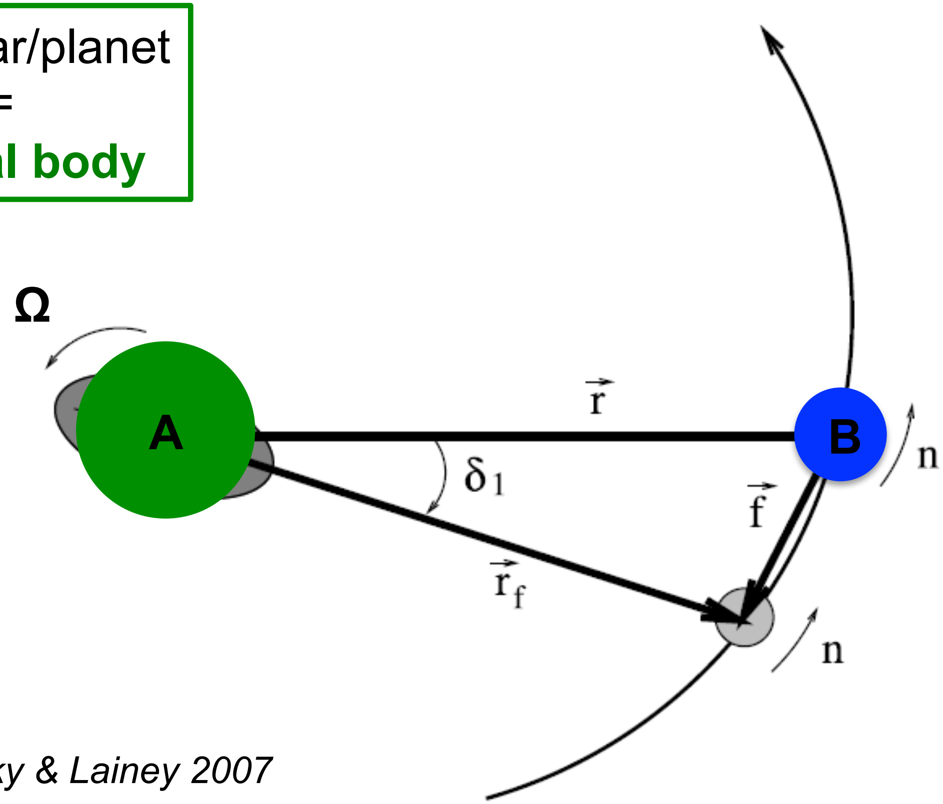
Ogilvie & Lin 2004: the case of Jupiter

Rieutord & Valdetarro 2010
See also *Ogilvie & Lin 2007*

The impact of tidal dissipation on the spin dynamics and on systems orbital architecture

The coplanar two-bodies system:

Host star/planet
=
Central body



Punctual planet/satellite
=
Perturber

Tidal frequency: $\chi = 2(n - \Omega)$

Efroimsky & Lainey 2007

The impact of tidal dissipation: the case of rocky bodies

An example: the Mars-Phobos system

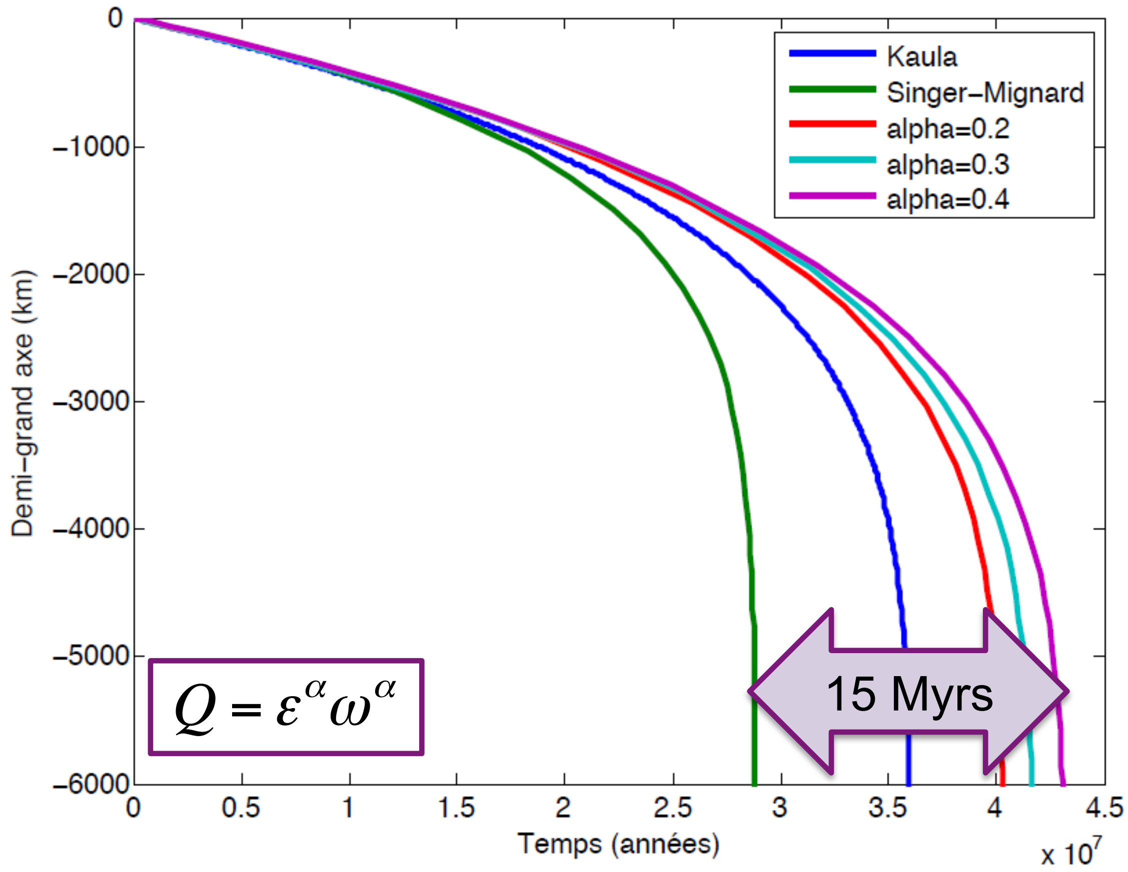
Regular evolution

Internal dissipation

↓

Rheology: ε & α

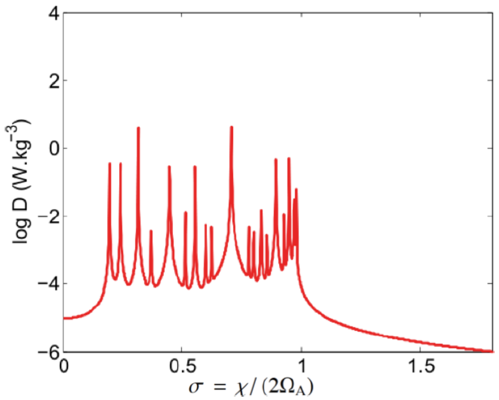
Efroimsky & Lainey 2007



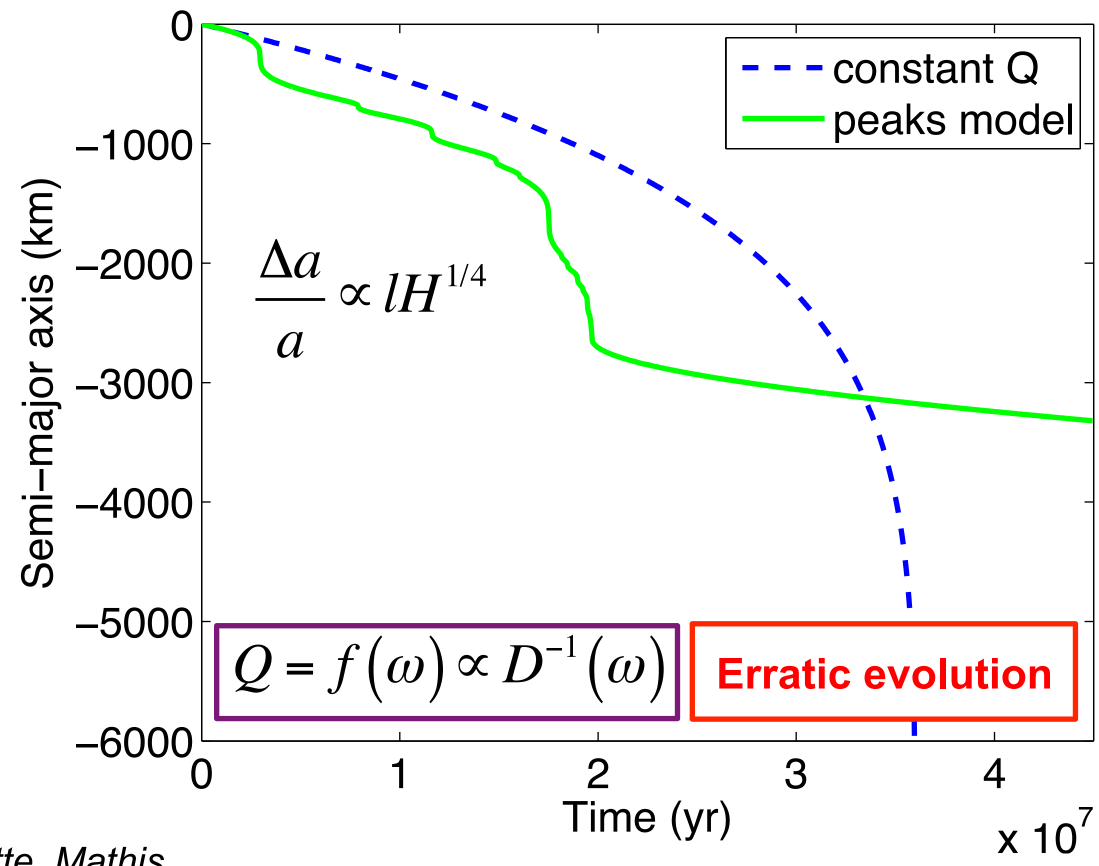
The impact of tidal dissipation: the case of fluid bodies

An example: a fully convective body of the mass of Mars-Phobos system

Internal dissipation

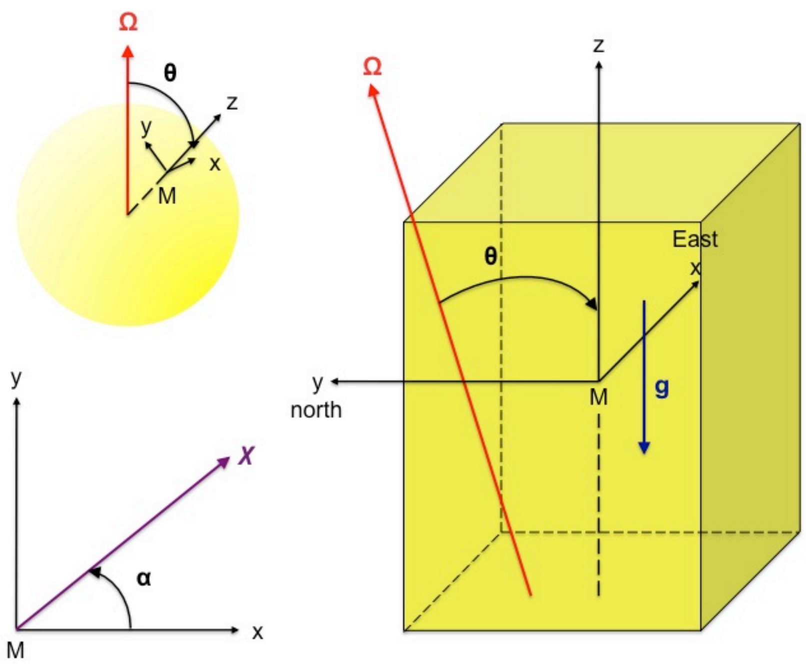


Fluid parameters



Auclair-Desrotour, Le Poncin-Lafitte, Mathis 2014a

A reduced local model to understand tidal dissipation in fluids



Ogilvie & Lin 2004

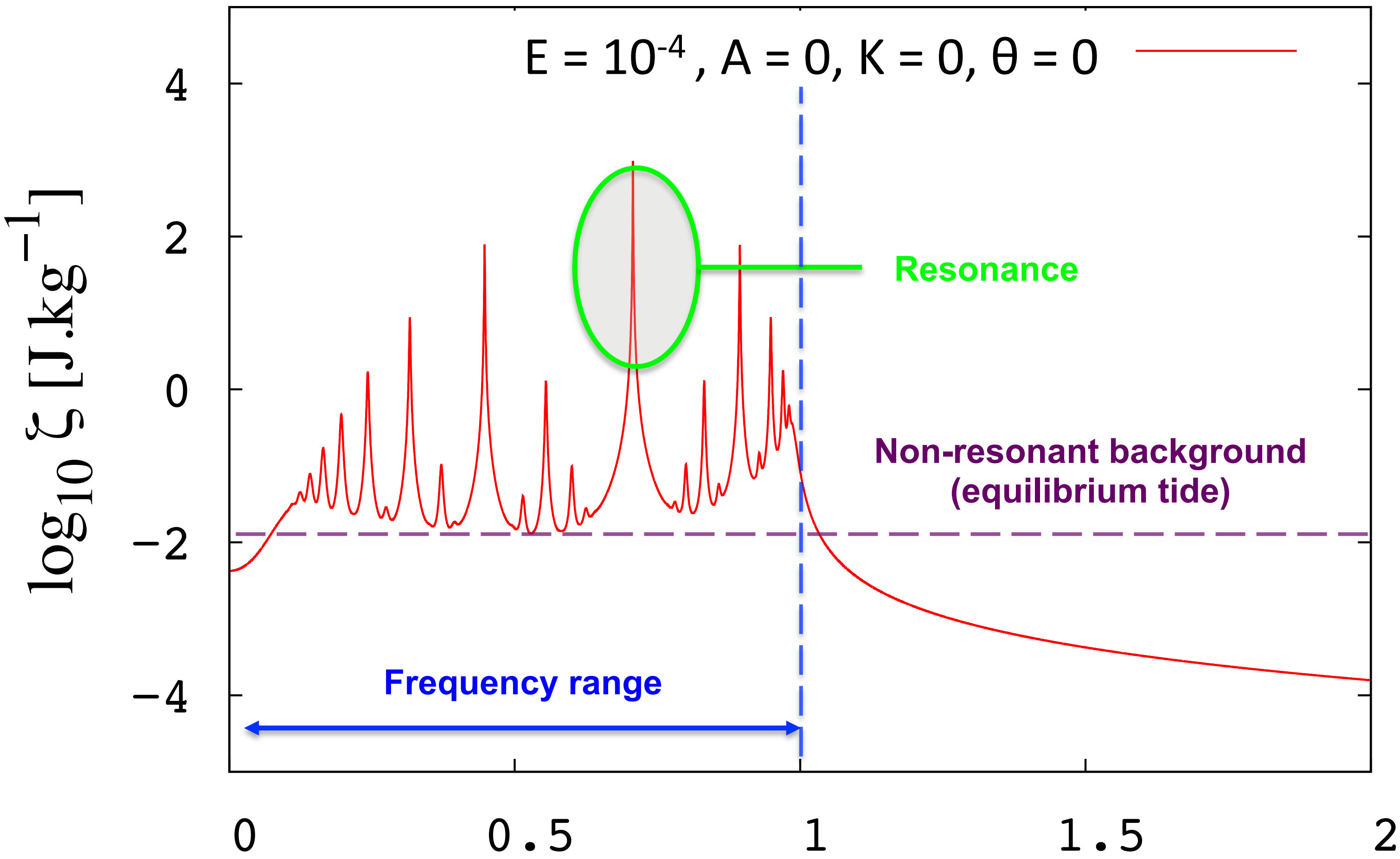
Auclair-Desrotour, Le Poncin-Lafitte, Mathis 2014b

- Cartesian geometry
- Rotating and inclined
- Possible stable stratification
- Viscous and thermal dissipation

Control parameters:

$A = \left(\frac{N}{2\Omega}\right)^2$	Stratification / Coriolis
$E = \frac{2\pi^2 \nu}{\Omega L^2}$	Viscous force / Coriolis
$K = \frac{2\pi^2 \kappa}{\Omega L^2}$	Thermal diffusivity / Coriolis

The complex erratic tidal dissipation spectrum

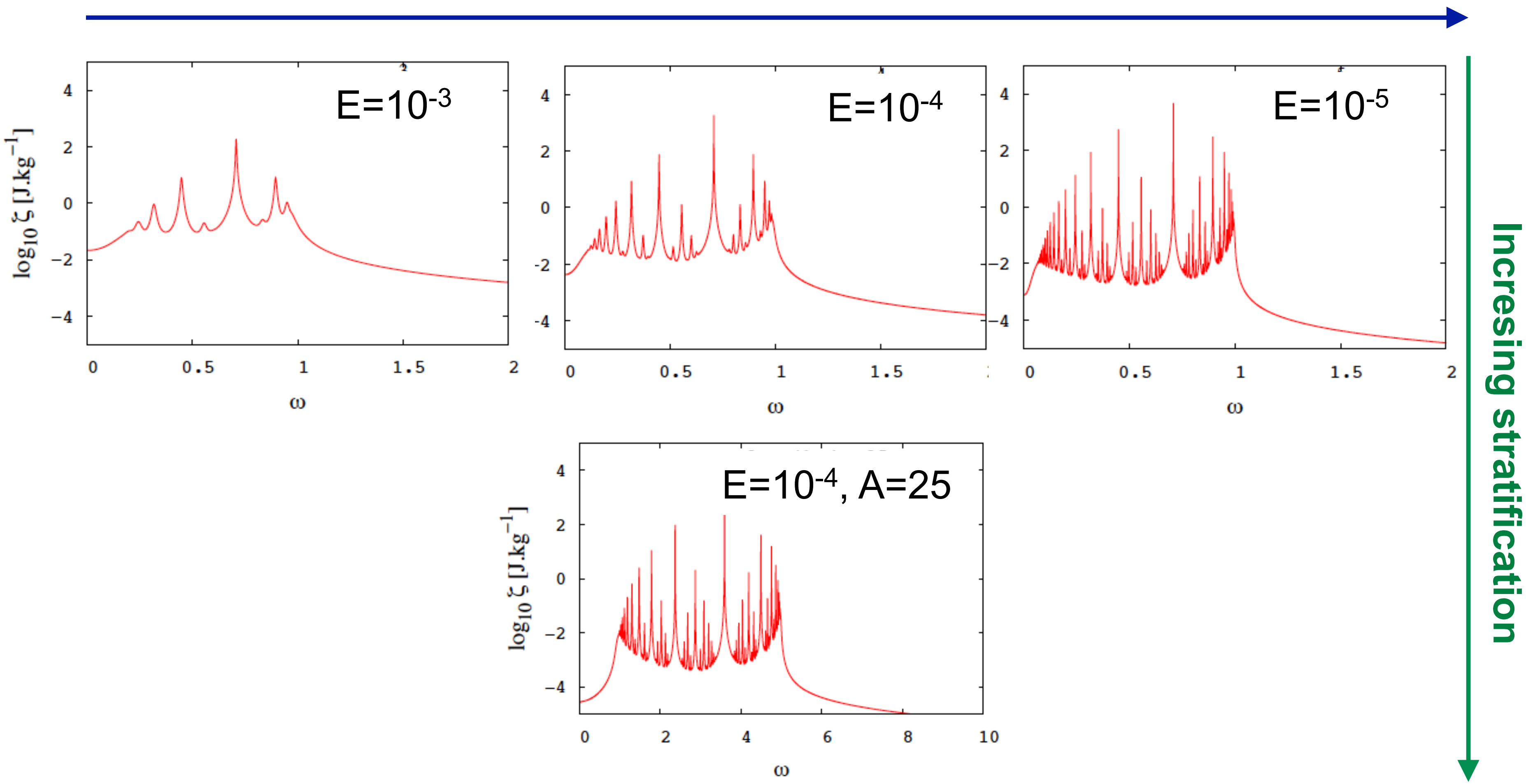


$$\omega = \frac{\chi}{2\Omega}$$

→ **Need to characterize spectra**

An evolving behaviour

Deacrising viscosity / increasing rotation

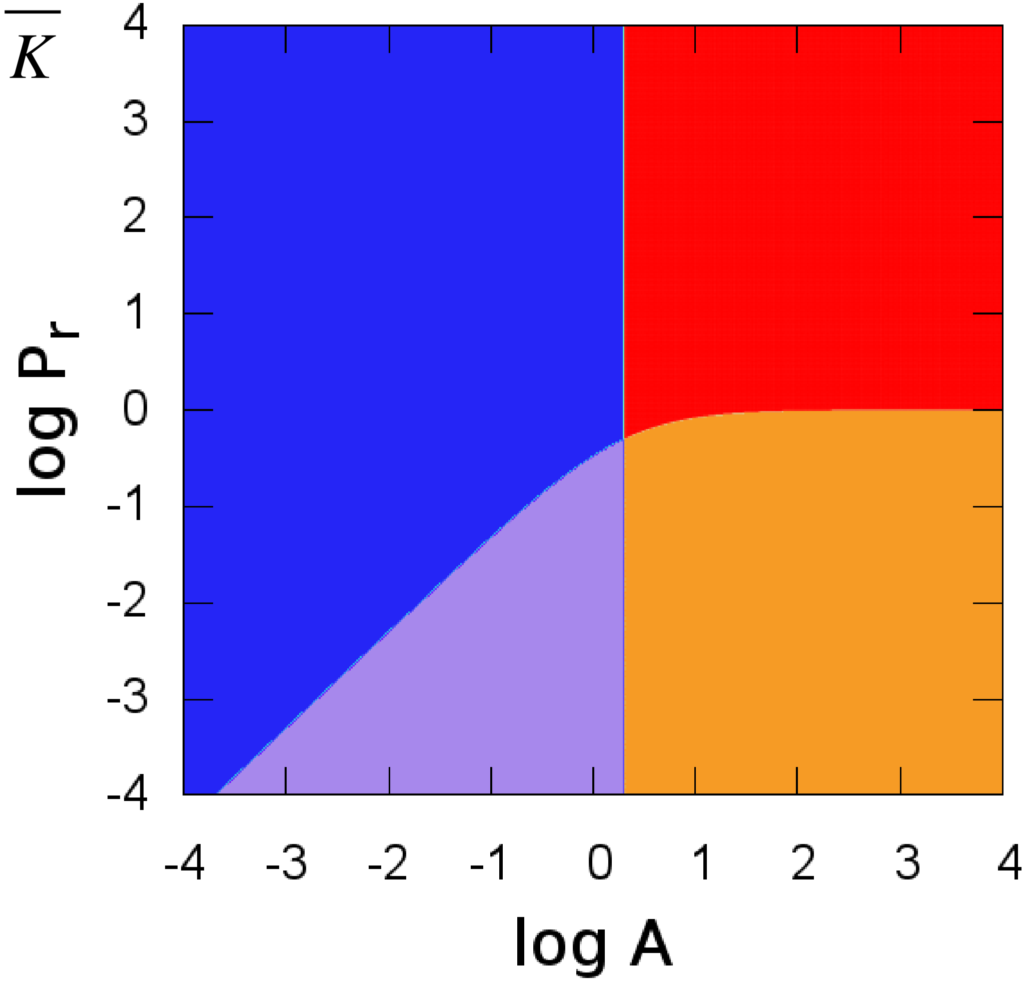


The four main regimes

$$\mathrm{Pr} = \frac{E}{K}$$

Viscously driven dissipation

Inertial waves *CZ*



Gravito-inertial waves *Stable Zone*

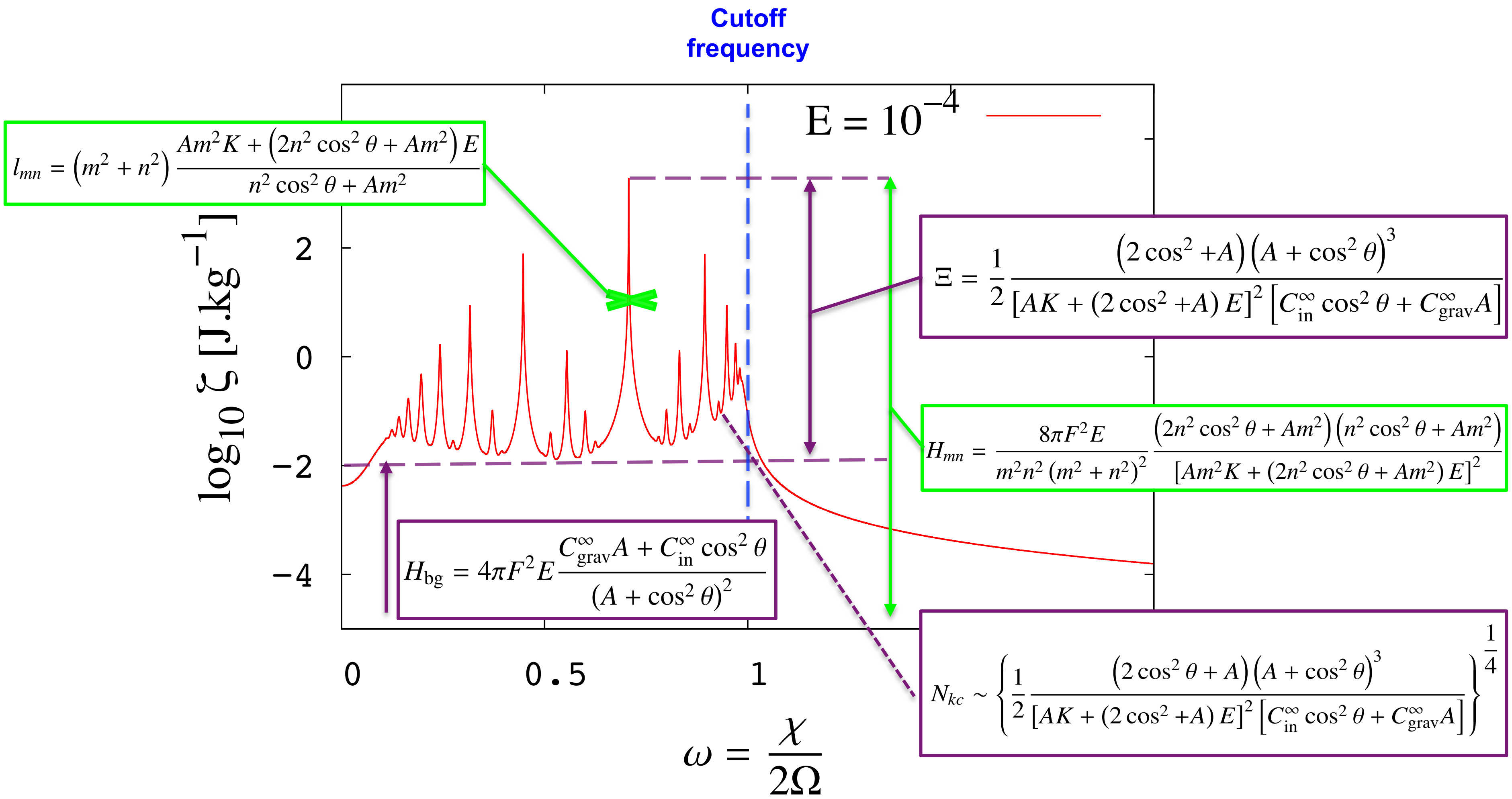
$$A_{mn}(\theta) = \frac{2n^2}{m^2}\cos^2\theta$$

$$Pr_{mn}(\theta, A) = \frac{A}{A + A_{mn}(\theta)}$$

$$A = \left(\frac{N}{2\Omega}\right)^2$$

Thermal diffusion driven dissipation

A complete characterization

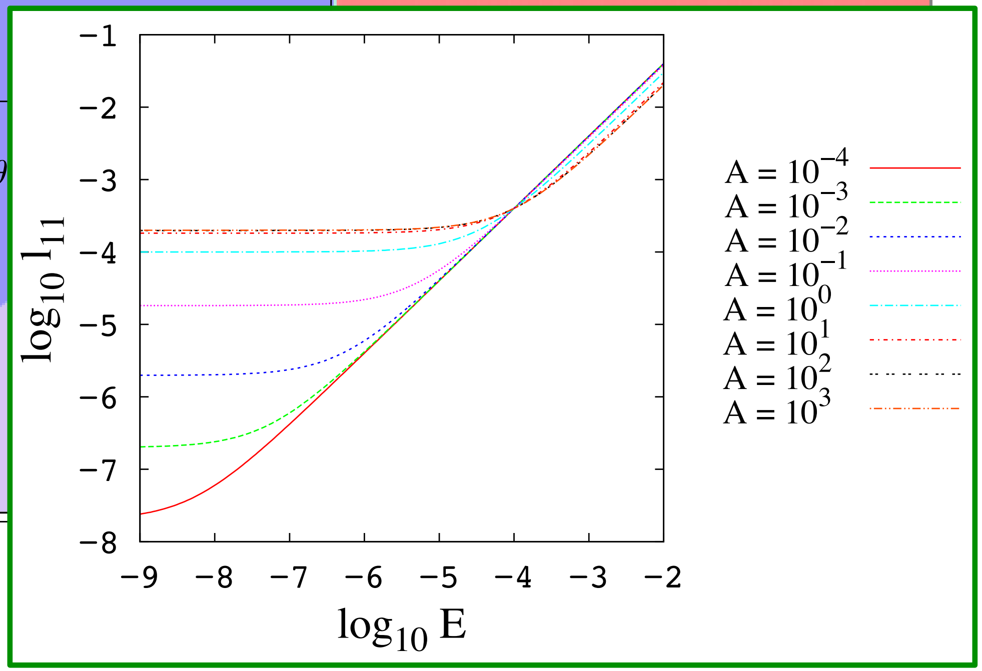


Asymptotic scaling laws

Domain	$A \ll A_{11}$		$A \gg A_{11}$	
$Pr \gg Pr_{11}$	$\omega_{mn} \sim \frac{n}{\sqrt{m^2+n^2}} \cos\theta$		$\omega_{mn} \sim \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$	
	$l_{mn} \sim E$	$k_c \sim E^{-1/4}$	$l_{mn} \sim E$	$k_c \sim A^{1/8} E^{-1/4}$
	$H_{mn} \sim F^2 E^{-1}$	$N_{kc} \sim E^{-1/2}$	$H_{mn} \sim F^2 E^{-1}$	$N_{kc} \sim A^{1/4} E^{-1/2}$
	$H_{bg} \sim F^2 E$	$\Xi \sim E^{-2}$	$H_{bg} \sim F^2 E A^{-1}$	$\Xi \sim A E^{-2}$
$Pr \ll Pr_{11}$	$\omega_{mn} \sim \frac{n}{\sqrt{m^2+n^2}} \cos\theta$		$\omega_{mn} \sim \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$	
	$l_{mn} \sim AK$	$k_c \sim A^{-1/4} K^{-1/4}$	$l_{mn} \sim K$	$k_c \sim A^{1/8} K^{-1/4}$
	$H_{mn} \sim F^2 A^{-2} E K^{-2}$	$N_{kc} \sim A^{-1/2} K^{-1/2}$	$H_{mn} \sim F^2 E K^{-2}$	$N_{kc} \sim A^{1/4} K^{-1/2}$
	$H_{bg} \sim F^2 E$	$\Xi \sim A^{-2} K^{-2}$	$H_{bg} \sim F^2 E A^{-1}$	$\Xi \sim A K^{-2}$

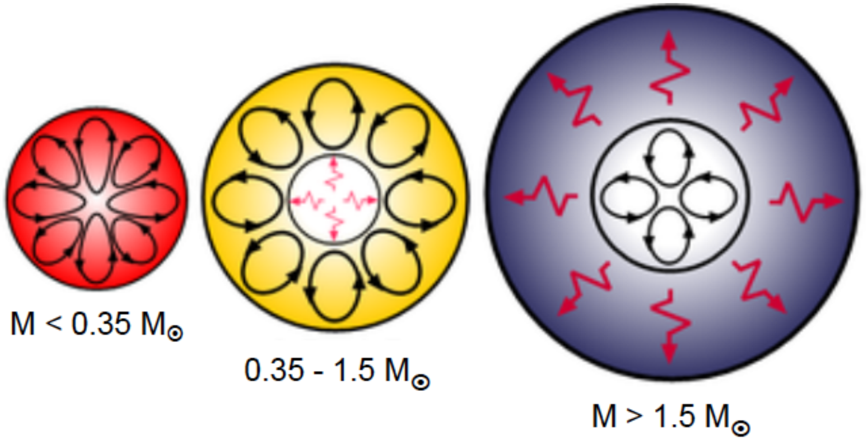
Asymptotic scaling laws

Domain	$A \ll A_{11}$	$A \gg A_{11}$
$Pr \gg Pr_{11}$	$\omega_{mn} \sim \frac{n}{\sqrt{m^2+n^2}} \cos\theta$ $l_{mn} \sim E$ $\quad k_c \sim E^{-1/4}$ $H_{mn} \sim F^2 E^{-1}$ $\quad N_{\rm kc} \sim E^{-1/2}$ $H_{\rm bg} \sim F^2 E$	$\omega_{mn} \sim \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$ $l_{mn} \sim E$ $\quad k_c \sim A^{1/8} E^{-1/4}$ $H_{mn} \sim F^2 E^{-1}$ $\quad N_{\rm kc} \sim A^{1/4} E^{-1/2}$
$Pr \ll Pr_{11}$	$\omega_{mn} \sim \frac{n}{\sqrt{m^2+n^2}} \cos\theta$ $l_{mn} \sim AK$ $H_{mn} \sim F^2 A^{-2} E K^{-2}$ $H_{\rm bg} \sim F^2 E$	

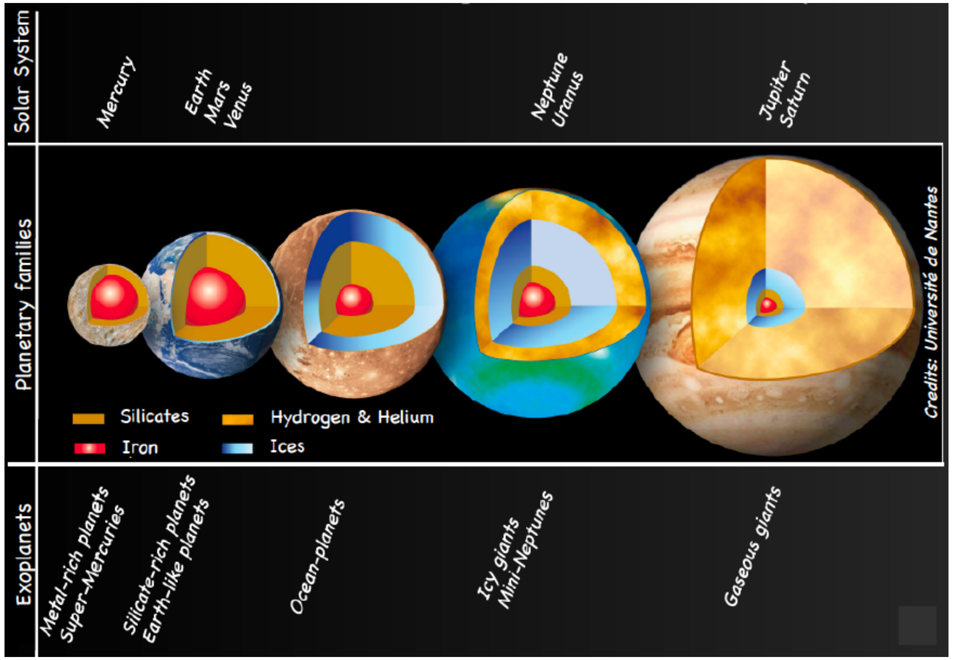


Towards global and multi-layer models

Host star (M in $M_\odot$)



Planets



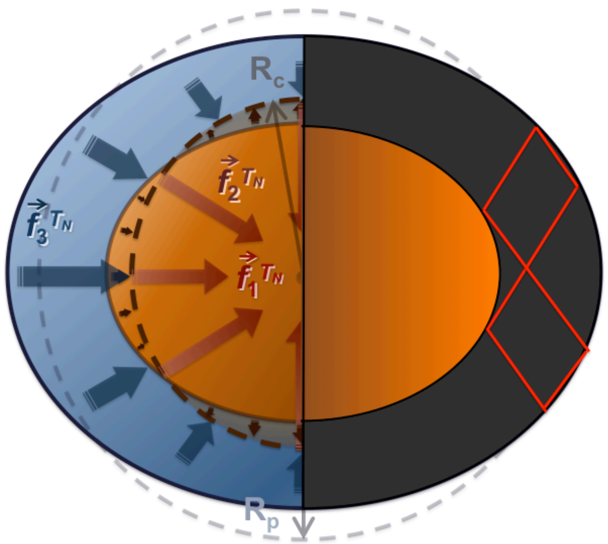
Their complex internal structure and rotation impact tidal dissipation
→ Need of an ab-initio physical modeling

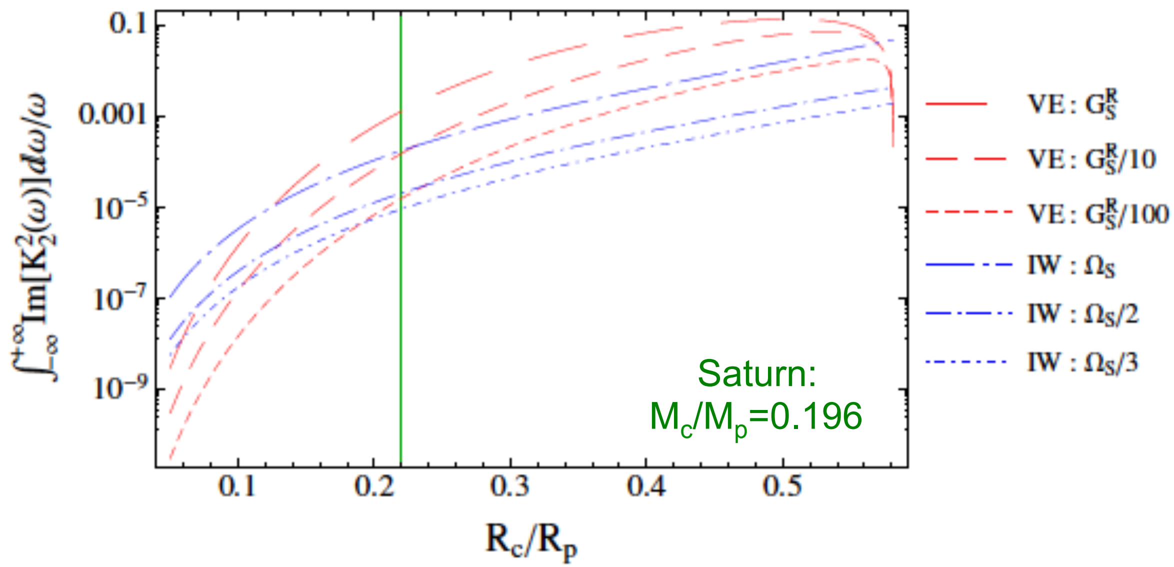
Frequency-averaged models

The example of a Saturn-like planet:

Remus, Mathis, Zahn & Lainey 2012

Ogilvie 2009, 2013





→ Integrated models needed for gaseous giant (and telluric) planets

→ Possibility of frequency-averaged grids as a function of stellar and planetary properties

Guenel, Mathis & Remus 2014

Conclusions & perspectives

- Dependence of the spin/orbital dynamics on the resonant tidal fluid dissipation :

  → *width, height, non-resonant background level*
- Dependence of the characteristics of these resonances on the physical parameters of the fluid :

  → *rotation, stratification, viscosity, thermal diffusivity, etc.*
- Local model : general method and qualitative results

  → *Need of global models (Guenel, Baruteau, Mathis & Rieutord; Ogilvie et al.); need to characterize the case of stratified convection (I. Baraffe's talk)*
- Generalization to magnetic stars and planets :

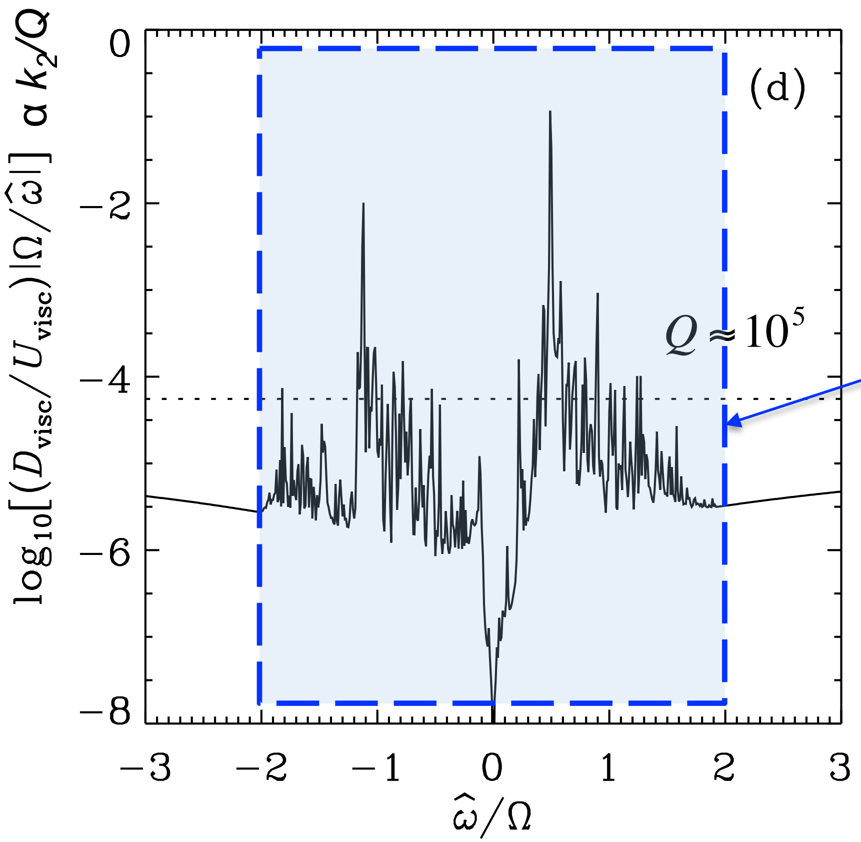
  → *Alfvén waves; new asymptotic behaviors (Mathis, Auclair-Desrotour, Guenel, Le Poncin-Lafitte)*

Spin/orbit — **Tidal dissipation** — **Internal structure**

APPENDIX

Tidal dissipation
in stars and fluid planetary layers

→ A resonant erratic tidal dissipation spectrum



Ogilvie & Lin 2004: the case of Jupiter
See also *Ogilvie & Lin 2007*; *Rieutord & Valdetarro 2010*

Tidal inertial waves (Coriolis acceleration)

$$Q = f(\omega) \propto D^{-1}(\omega)$$

II. What is the impact of tidal dissipation on the orbital evolution ?

- Orbital dynamics

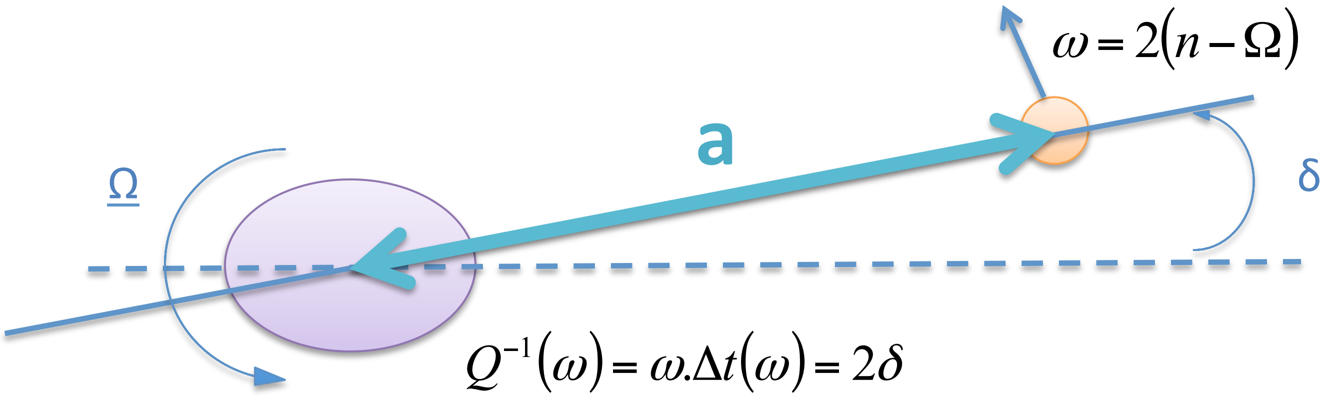
$$\frac{da}{dt} = -\frac{3k_2 R_A^5 G n M_B}{M_A a^4} Q^{-1}(\omega)\,\mathrm{sgn}(\omega)$$

Relative motion

Dissipation

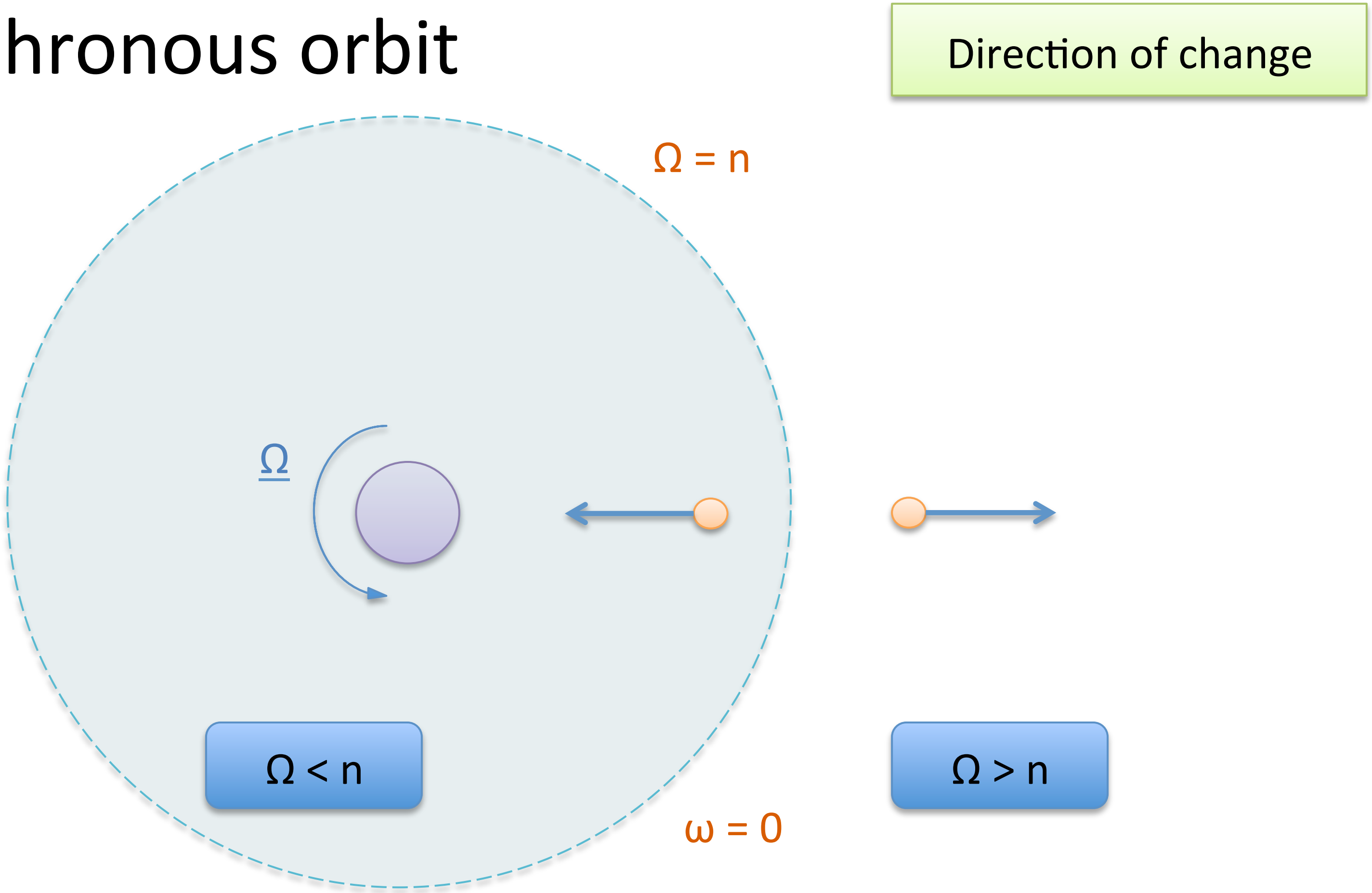
Direction of change

Velocity of change



II. What is the impact of tidal dissipation on the orbital evolution ?

- The synchronous orbit



II. What is the impact of tidal dissipation on the orbital evolution ?

- What about fluid bodies ?

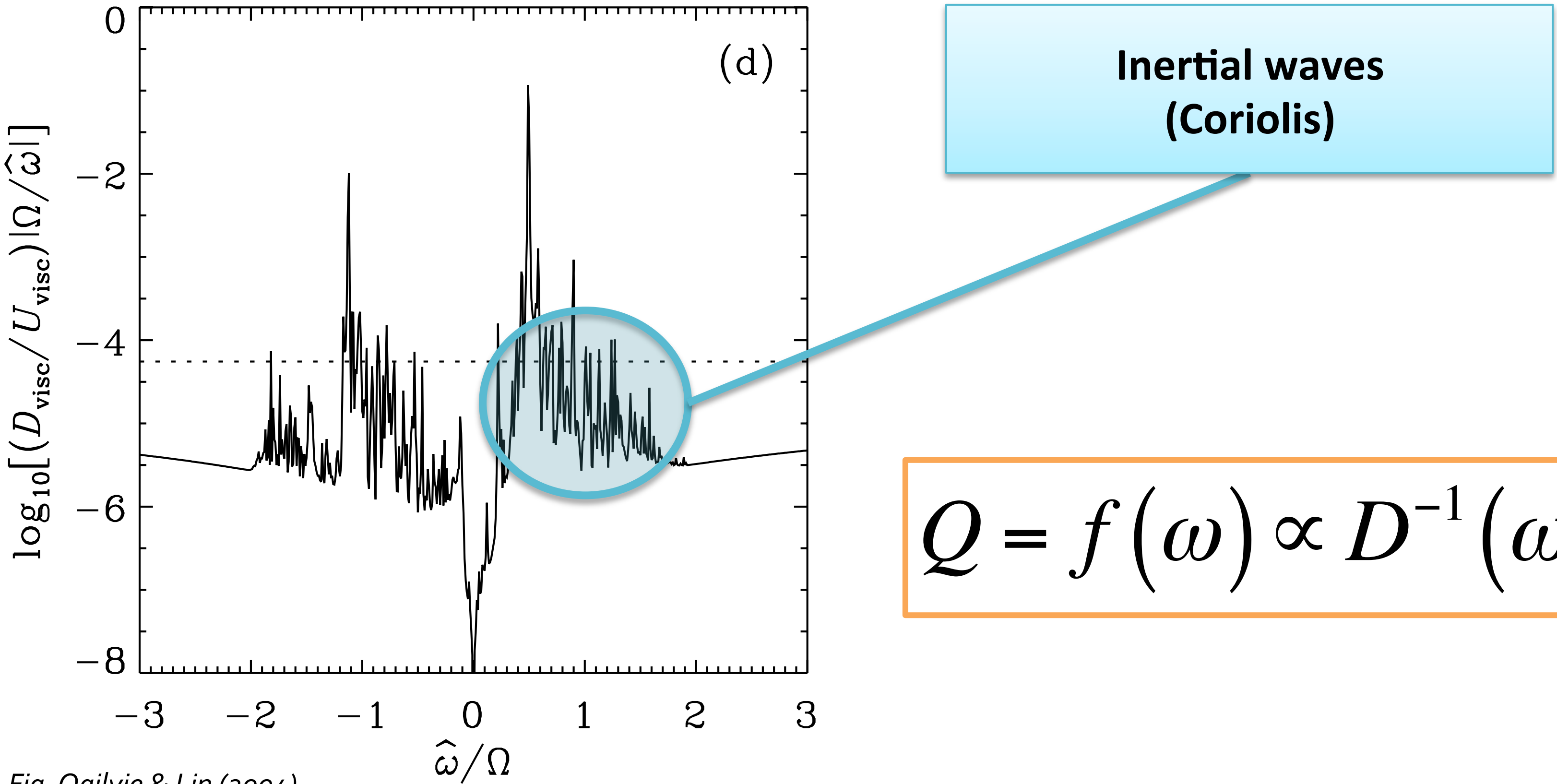
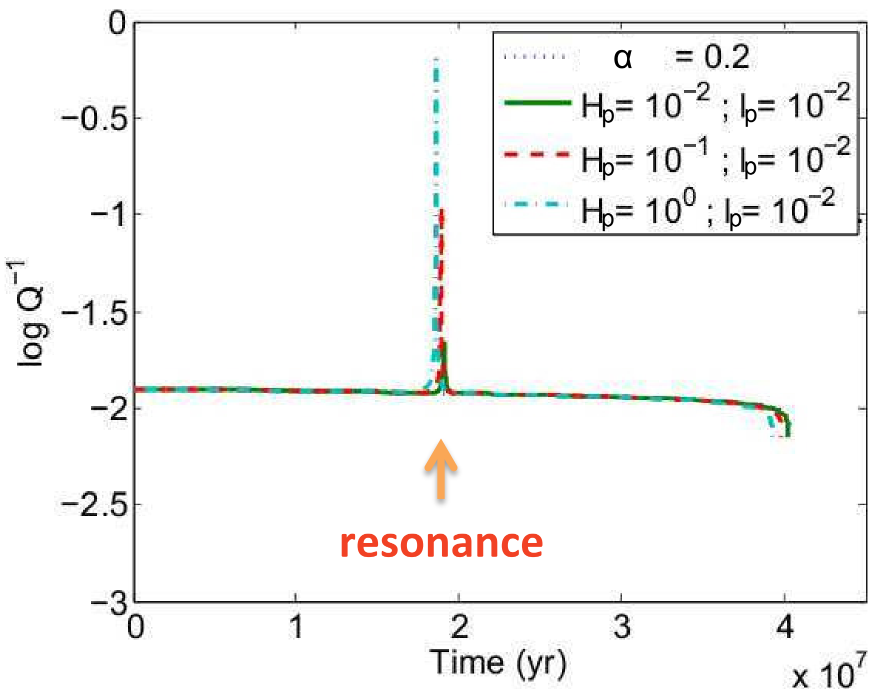


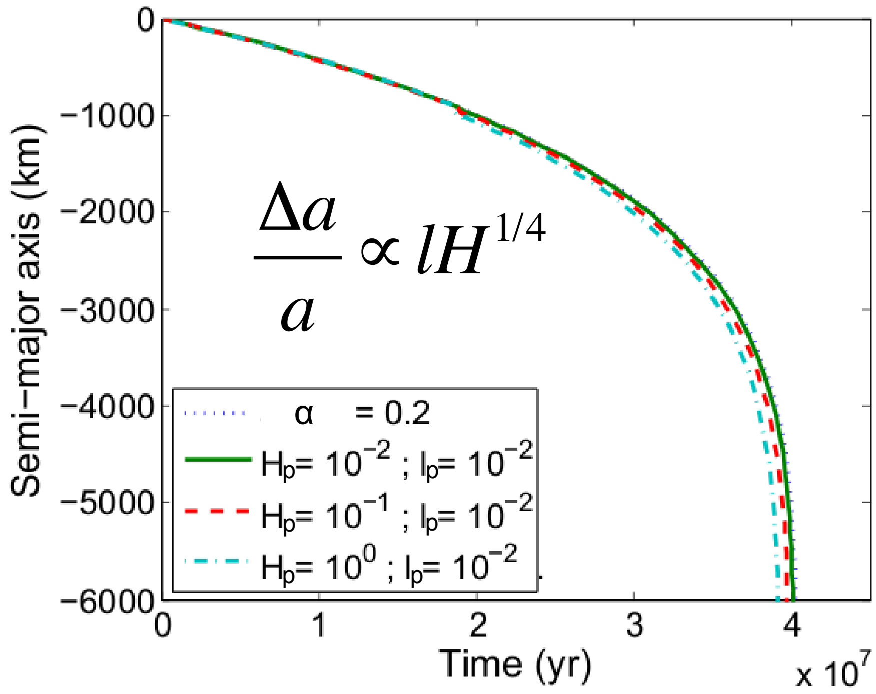
Fig. Ogilvie & Lin (2004)

$$Q = f(\omega) \propto D^{-1}(\omega)$$

II. What is the impact of tidal dissipation on the orbital evolution ?

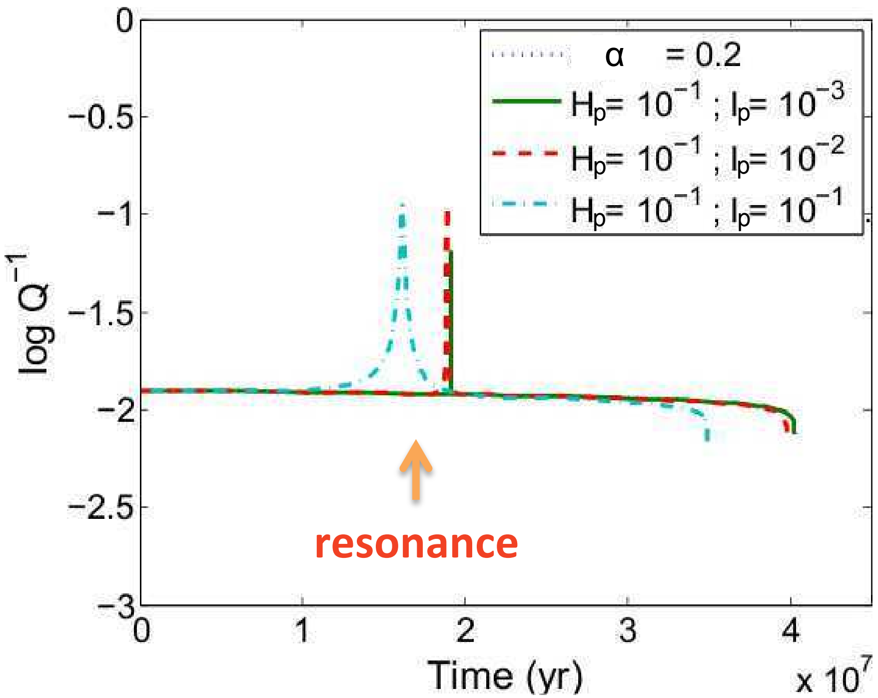
- Dependence on the height $Q_p^{-1}(\omega) = \dfrac{H_p}{\left[4\left(\sqrt{2}-1\right)\left(\frac{\omega-\omega_p}{l_p}\right)^2+1\right]^2}$

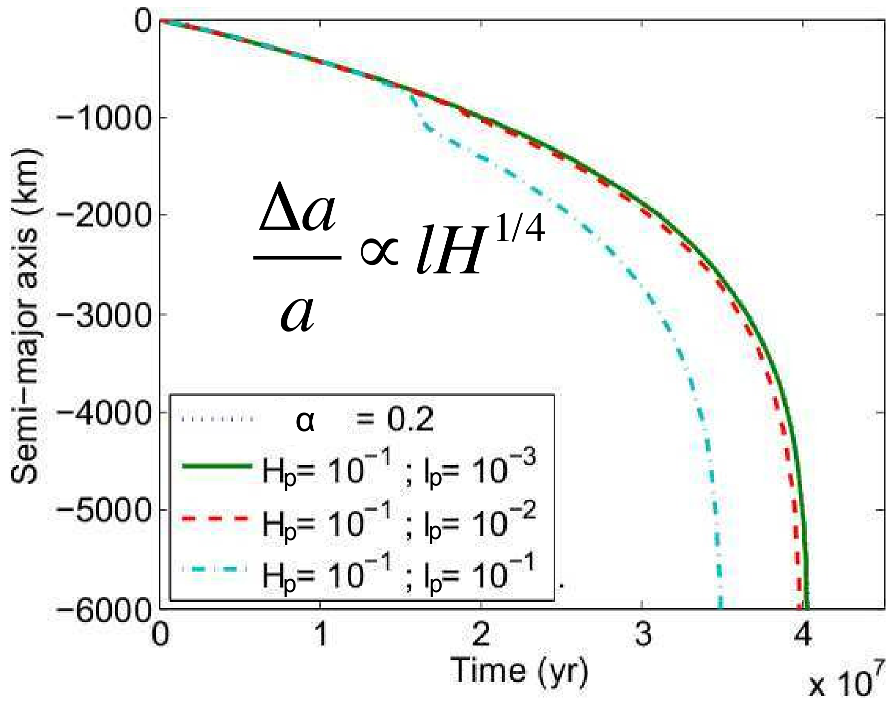




II. What is the impact of tidal dissipation on the orbital evolution ?

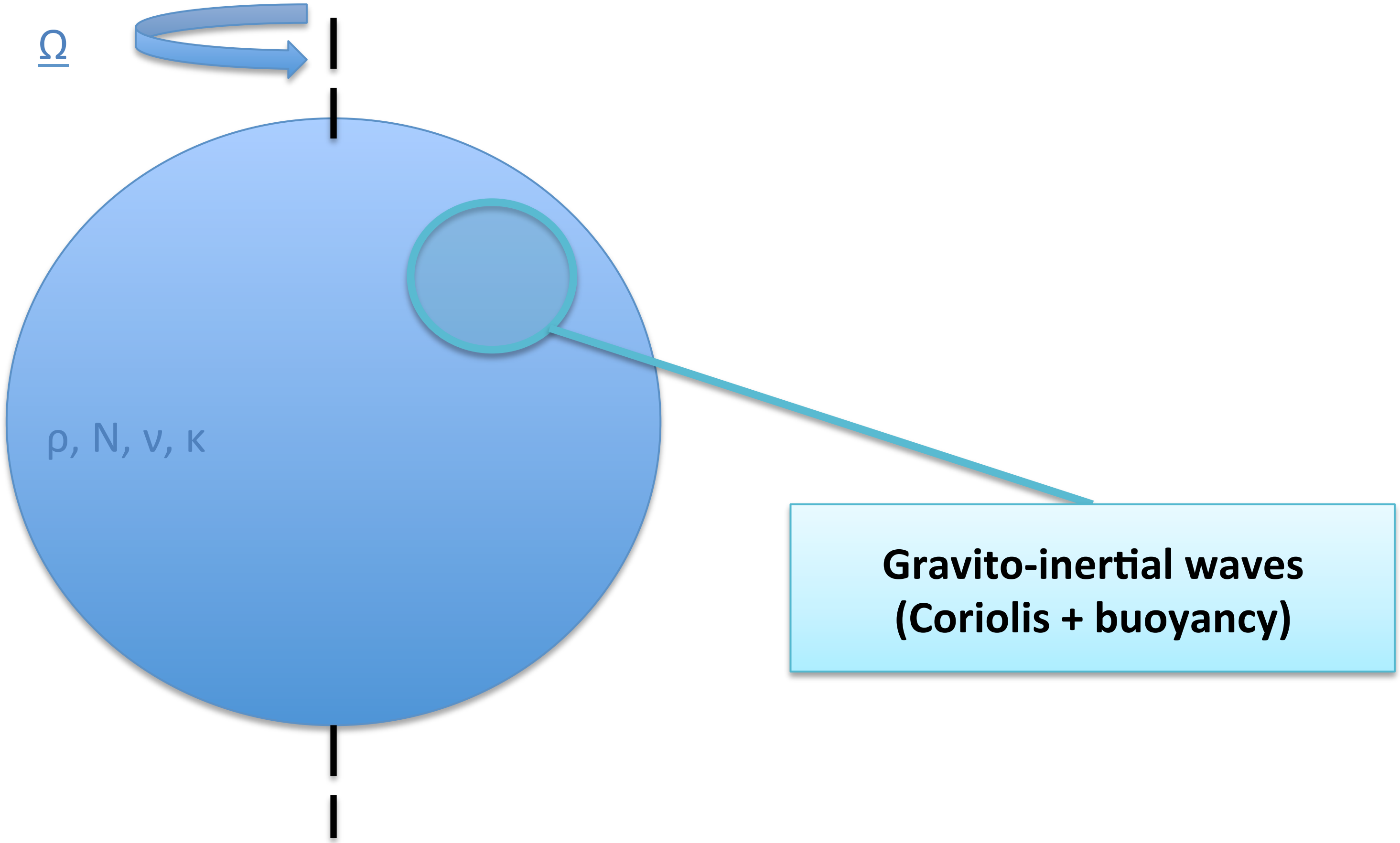
- Dependence on the width $Q_p^{-1}(\omega) = \dfrac{H_p}{\left[4\left(\sqrt{2}-1\right)\left(\frac{\omega-\omega_p}{l_p}\right)^2+1\right]^2}$





III. How does tidal dissipation depend on the internal physics of the bodies ?

- A local model for a fluid planet



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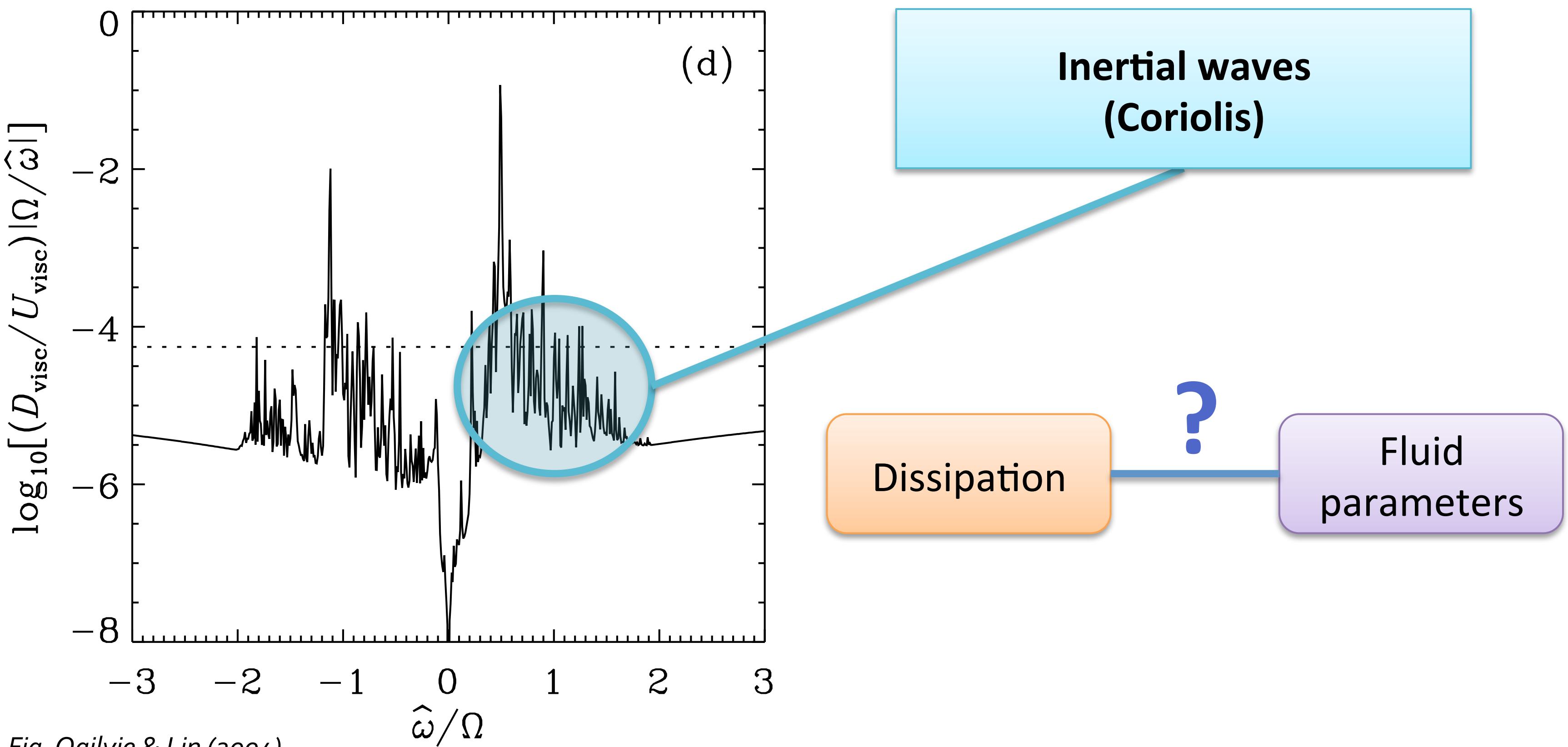


Fig. Ogilvie & Lin (2004)

III. How does tidal dissipation depend on the internal physics of the bodies ?

- Equations

Coriolis — Viscous friction — Buoyancy — Perturbation

$$\left\{ \begin{array}{l} \frac{\partial \mathbf{u}}{\partial t} + \mathbf{e}_z \wedge \mathbf{u} + Ro\,(\mathbf{u}\cdot\boldsymbol{\nabla})\,\mathbf{u} = -\frac{1}{2\Omega L\rho}\boldsymbol{\nabla}p + N_{\mathrm{Ek}}\nabla^2\mathbf{u} + \mathbf{B} + \mathbf{f} \\ \nabla\cdot\mathbf{u} = 0 \\ \partial_t B + Aw = \frac{\kappa}{2\Omega L^2}\nabla^2 B \end{array} \right.$$

Dynamics

Mass conservation

Heat transport

Stratification — Thermal diffusion

$$A = \left(\frac{N}{2\Omega}\right)^2 \qquad N^2 = -\frac{g}{\rho}\left[\frac{d\rho}{dz} + \frac{g\rho^2}{\gamma P}\right]$$

III. How does tidal dissipation depend on the internal physics of the bodies ?

- Control parameters

θ	Colatitude	Position
$A = \left(\frac{N}{2\Omega}\right)^2$	Frequency ratio	Nature of the waves
$E = \frac{2\pi^2\nu}{\Omega L^2}$	Ekman	Influence of viscous friction compared to Coriolis effects
$K = \frac{2\pi^2\kappa}{\Omega L^2}$	Thermal diffusivity	Influence of thermal diffusion compared to Coriolis effects

III. How does tidal dissipation depend on the internal physics of the bodies ?

- Periodic velocity field $u = \sum u_{mn} e^{i2\pi(mx+nz)}$

$$
\begin{cases}
u_{mn} = n \dfrac{i\tilde{\omega}\left(n f_{mn} - m h_{mn}\right) - n\cos\theta g_{mn}}{\left(m^2+n^2\right)\tilde{\omega}^2 - n^2\cos^2\theta - A m^2 \dfrac{\tilde{\omega}}{\hat{\omega}}}, \\[2ex]
v_{mn} = \dfrac{n\cos\theta\left(n f_{mn} - m h_{mn}\right) + i\left[\left(m^2+n^2\right)\tilde{\omega} - \dfrac{A m^2}{\hat{\omega}}\right] g_{mn}}{\left(m^2+n^2\right)\tilde{\omega}^2 - n^2\cos^2\theta - A m^2 \dfrac{\tilde{\omega}}{\hat{\omega}}} \\[2ex]
w_{mn} = -m \dfrac{i\tilde{\omega}\left(n f_{mn} - m h_{mn}\right) - n\cos\theta g_{mn}}{\left(m^2+n^2\right)\tilde{\omega}^2 - n^2\cos^2\theta - A m^2 \dfrac{\tilde{\omega}}{\hat{\omega}}},
\end{cases}
$$

Influence of the perturbation

Inertial part

Viscous diffusivity

$$
\begin{cases}
\tilde{\omega} = \omega + iE\left(m^2+n^2\right) \\
\hat{\omega} = \omega + iK\left(m^2+n^2\right)
\end{cases}
$$

Thermal diffusivity

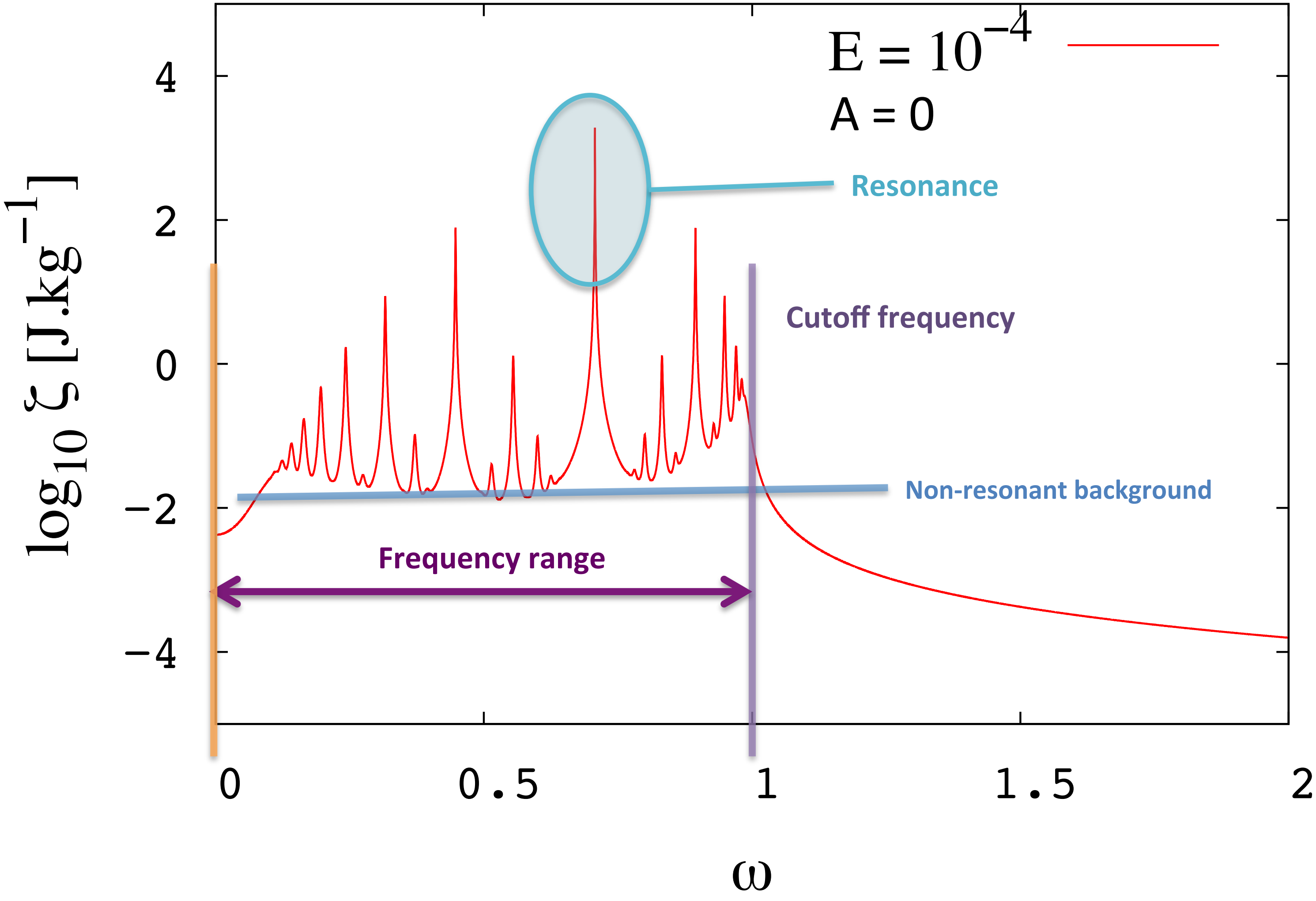
III. How does tidal dissipation depend on the internal physics of the bodies ?

- Energy dissipated by viscous friction

$$D = \int_0^1 \int_0^1 \left\langle -\mathbf{u} \cdot \nu \nabla^2 \mathbf{u} \right\rangle dx dz$$

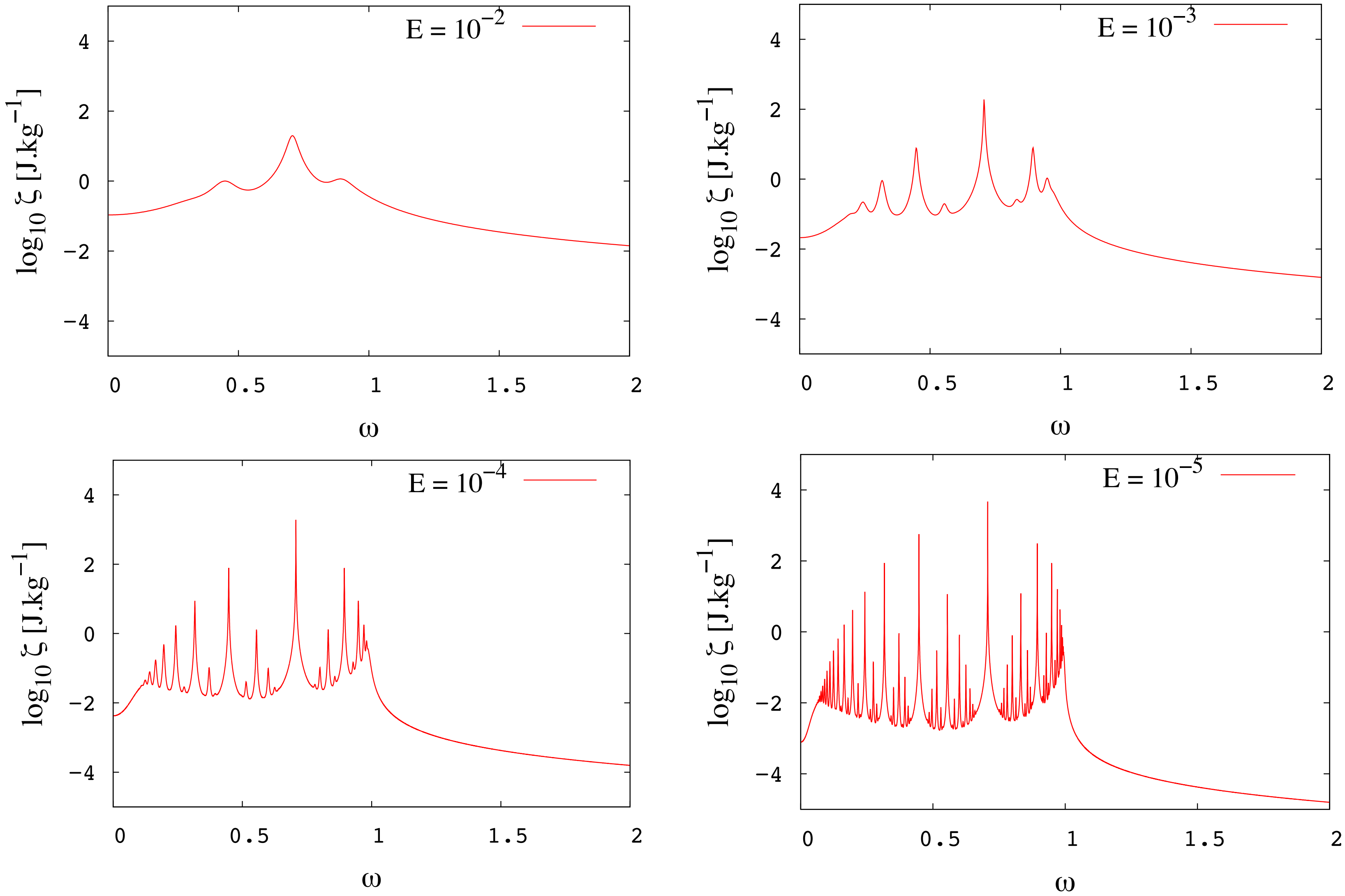
$$\zeta = \frac{2\pi}{\Omega} D = 2\pi E \sum_{(m,n) \in \mathbb{Z}^{*2}} \left(m^2 + n^2\right)\left(\left|u_{mn}^2\right| + \left|v_{mn}^2\right| + \left|w_{mn}^2\right|\right)$$

III. How does tidal dissipation depend on the internal physics of the bodies ?



$$\omega = \frac{\chi}{2\Omega}$$

III. How does tidal dissipation depend on the internal physics of the bodies ?



$\theta = 0$

$A = 0$

$K = 0$

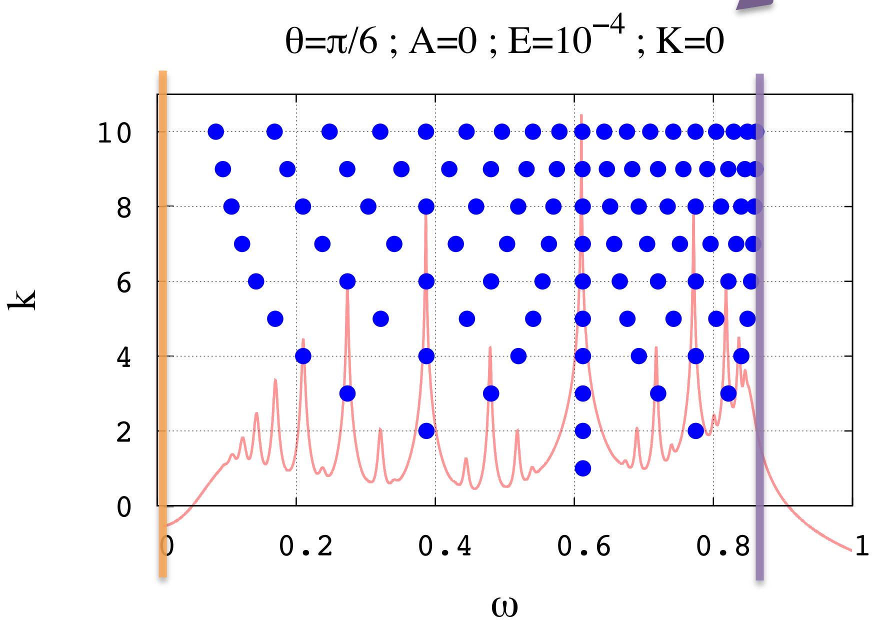
III. How does tidal dissipation depend on the internal physics of the bodies ?

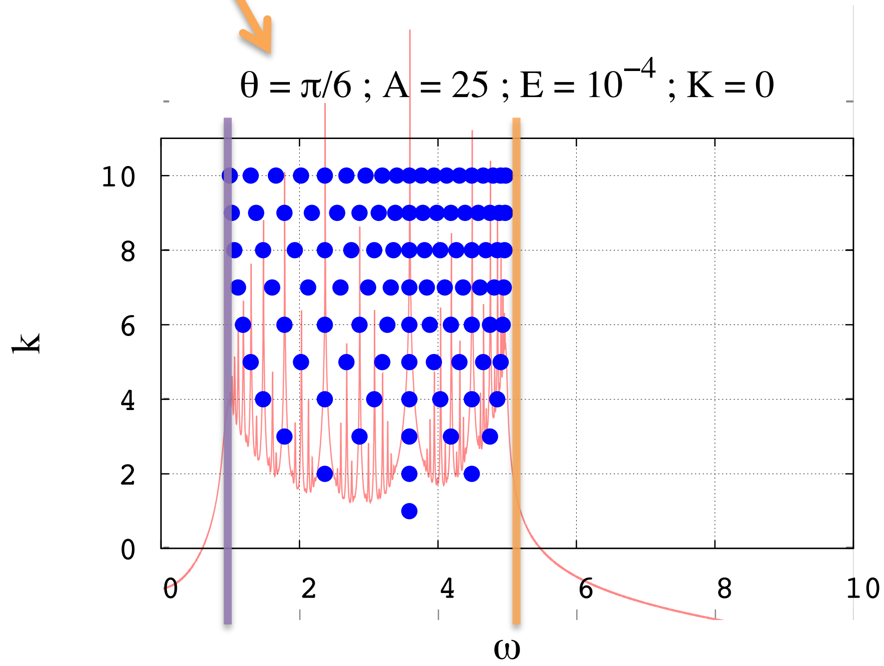
- Positions of the resonances

Inertial waves

$$\omega_{mn} = \sqrt{\frac{n^2 \cos^2\theta + A m^2}{m^2 + n^2}}$$

Gravito-inertial waves





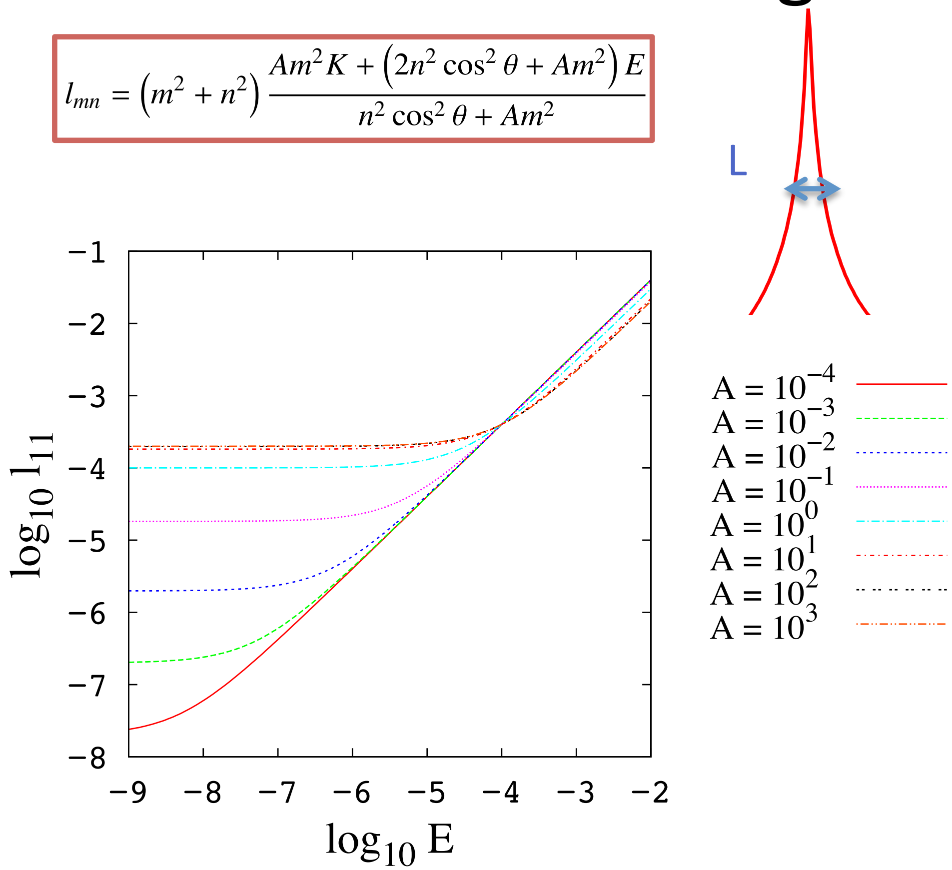
$$k = \max\{|m|, |n|\}$$

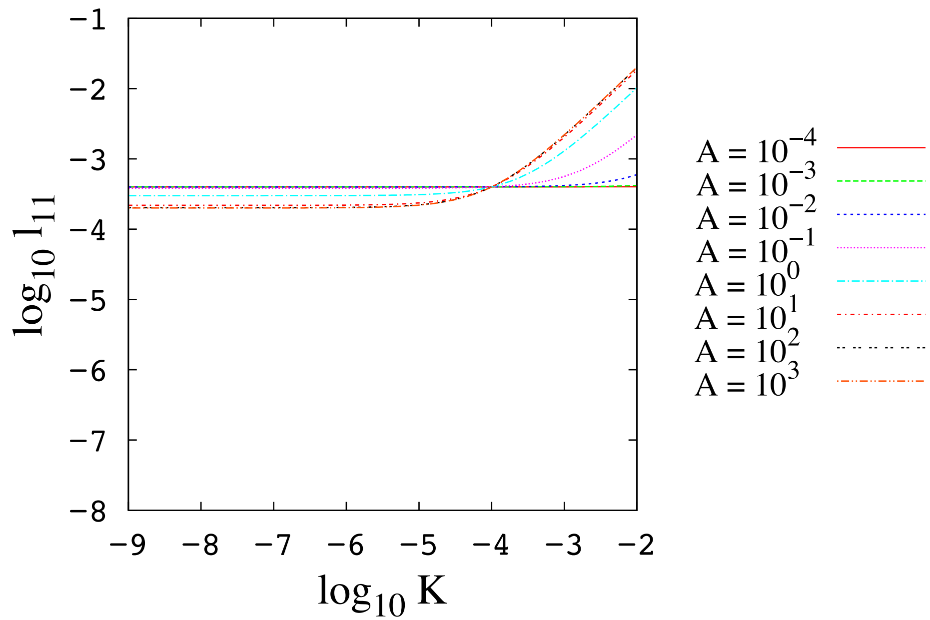
III. How does tidal dissipation depend on the internal physics of the bodies ?

- Width at mid-height

$$l_{mn} = \left(m^2+n^2\right)\frac{Am^2K+\left(2n^2\cos^2\theta+Am^2\right)E}{n^2\cos^2\theta+Am^2}$$

Domain	$A \ll A_{mn}$	$A \gg A_{mn}$
$Pr \gg Pr_{mn}$	$2E\left(m^2+n^2\right)$	$E\left(m^2+n^2\right)$
$Pr \ll Pr_{mn}$	$AK\dfrac{m^2\left(m^2+n^2\right)}{n^2\cos^2\theta}$	$K\left(m^2+n^2\right)$



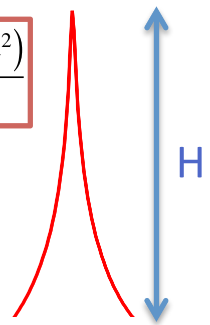


III. How does tidal dissipation depend on the internal physics of the bodies ?

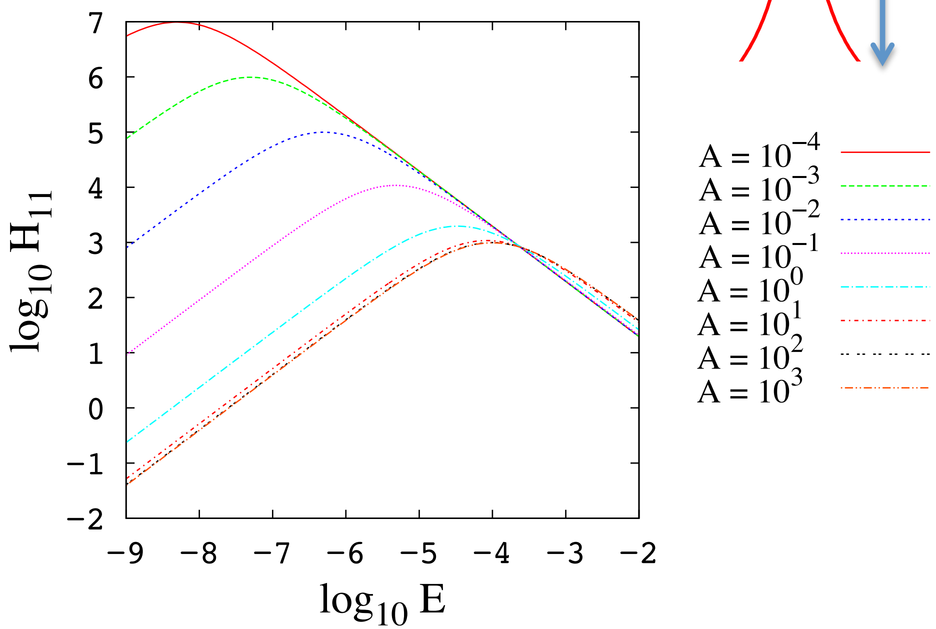
- Height

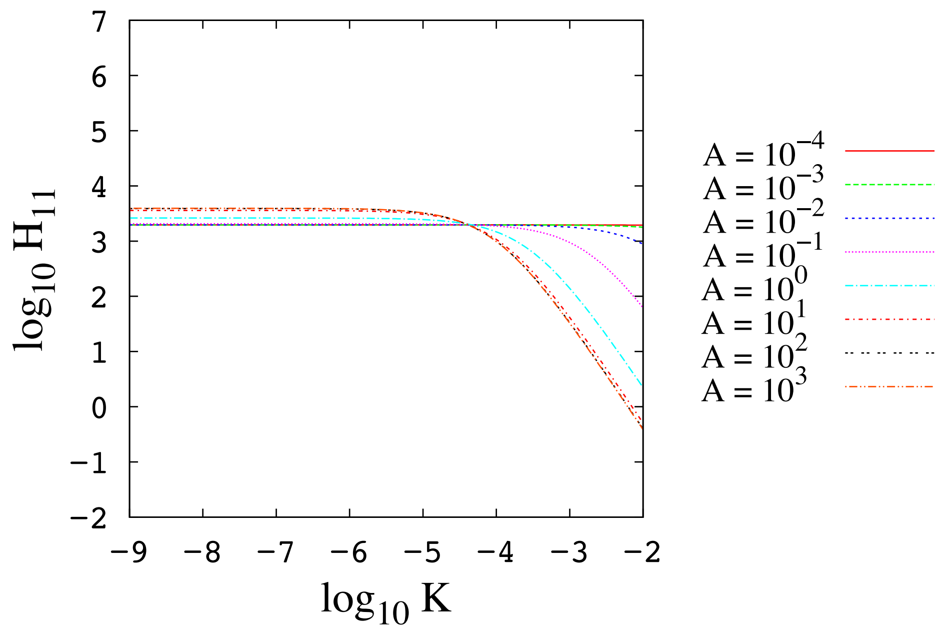
$$H_{mn} = \frac{8\pi F^2 E}{m^2 n^2 \left(m^2+n^2\right)^2} \frac{\left(2n^2\cos^2\theta + Am^2\right)\left(n^2\cos^2\theta + Am^2\right)}{\left[Am^2 K + \left(2n^2\cos^2\theta + Am^2\right)E\right]^2}$$

$$f_{mn} = i\frac{F}{|m|\,n^2}, \quad g_{mn} = 0, \quad h_{mn} = 0$$

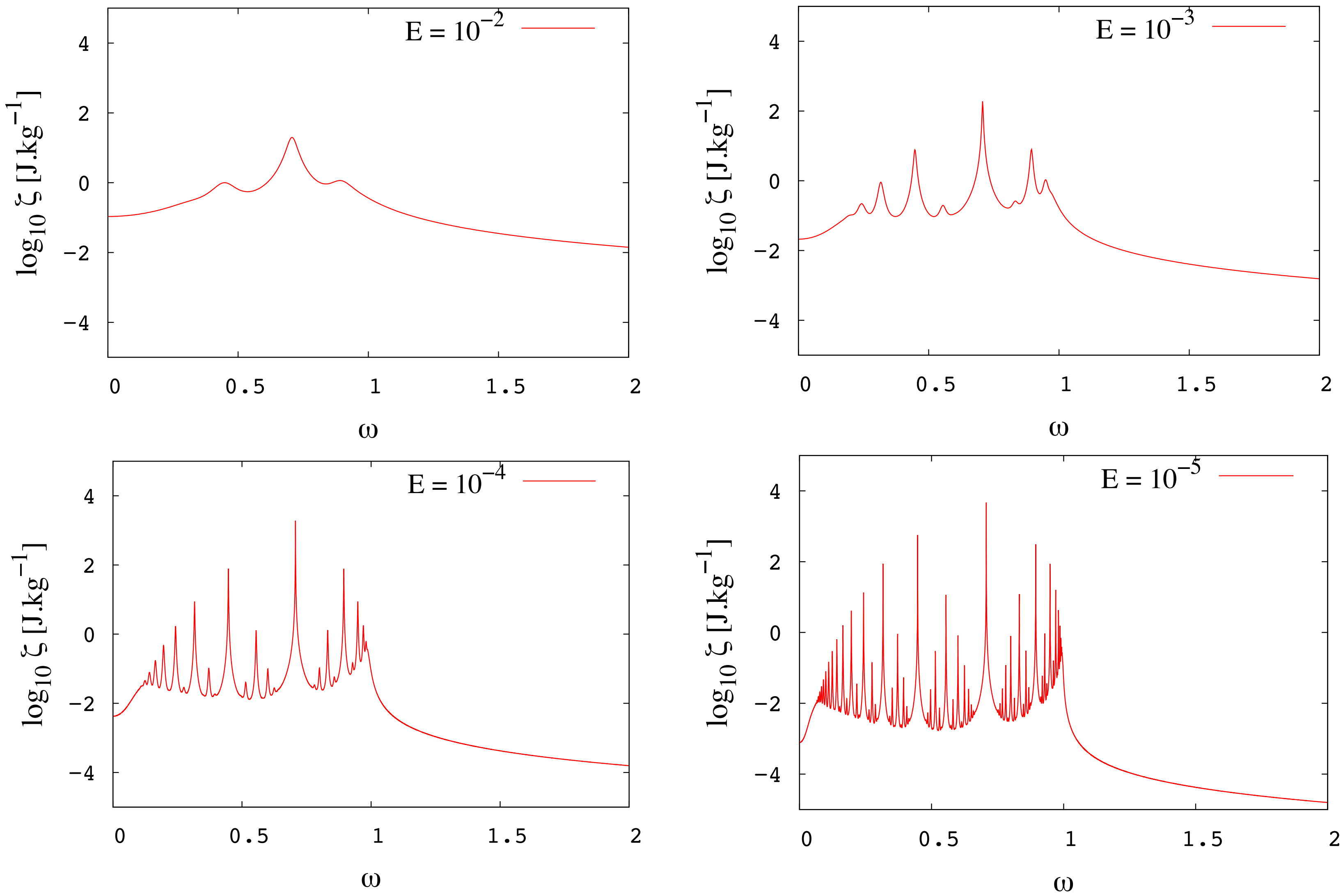


Domain	$A \ll A_{mn}$	$A \gg A_{mn}$
$Pr \gg Pr_{mn}$	$\frac{4\pi F^2}{m^2 n^2 \left(m^2+n^2\right)^2 E}$	$\frac{8\pi F^2}{m^2 n^2 \left(m^2+n^2\right)^2 E}$
$Pr \ll Pr_{mn}$	$\frac{16\pi F^2 n^2}{m^6 \left(m^2+n^2\right)^2} \frac{E\cos^4\theta}{A^2 K^2}$	$\frac{8\pi F^2}{m^2 n^2 \left(m^2+n^2\right)^2} \frac{E}{K^2}$





III. How does tidal dissipation depend on the internal physics of the bodies ?



$\theta = 0$

$A = 0$

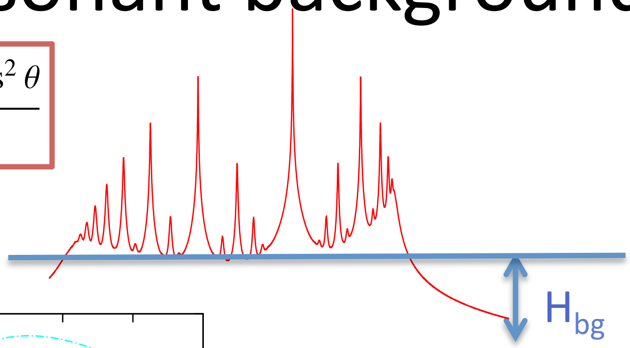
$K = 0$

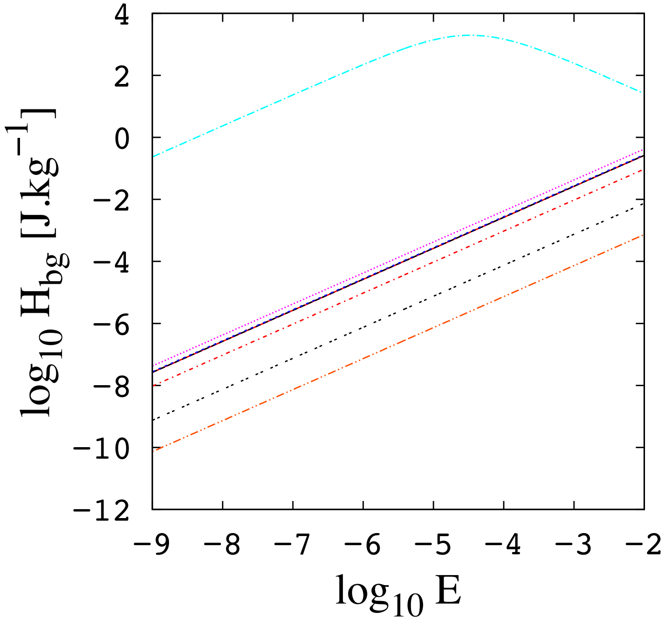
III. How does tidal dissipation depend on the internal physics of the bodies ?

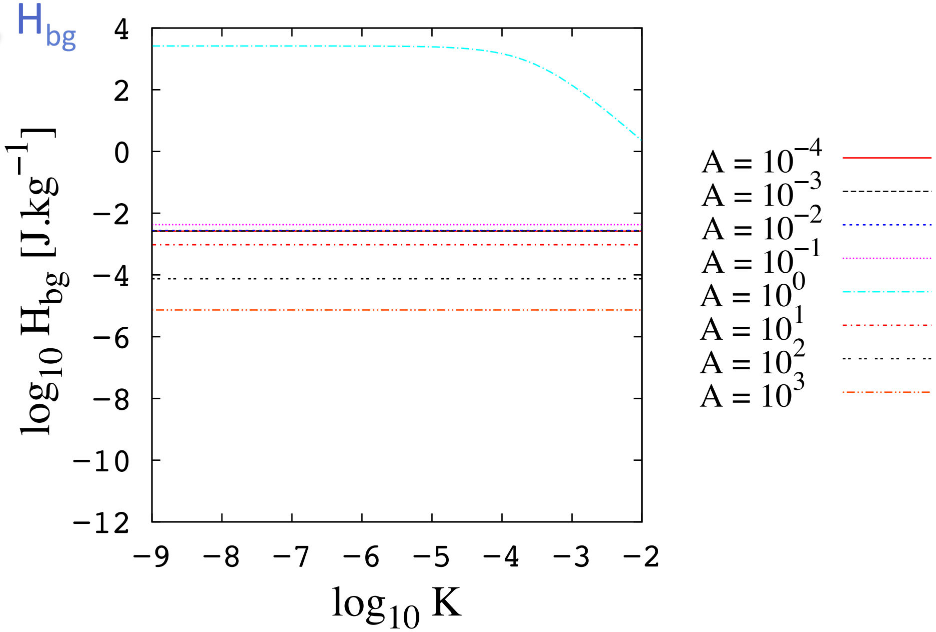
- Non-resonant background

$$H_{bg} = 4\pi F^2 E \frac{C^{\infty}_{grav} A + C^{\infty}_{in} \cos^2\theta}{(A + \cos^2\theta)^2}$$

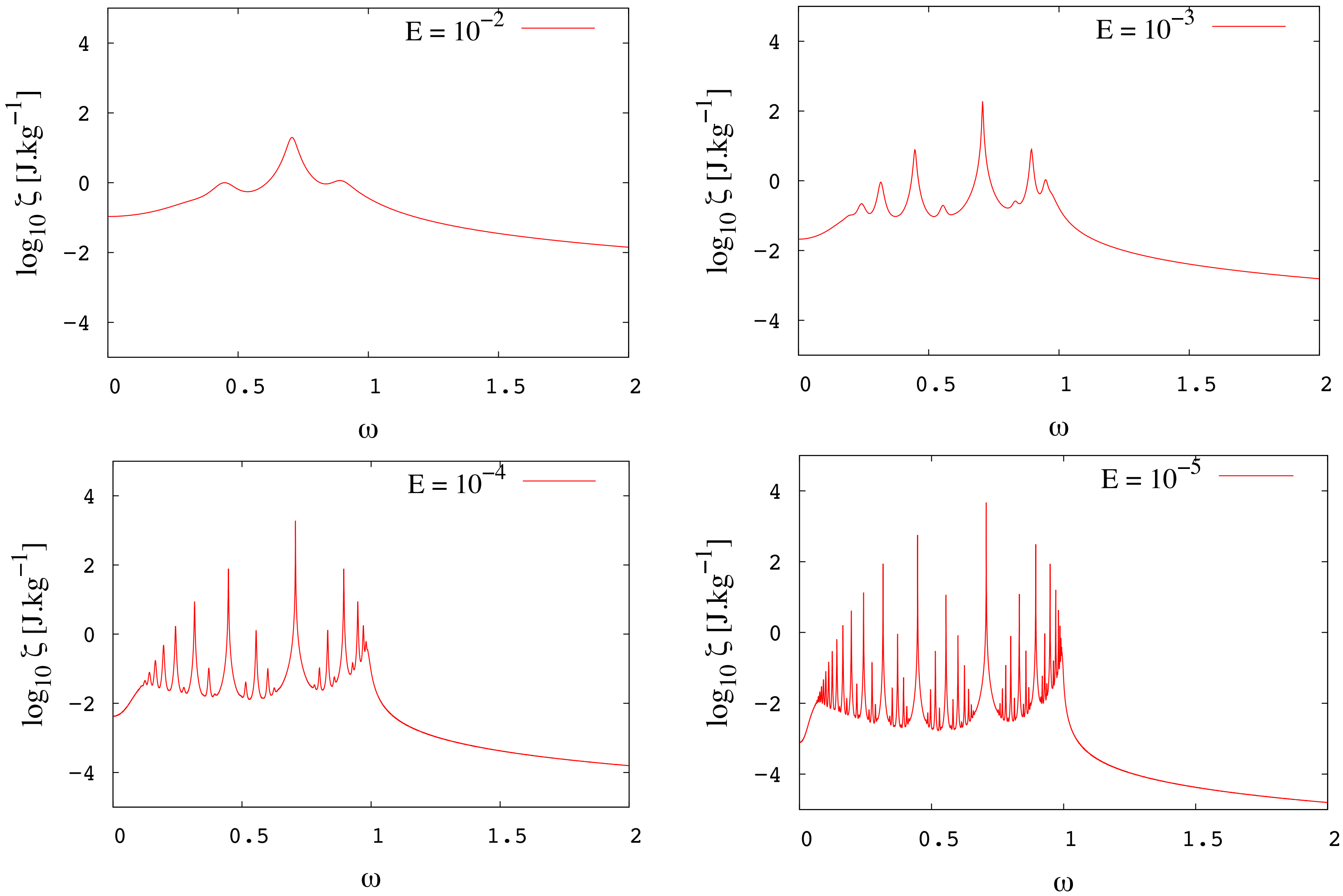
$A \ll \cos^2\theta$	$A \gg \cos^2\theta$
$4\pi C^{\infty}_{in} F^2 \dfrac{E}{\cos^2\theta}$	$4\pi C^{\infty}_{grav} F^2 \dfrac{E}{A}$







III. How does tidal dissipation depend on the internal physics of the bodies ?



$\theta = 0$

$A = 0$

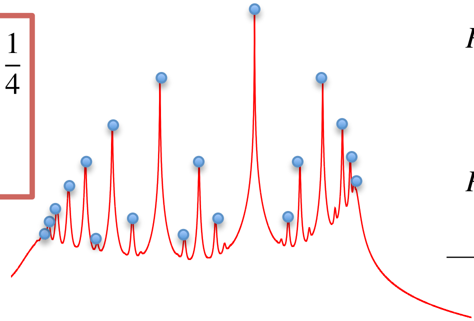
$K = 0$

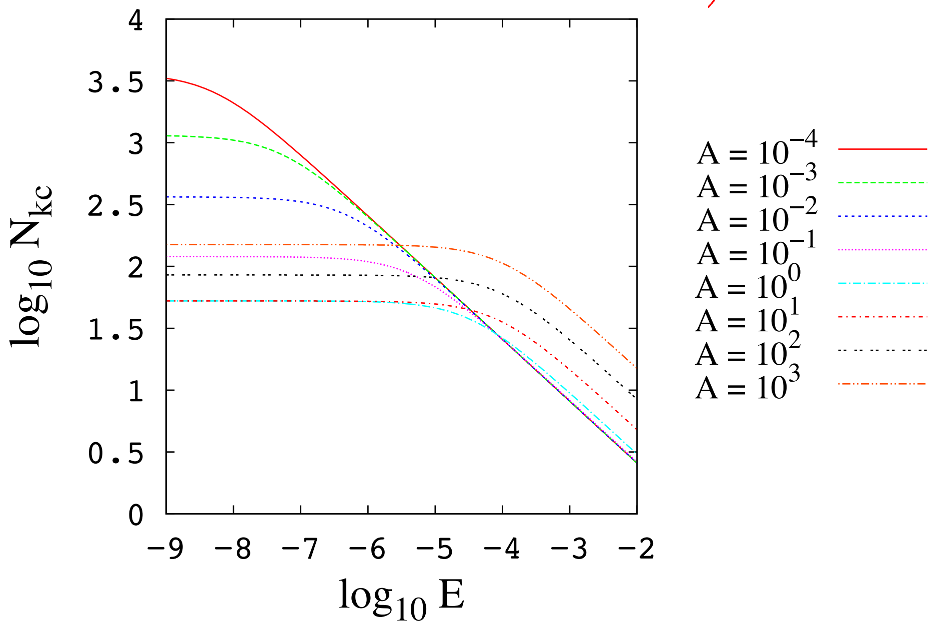
III. How does tidal dissipation depend on the internal physics of the bodies ?

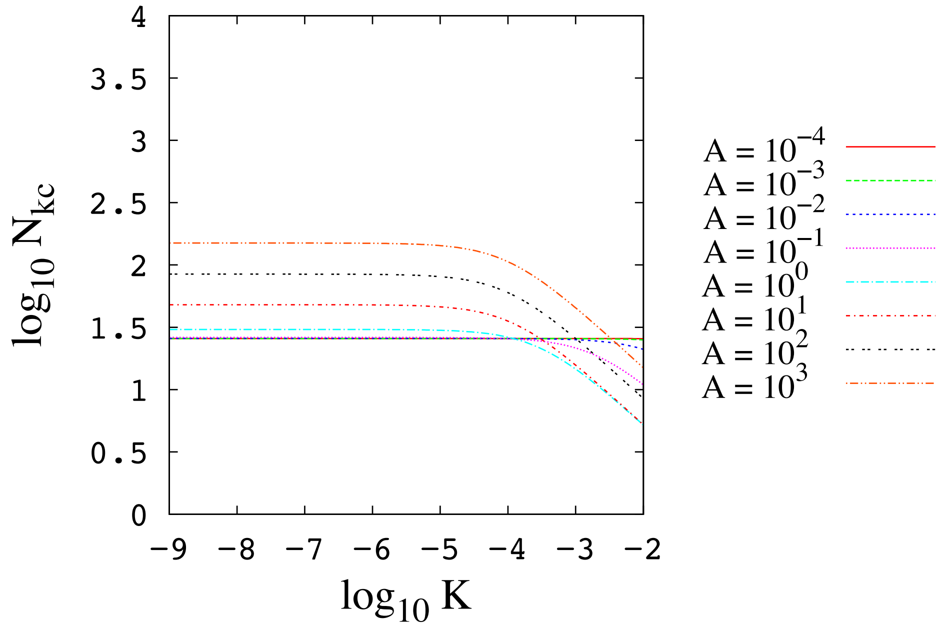
- Number of resonances

$$N_{kc} \sim \left\{ \frac{1}{2} \frac{\left(2\cos^2\theta + A\right)\left(A + \cos^2\theta\right)^3}{\left[AK + \left(2\cos^2 + A\right)E\right]^2 \left[C_{\rm in}^{\infty}\cos^2\theta + C_{\rm grav}^{\infty}A\right]} \right\}^{\frac{1}{4}}$$

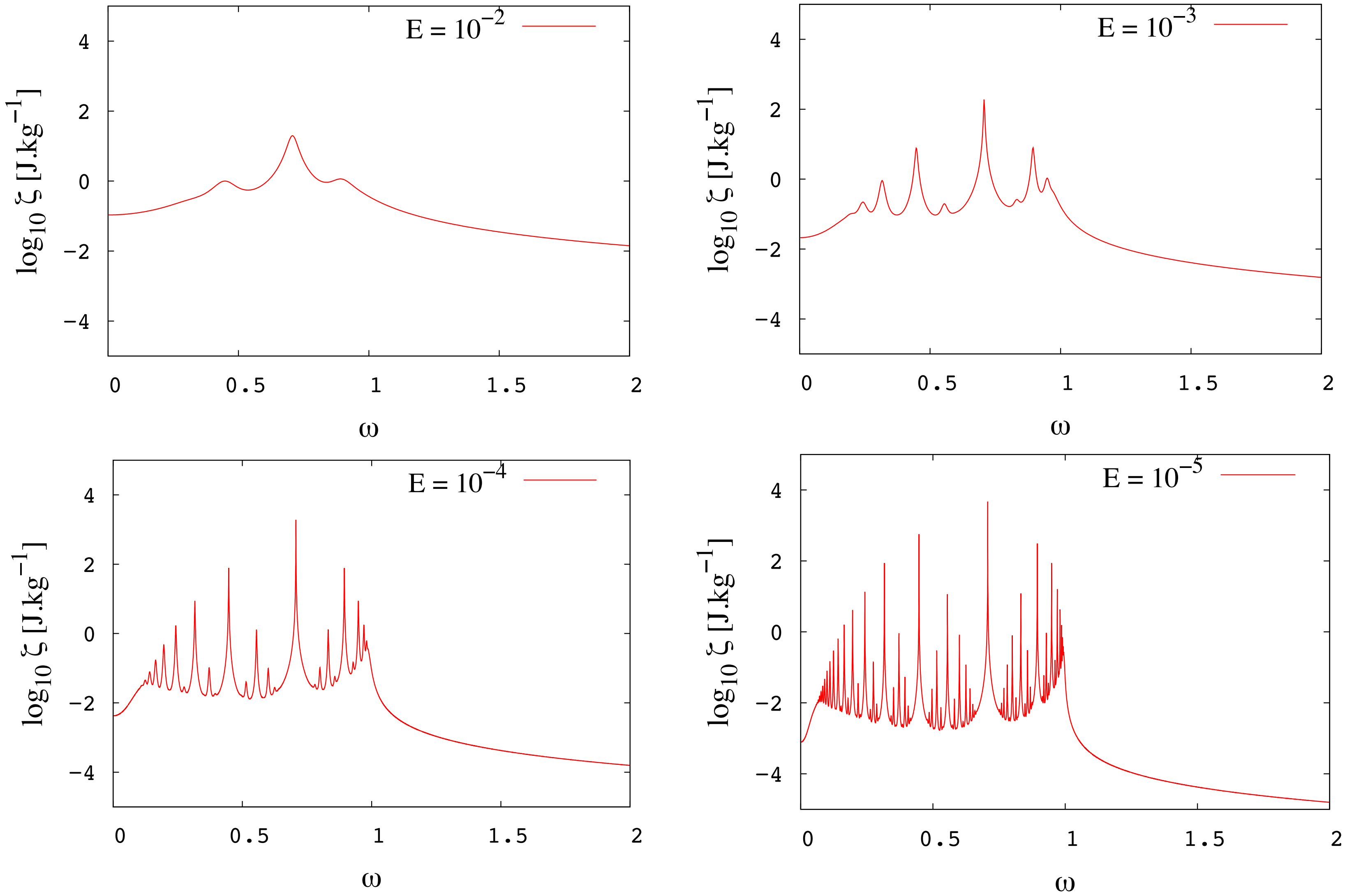
Domain	$A \ll A_{11}$	$A \gg A_{11}$
$Pr \gg Pr_{11}$	$N_{kc} \sim \left(\frac{\cos^2\theta}{4C_{\rm in}^{\infty}E^2}\right)^{\frac{1}{4}}$	$N_{kc} \sim \left(\frac{A}{2C_{\rm grav}^{\infty}E^2}\right)^{\frac{1}{4}}$
$Pr \ll Pr_{11}$	$N_{kc} \sim \left(\frac{\cos^6\theta}{C_{\rm in}^{\infty}A^2K^2}\right)^{\frac{1}{4}}$	$N_{kc} \sim \left(\frac{A}{2C_{\rm grav}^{\infty}K^2}\right)^{\frac{1}{4}}$







III. How does tidal dissipation depend on the internal physics of the bodies ?



$\theta = 0$

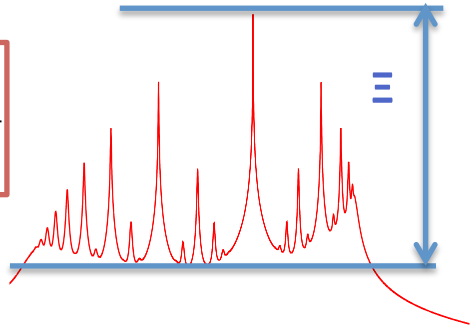
$A = 0$

$K = 0$

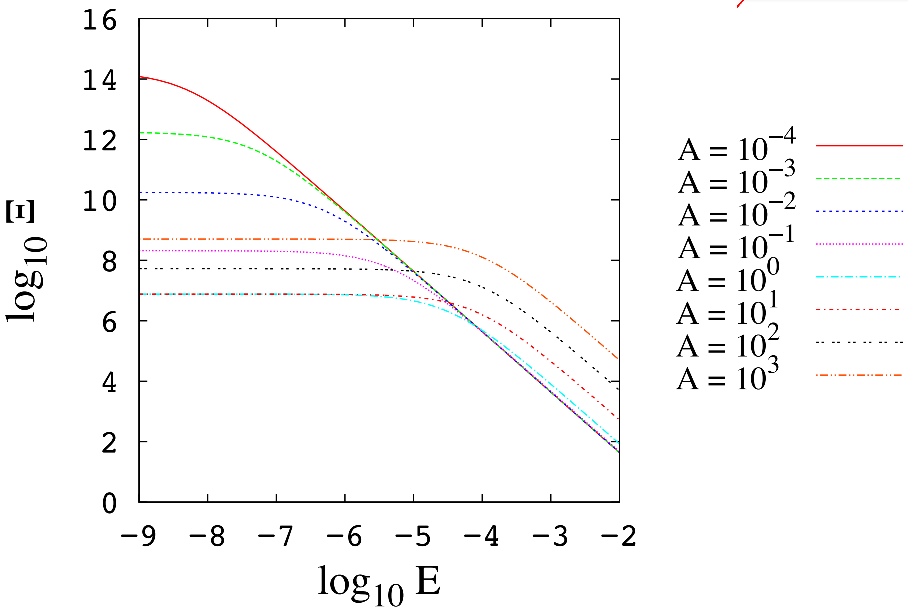
III. How does tidal dissipation depend on the internal physics of the bodies ?

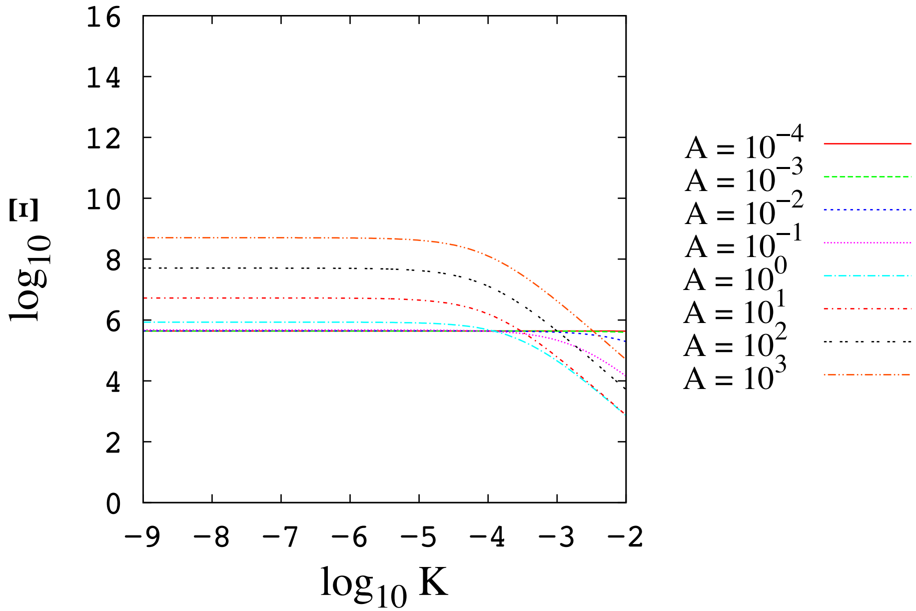
- Sharpness ratio

$$\Xi = \frac{1}{2}\frac{\left(2\cos^2 + A\right)\left(A+\cos^2\theta\right)^3}{\left[AK + \left(2\cos^2 + A\right)E\right]^2\left[C_{\rm in}^{\infty}\cos^2\theta + C_{\rm grav}^{\infty}A\right]}$$

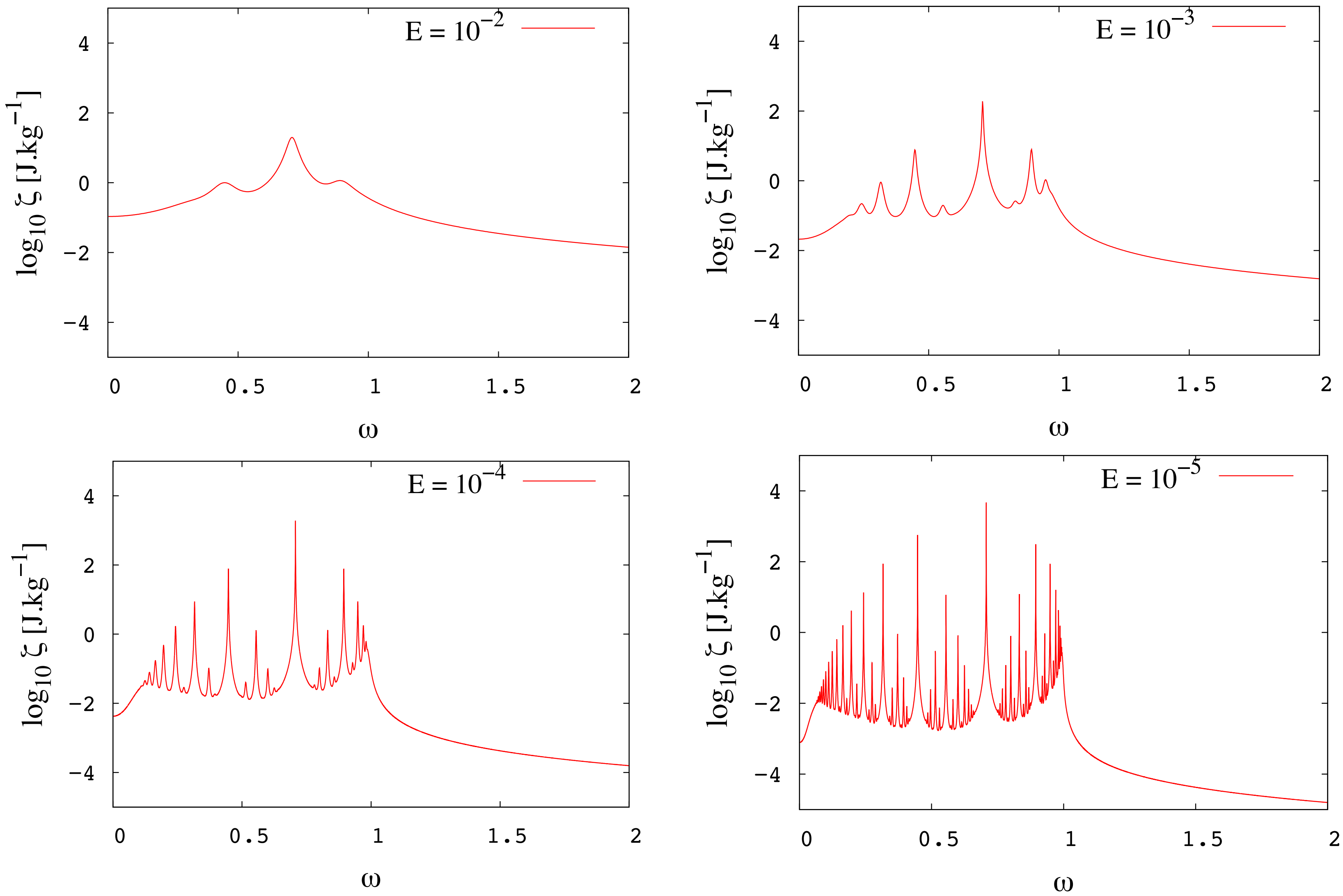


DOMAIN	$A \ll A_{11}$	$A \gg A_{11}$
$Pr \gg Pr_{11}$	$\dfrac{\cos^2\theta}{4C_{\rm in}^{\infty}E^2}$	$\dfrac{A}{2C_{\rm grav}^{\infty}E^2}$
$Pr \ll Pr_{11}$	$\dfrac{\cos^6\theta}{C_{\rm in}^{\infty}A^2K^2}$	$\dfrac{A}{2C_{\rm grav}^{\infty}K^2}$





III. How does tidal dissipation depend on the internal physics of the bodies ?



$\theta = 0$

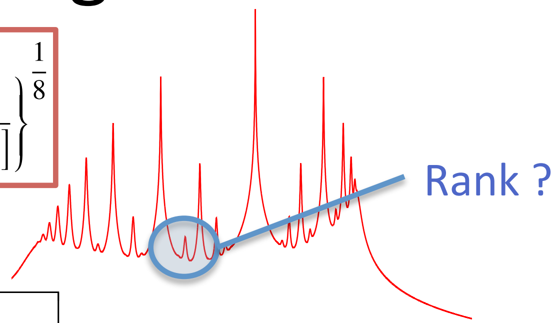
$A = 0$

$K = 0$

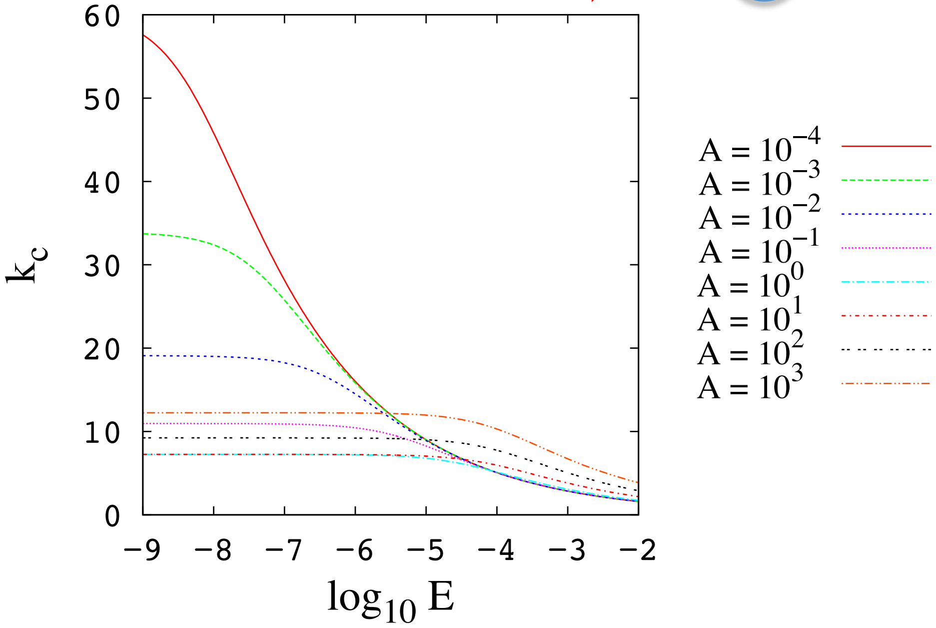
III. How does tidal dissipation depend on the internal physics of the bodies ?

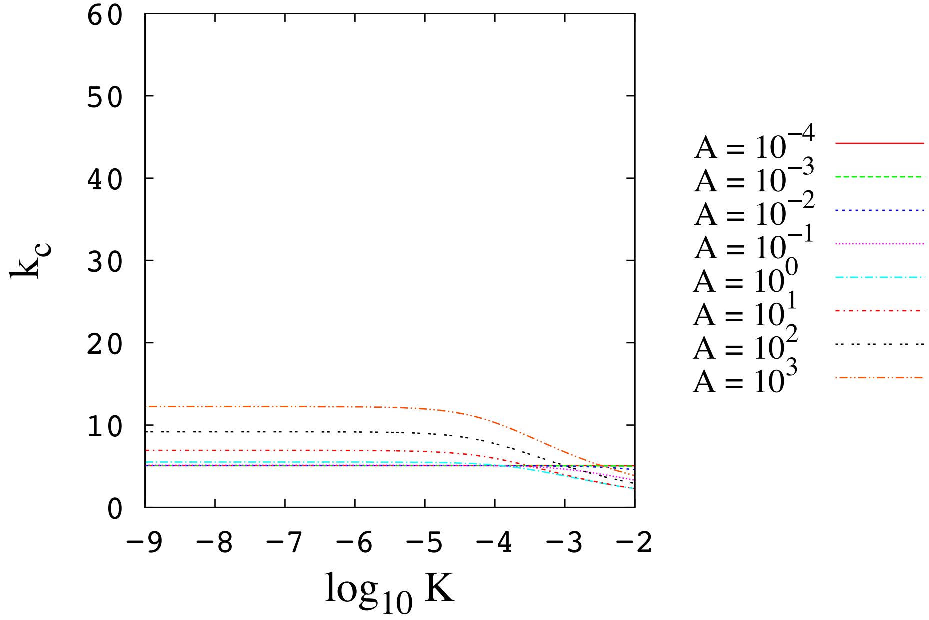
- Rank of the highest harmonics

$$k_c \sim \left\{ \frac{1}{2} \frac{\left(2\cos^2\theta + A\right)\left(A+\cos^2\theta\right)^3}{\left[AK + \left(2\cos^2 + A\right)E\right]^2 \left[C_{\rm in}^{\infty}\cos^2\theta + C_{\rm grav}^{\infty} A\right]} \right\}^{\frac{1}{8}}$$



Domain	$A \ll A_{11}$	$A \gg A_{11}$
$Pr \gg Pr_{11}$	$k_c \sim \left(\frac{\cos^2\theta}{4C_{\rm in}^{\infty}E^2}\right)^{\frac{1}{8}}$	$k_c \sim \left(\frac{A}{2C_{\rm grav}^{\infty}E^2}\right)^{\frac{1}{8}}$
$Pr \ll Pr_{11}$	$k_c \sim \left(\frac{\cos^6\theta}{C_{\rm in}^{\infty}A^2K^2}\right)^{\frac{1}{8}}$	$k_c \sim \left(\frac{A}{2C_{\rm grav}^{\infty}K^2}\right)^{\frac{1}{8}}$





Synopsis

I. Tidal dissipation ?

II. Tidal gravito-inertial waves ?

III. Impact of tidal dissipation on the spin/orbital evolution ?

IV. How does tidal dissipation depend on the internal physics of the bodies ?