Modelling Transport Processes in Stellar Radiative Interiors

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What transport processes are we talking about ?

Transport of angular momentum

Rotation induced processes (meridional circulation, turbulence)

Winds

Internal Gravity Waves

Magnetic Fields

Transport of nucleides

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Magnetic Fields

Winds

Double-diffusive convection

Atomic Diffusion

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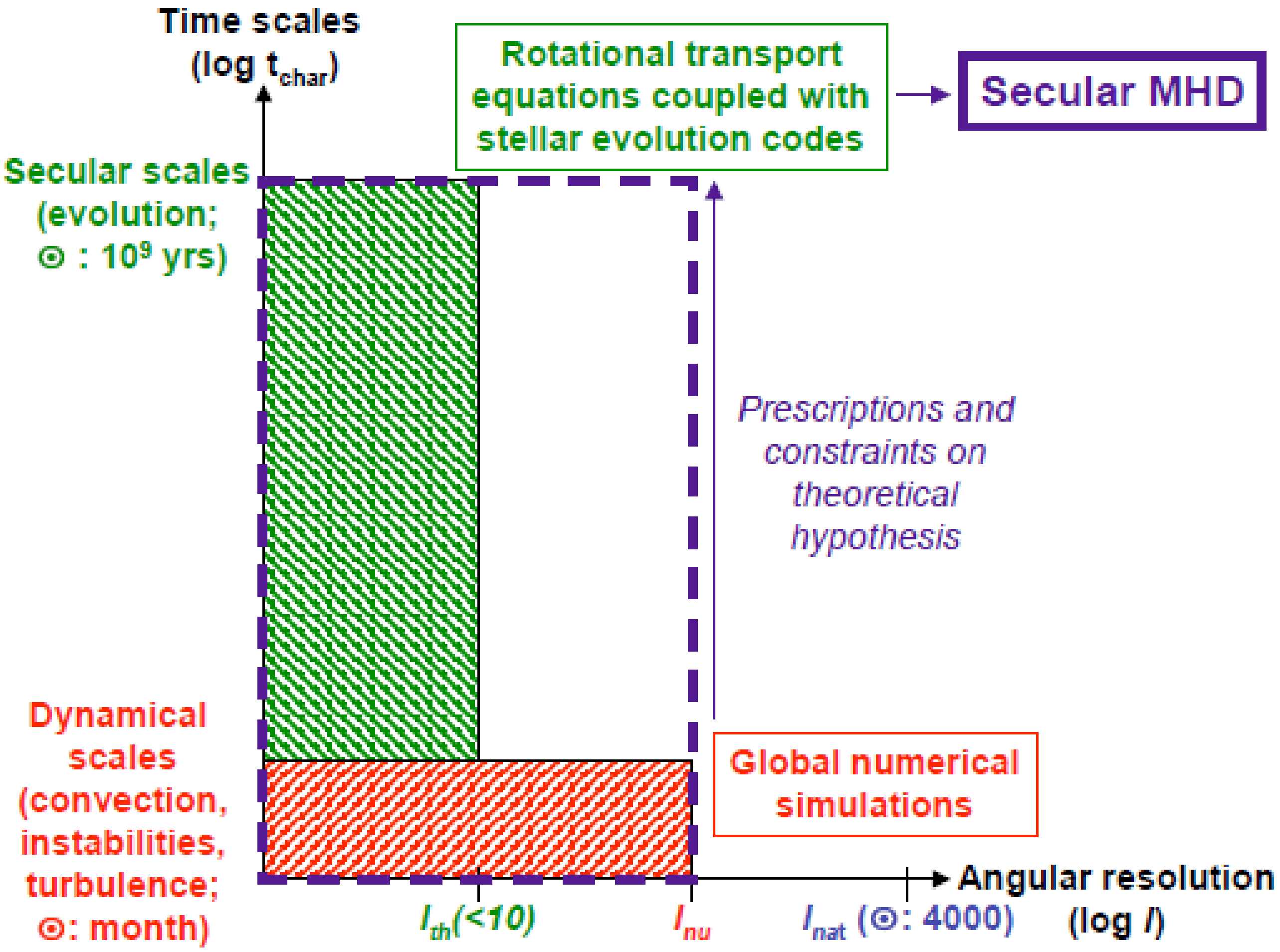
Winds

Strongly related to rotation

Double-diffusive convection

Atomic Diffusion

The many scales of angular momentum transport processes



Time scales
(log t_{char})

Rotational transport
equations coupled with
stellar evolution codes

Secular MHD

Secular scales
(evolution;
$\odot$: 10^9 yrs)

*Prescriptions and
constraints on
theoretical
hypothesis*

Dynamical
scales
(convection,
instabilities,
turbulence;
$\odot$: month)

Global numerical
simulations

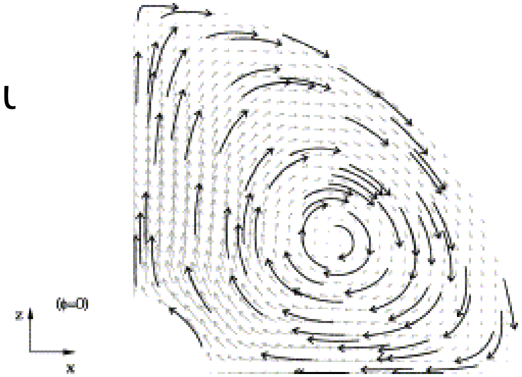
Angular resolution
(log l)

$l_{th}(<10)$ l_{nu} l_{nat} ($\odot$: 4000)

Decressin et al. 2009

Transport of angular momentum

Rotation induced mixing : meridional circulation and hydrodynamical instabilities



Meridional circulation induced by gain or loss of angular momentu

(Busse 1981, Zahn 1992, Rieutord 2008)

Hydrodynamical instabilities induced by differential rotation → turbulent transport

(baroclinic instabilities, double-diffusive instabilities)

Small viscosity of stellar interiors → ∃ always a scale for which horizontal shear becomes turbulent

Stable stratification in radiative interiors
→ anisotropic transport ($\nu v \ll \nu h$, $Dv \ll Dh$) with horizontal gradients smaller than vertical ones.
Ω constant over an isobar → shellular rotation

2-D retroacting problem → 1.5 D problem

$$\rho \frac{d}{dt}\left(r^2\Omega\right) = \boxed{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4 \Omega U\right)} + \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4 \rho \nu_v \frac{\partial \Omega}{\partial r}\right)}$$

advection **diffusion**

Transport of angular momentum

Winds → Angular momentum losses and transport trigger

Different physical mechanisms according to the initial mass and evolutionary phase

- magnetized stellar and disc winds during the PMS evolution of low-mass stars

(Kawaler, Krishnamurti, Matt et al.)

$$\tau_{\mathrm{w}} = K_1^2 B_*^{4m} \dot{M}_{\mathrm{w}}^{1-2m} R_*^{4m+2} \frac{\Omega_*}{(K_2^2 v_{\mathrm{esc}}^2 + \Omega_*^2 R_*^2)^m}$$

- Von Zeipel theorem and radiative winds inducing mass and angular momentum losses in rotating hot massive stars

$$\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \frac{(1-\Gamma)^{\frac{1}{\alpha}-1}}{\left[1-\frac{\Omega^2}{2\pi G\rho_{\mathrm{m}}}-\Gamma\right]^{\frac{1}{\alpha}-1}}$$

(Maeder & Meynet 2000)

Transport of angular momentum

Combined treatment of rotational transport and winds

- perturbative approach → several formalisms exist

Endal & Sofia 1978, Pinsonneault et al. 1989, Heger et al. 2000

$$\frac{d}{dt}(r^2\Omega) = \underbrace{\boxed{\frac{\partial}{\partial M_r}\left[(4\pi r^2\rho)^2 r^2 \nu \frac{\partial \Omega}{\partial M_r}\right]}}_{\text{diffusion}} \quad \underbrace{\boxed{+\ \dot{\tau}_{wind}}}_{\text{wind torque}}$$

Zahn 1992, Maeder & Zahn 1998 → shellular rotation hypothesis

$$\rho\frac{d}{dt}\left(r^2\Omega\right) = \underbrace{\boxed{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4 \Omega U\right)}}_{\text{advection}} + \underbrace{\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4 \rho \nu_v \frac{\partial \Omega}{\partial r}\right)}}_{\text{diffusion}} \quad \underbrace{\boxed{+\ \dot{\tau}_{wind}}}_{\text{wind torque}}$$

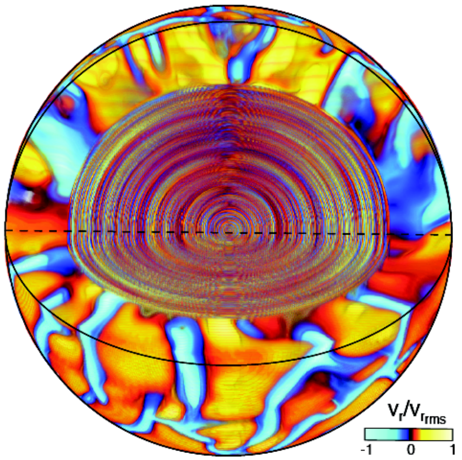
Transport of angular momentum

Internal Gravity Waves

- Propagate in highly stratified media along a favoured direction
- Excitation mechanism : turbulent motions (convective zones edges) / internal stresses
- Conserve angular momentum if not dissipated

IGW generation by overshooting convective plumes in multi-D simulations.

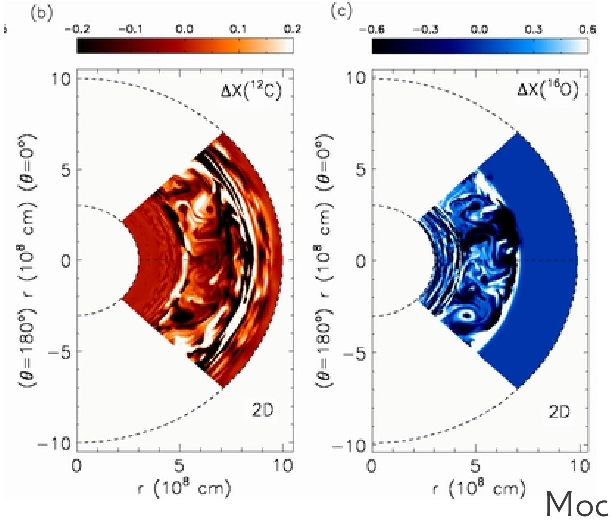
Solar type star at $\Omega=\Omega_\odot$



Alvan et al., 2014, A&A 565, A42

core C flash

O shell burning



IGW in off-center ignition configurations

Mocak et al., 2011, ApJ 743, 55

Transport of angular momentum

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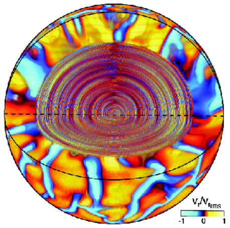
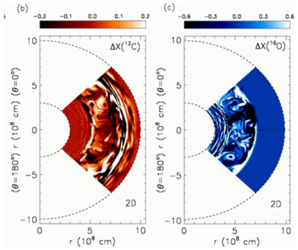




Introduction in stellar evolution codes through the waves luminosity

$$\rho \frac{d}{dt}\left(r^2\Omega\right) = \underbrace{\boxed{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4 \Omega U\right)}}_{\text{advection}} + \underbrace{\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4 \rho \nu_v \frac{\partial \Omega}{\partial r}\right)}}_{\text{diffusion}} + \underbrace{\boxed{\frac{3}{8\pi r^2}\frac{\partial}{\partial r}\left(\mathcal{L}_J\right)}}_{\text{IGW}} \quad \underbrace{\boxed{+\ \dot{\tau}_{wind}}}_{\text{wind torque}}$$

Goldreich et al. 1994, Kumar et al. 1997, Talon & Zahn 1998,
Talon & Charbonnel 2003, 2005, Charbonnel et al. 2013

Transport of angular momentum

Magnetic fields

The magnetic fields affect the transport of both angular momentum and nucleides:

- via magneto-hydrodynamical instabilities

  Magnetic shear instability, Tayler instability

  (Spruit 1999, 2002, Maeder & Meynet 2003)

- via a magnetic torque introduced by the Lorentz couple

$$\frac{\partial(\rho r^2\Omega)}{\partial t} = \underbrace{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4\Omega\left[U - 5\dot{r}\right]\right)}_{\text{Advection}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4\nu_v\frac{\partial\Omega}{\partial r}\right)}_{\text{Diffusion}} + \underbrace{\Gamma_{\mathcal{L},0}(r) - \frac{1}{5}\Gamma_{\mathcal{L},2}(r)}_{\text{Magnetic Torque}}$$

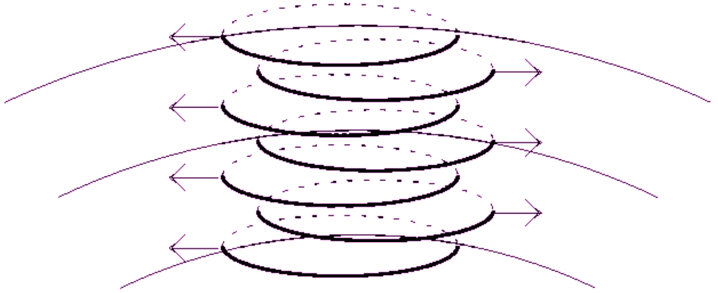
Transport of angular momentum

Magnetic fields

Magnetic fields in stellar stably stratified interiors are thought to be unstable with respect to axisymmetric perturbations.

→ development of magnetohydrodynamical instabilities

The Pitts & Tayler instability is suggested to be the strongest (Spruit 1999).
→ produces a poloïdal field that retroacts to decrease differential rotation



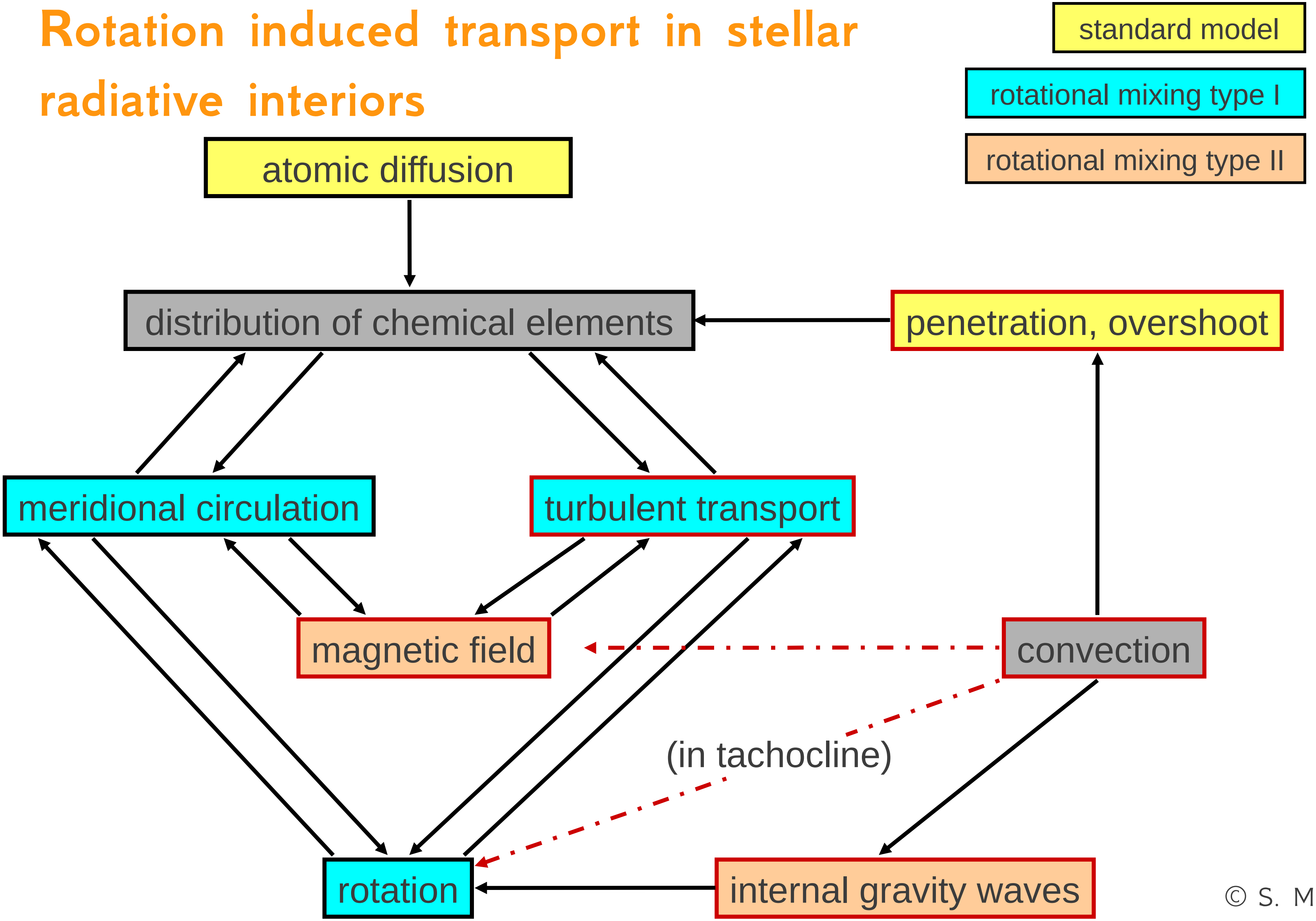
Spruit, 1999, A&A 349, 189 ↓ g

Azimuthal field has unstable displacements along horizontal surfaces

This instability has been claimed to generate dynamo in the radiative interiors and contribute to transport of AM and chemicals (Spruit 1999, 2002, 2006).

This hypothesis is compromised by analytical work, 3D simulations (Zahn, Brun & Mathis 2007) and some confrontations to observations (Maeder & Meynet 2004).

Rotation induced transport in stellar radiative interiors



standard model

rotational mixing type I

rotational mixing type II

atomic diffusion

distribution of chemical elements

penetration, overshoot

meridional circulation

turbulent transport

magnetic field

convection

(in tachocline)

rotation

internal gravity waves

© S. Mathis

Transport of nucleides

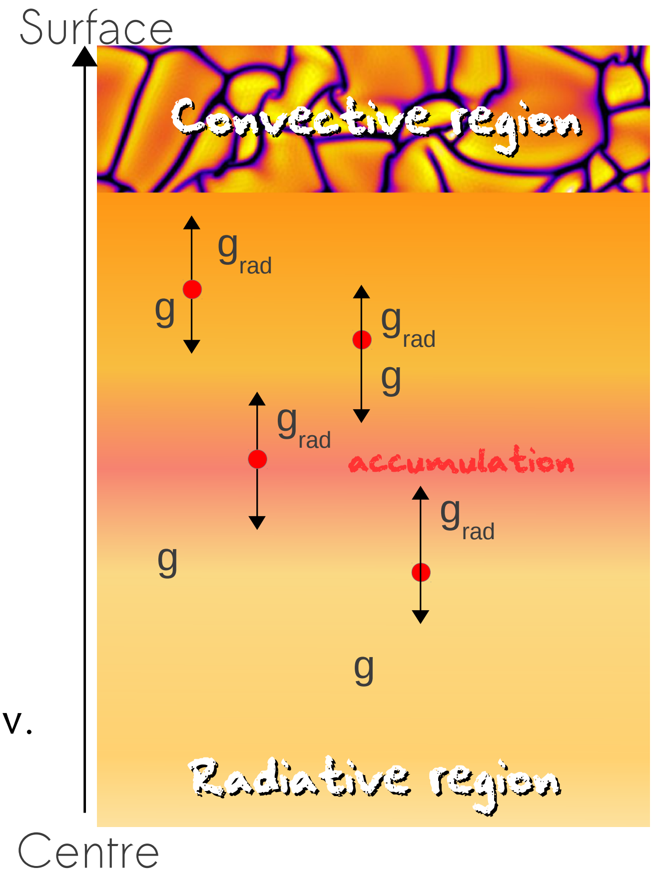
Atomic Diffusion → transport of nucleides

Multicomponent gas

Stratification maintly due to competition between gravitational settling and radiative levitation

Competing transport processes may hinder diffusion

Atomic diffusion coefficient → collision integrals
Atomic diffusion velocity = grav. sett. + thermal diff. + rad. lev.

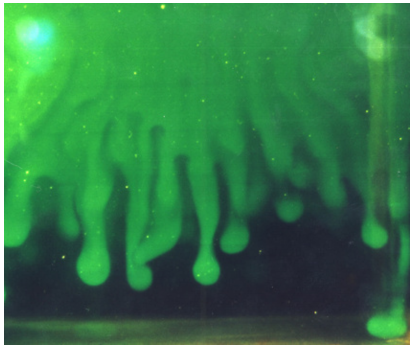


Full equation for the evolution of the concentration of nucleide i

$$\rho\frac{\partial c_i}{\partial t} = -\nabla\cdot\Big(\overset{\text{atomic diffusion}}{\boxed{-\rho D_{\mathrm{mic}}\nabla\ln c_i + \rho v_{\mathrm{diff,i}} c_i}} + \overset{\text{competing processes}}{\boxed{\rho v c_i + \rho(D_{\mathrm{conv}} + D_{\mathrm{turb}})\nabla\ln c_i\Big)}} - \overset{\text{nuclear}}{\boxed{\rho\lambda_i c_i}}$$

Adapted from O. Richard 2014

Transport of nucleides



Double-diffusive (thermohaline) instability → transport of nucleides

Develops in the astrophysical context in regions of stably startified entropy and unstably stratified nucleides

$$\nabla - \nabla_{ad} < 0 \text{ and } \nabla_{\mu} > 0$$

Occurs in regions where ^{3}He(^{3}He,2p)^{4}He and where heavy elements accumulate (A-type stars)

Ulrich 1972 / Kippenhahn 1980 / Charbonnel & Zahn 2007

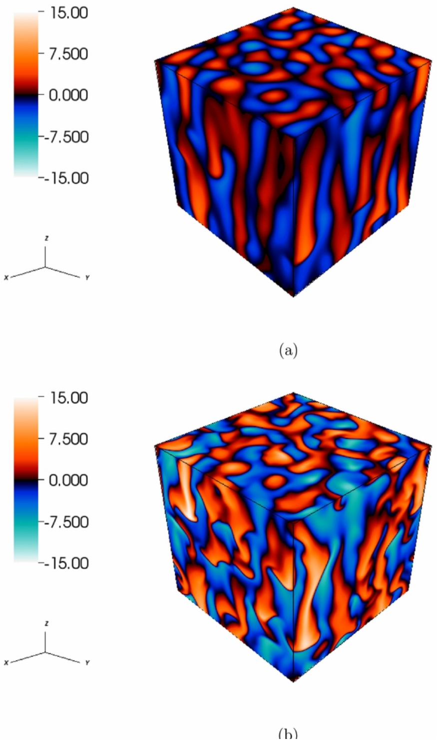
$$D_{thm} = C_t K \frac{\varphi}{\delta} \frac{-\nabla_{\mu}}{\nabla - \nabla_{add}} \quad \text{and} \quad K = \frac{4acT^3}{3\kappa\rho^2 c_P} \quad \text{with} \quad C_t = \frac{8}{3}\pi^2\alpha^2$$

α = aspect ratio of the fingers

Brown et al. 2013

$$D_{\mu} = Nu_{\mu}\, \kappa_{\mu}$$

Nusselt number = total vertical flux / diffused flux of nucleides
depends on the Prandt number, on the ratio btw thermal and compositional diffusivity, on the gradients ratio and on a parameter C of order 10.



Brown, Garaud & Stellmach 2013

Transport of nucleides

Winds → transport of nucleides

In stars with strong impact of atomic diffusion (A-type)

Weak mass loss → appearance of an outwardflowing wind which is represented as an advection term in the transport equation.

Charbonneau 1993, ApJ 405, 720
Vick et al., 2010, 2011, 2013

In rotating massive stars with strong radiative winds

$$g_{\mathrm{tot}} = g_{\mathrm{eff}} + g_{\mathrm{rad}} = g_{\mathrm{grav}} + g_{\mathrm{rot}} + g_{\mathrm{rad}}$$

Mass loss is globally enhanced by rotation → stronger torques
→ stronger differential rotation → impact on turbulent transport of nucleides

$$\boxed{\frac{\dot{M}(\Omega)}{\dot{M}(0)} = \frac{(1-\Gamma)^{\frac{1}{\alpha}-1}}{\left[1-\frac{\Omega^2}{2\pi G\rho_{\mathrm{m}}}-\Gamma\right]^{\frac{1}{\alpha}-1}}}$$

Maeder & Meynet 2000

Adapted from O. Richard 2014

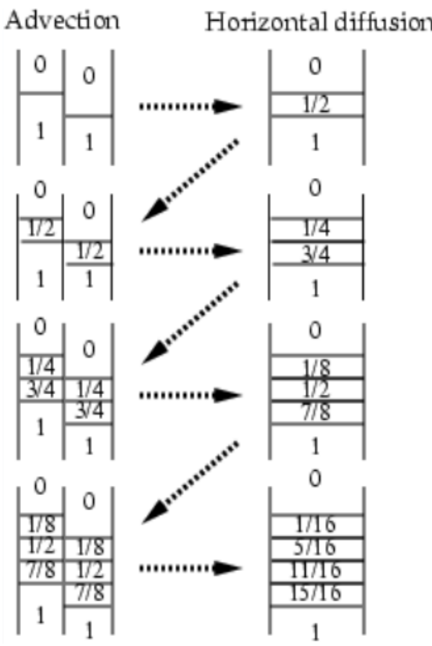
Transport of nucleides

Rotation → transport of nucleides

Rotation induced mixing (meridional circulation, instabilities) modelled as diffusive processes

$$\rho\left(\frac{d\overline{c_i}}{dt}\right)_{M_r} = \boxed{\dot{c_i}} + \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho V_{mic}\overline{c_i}\right]} + \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho\left(D_{\rm eff}+D_v\right)\frac{\partial\overline{c_i}}{\partial r}\right]}$$

nuclear microscopic processes Turbulence + merid. circ.



(Chaboyer & Zahn 1992, Zahn 1992, Maeder & Zahn 1998)

$$\rho r^2\frac{dX_i}{dt} = \boxed{f_c f_\omega \frac{d}{dr}\left(\rho r^2 D\frac{dX_i}{dr}\right)}$$

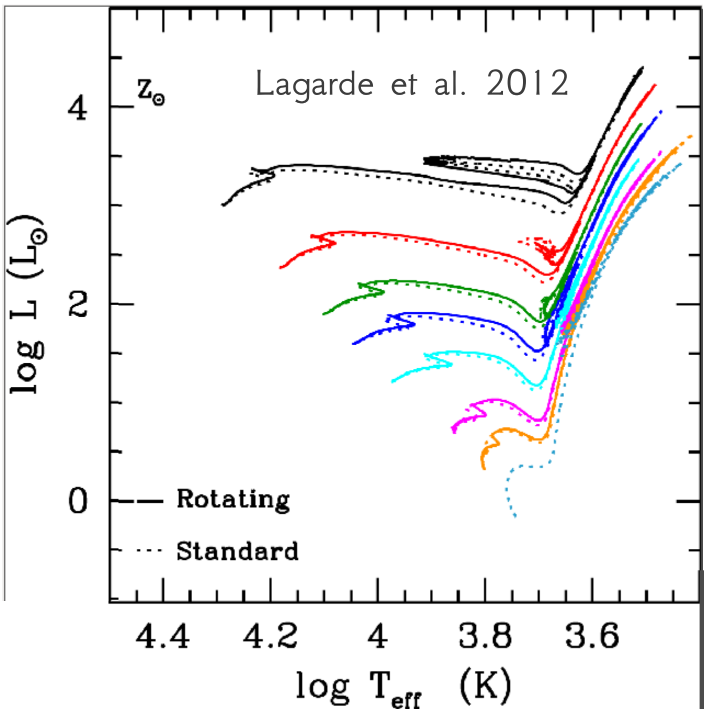
hydrodynamical instabilities

(Pinsonneault et al. 1989, Heger et al. 2000)

Application to stellar evolution modelling

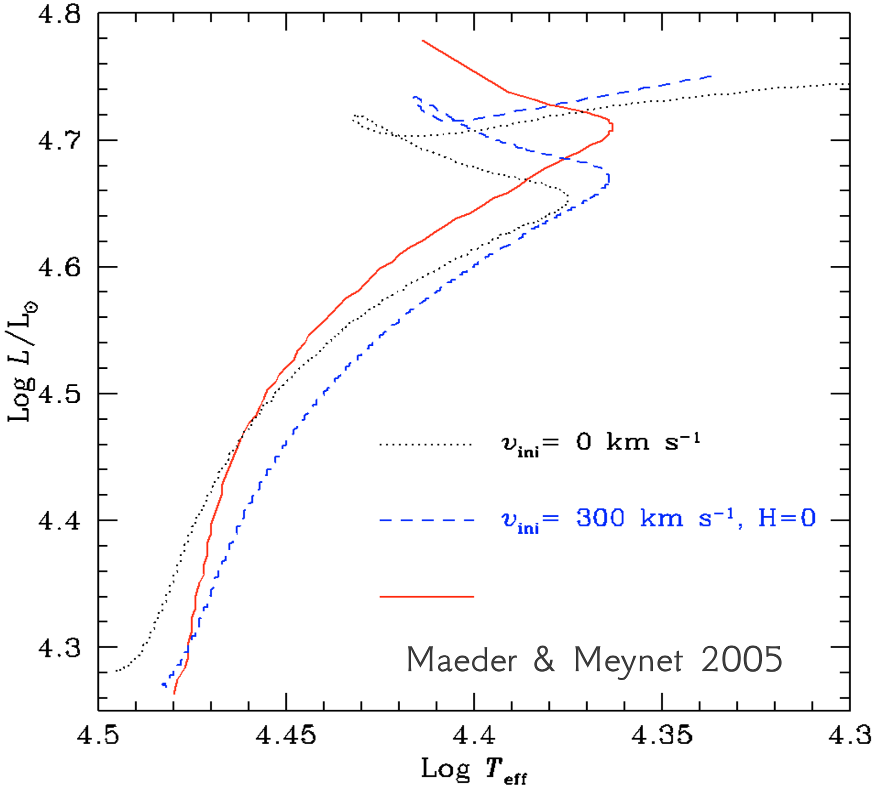
Focus on AM evolution in solar-type stars

Effect on stellar structure and evolution



Lagarde et al. 2012

Type I rotational mixing



Maeder & Meynet 2005

Type II rotational mixing with magnetic fields (massive star)

The lifetimes of rotating models on the main sequence are longer than those of the standard models. Indeed rotation-induced mixing brings fresh hydrogen fuel into the stellarcore during that phase. As a consequence, the exhaustion ofhydrogen in the central region is delayed and the lifetime on the main sequence lengthens; in addition, the mass ofthe helium-core is larger at the end of the main sequence when rotation is accounted for

Abundance patterns in the light of extra-mixing processes

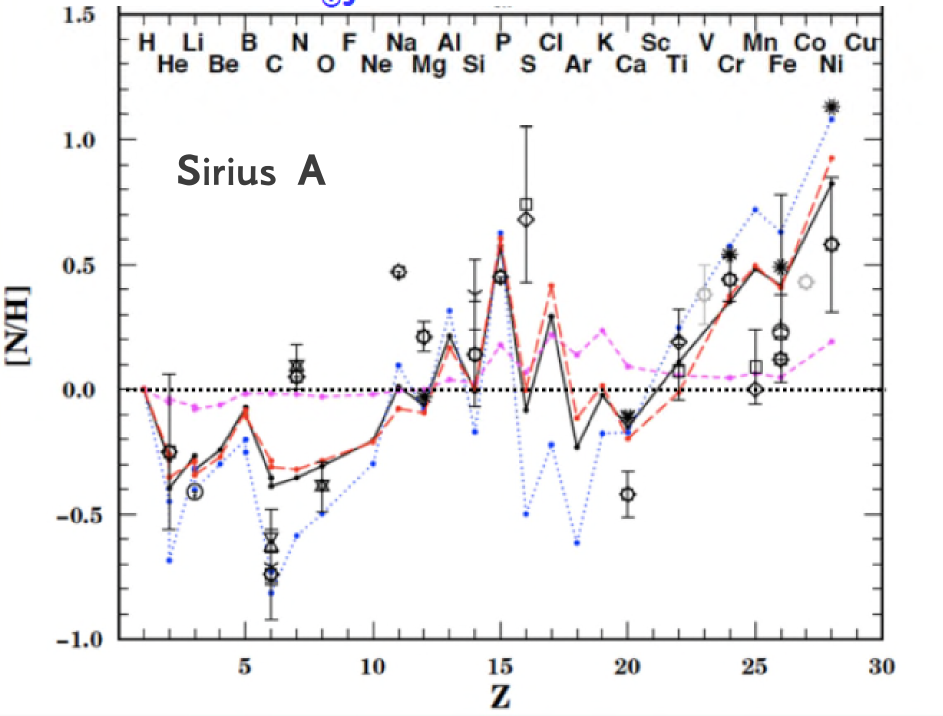
mass loss rates:

$1 \times 10^{-12}\ M_{\odot} yr^{-1}$;

$1 \times 10^{-13}\ M_{\odot} yr^{-1}$;

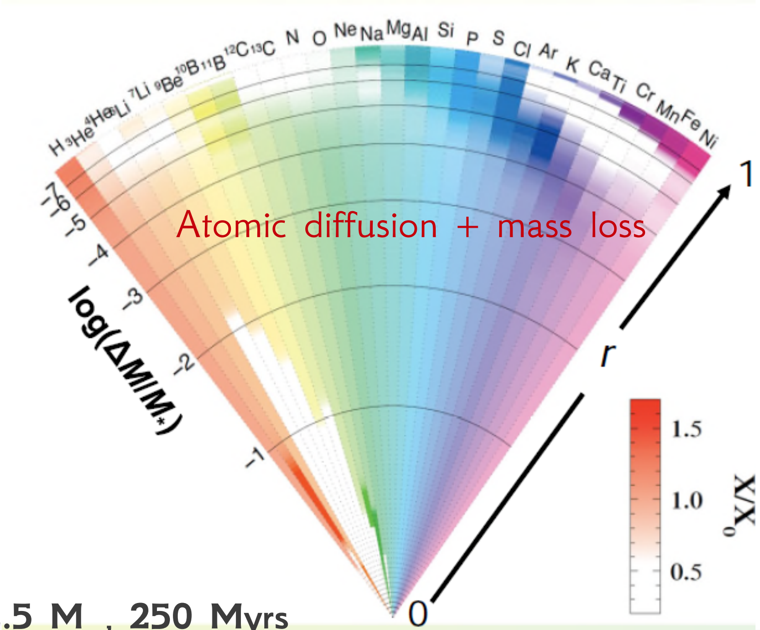
$5 \times 10^{-14}\ M_{\odot} yr^{-1}$

Mixing T5.20D1M-4



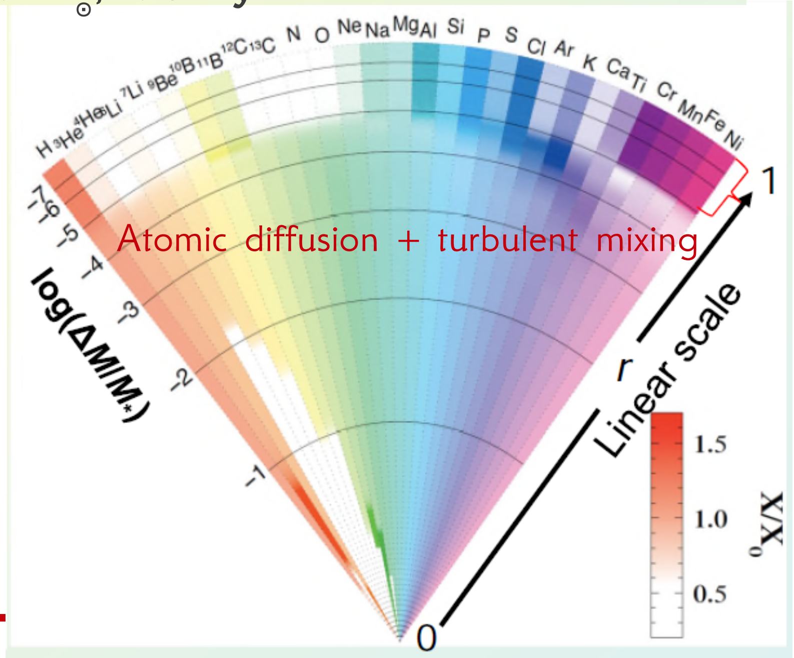
Atomic diffusion

Vick et al. 2010



Atomic diffusion + mass loss

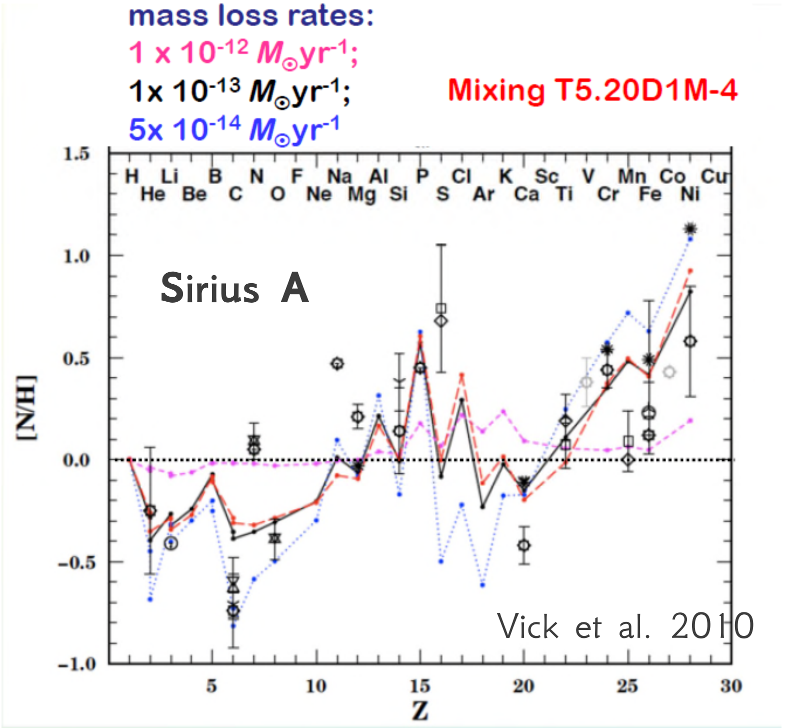
2.5 $M_{\odot}$, 250 Myrs



Atomic diffusion + turbulent mixing

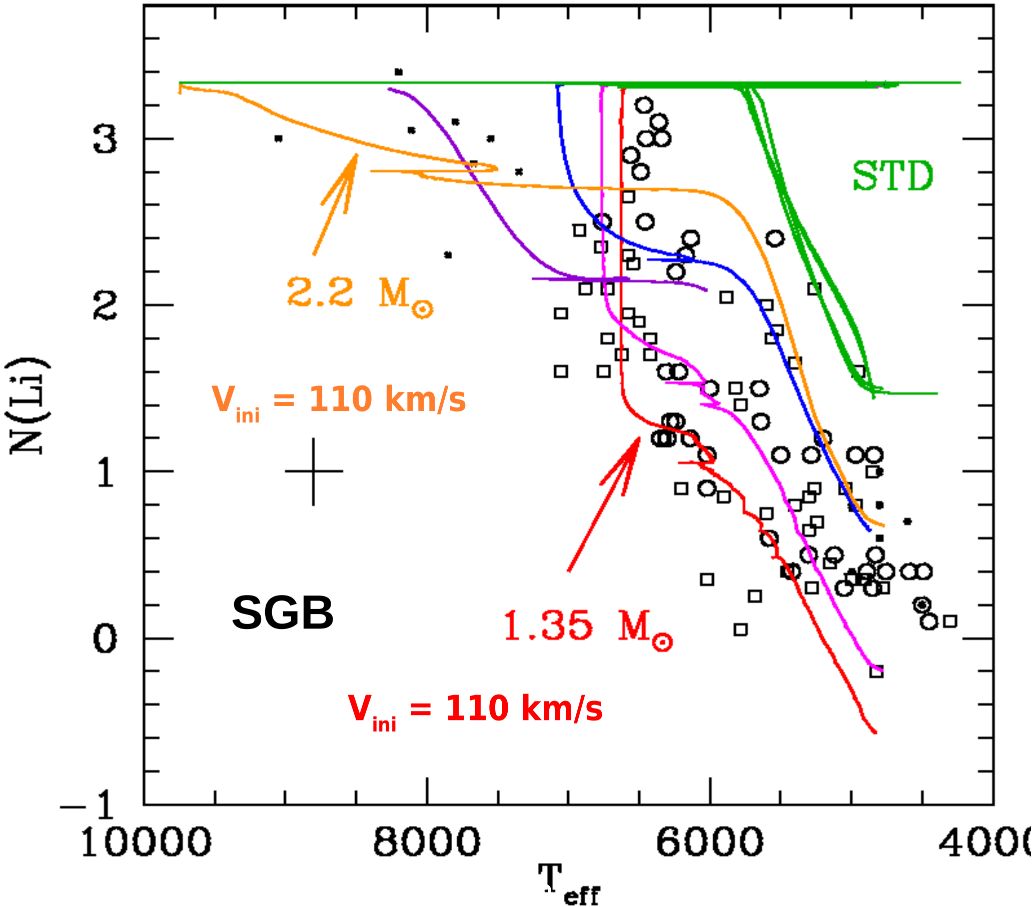
© O. Richard

Abundance patterns in the light of extra-mixing processes



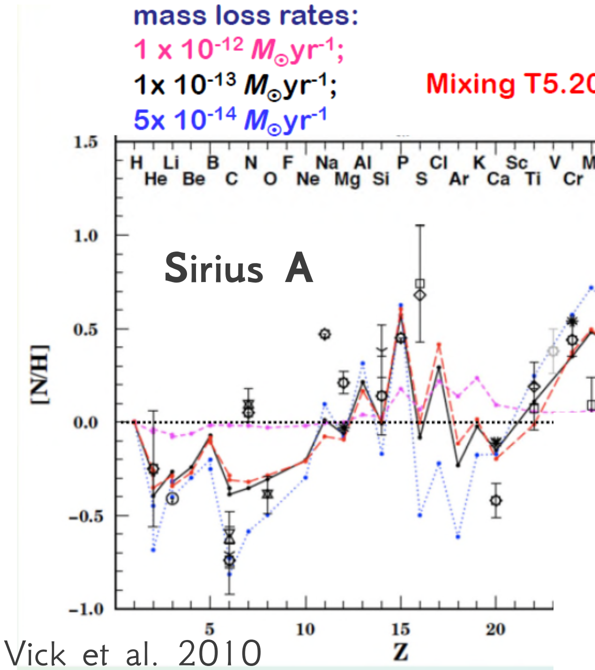
Atomic diffusion

Type I rotational mixing



Palacios et al. 2003

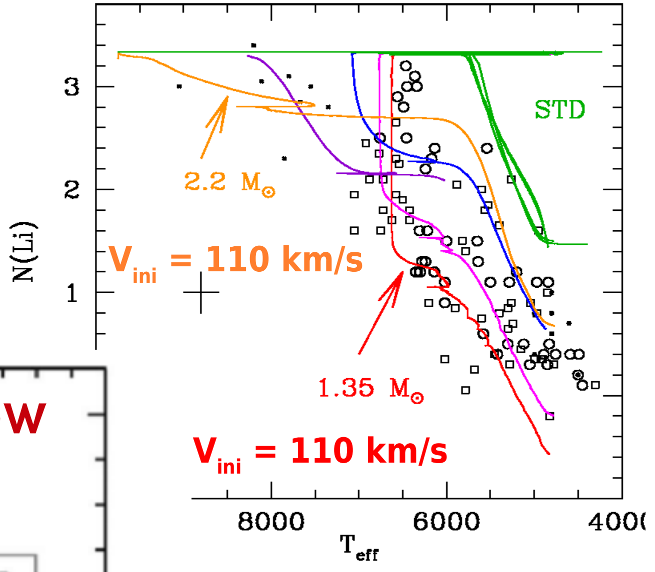
Abundance patterns in the light of extra-mixing processes



Vick et al. 2010

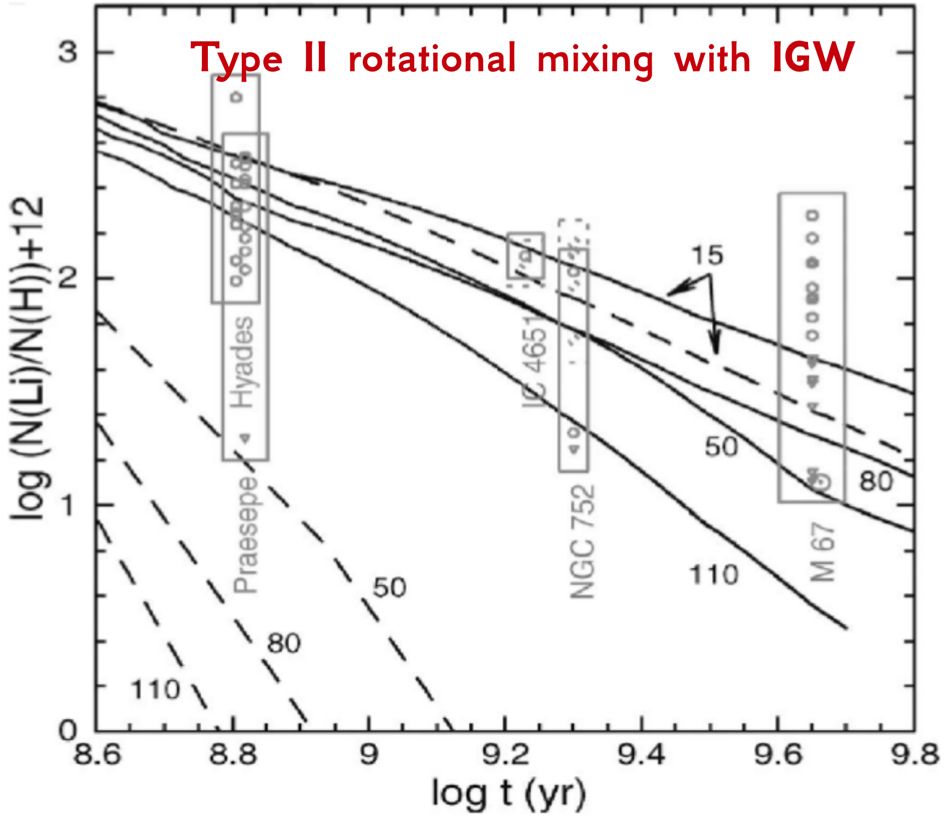
Atomic diffusion

Type I rotational mixing



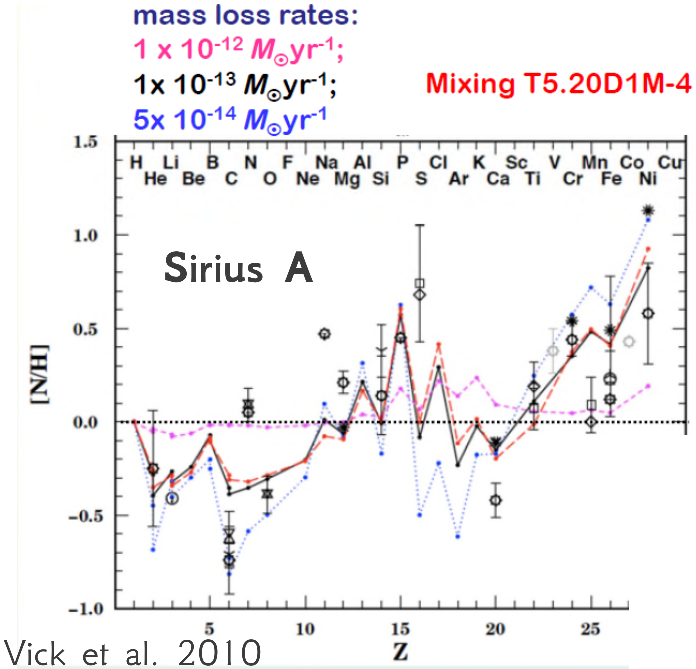
Palacios et al. 2003

Type II rotational mixing with IGW



Charbonnel & Talon 2005

Abundance patterns in the light of extra-mixing processes



Vick et al. 2010

Atomic diffusion

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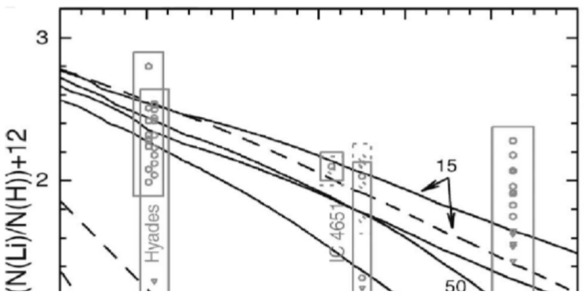
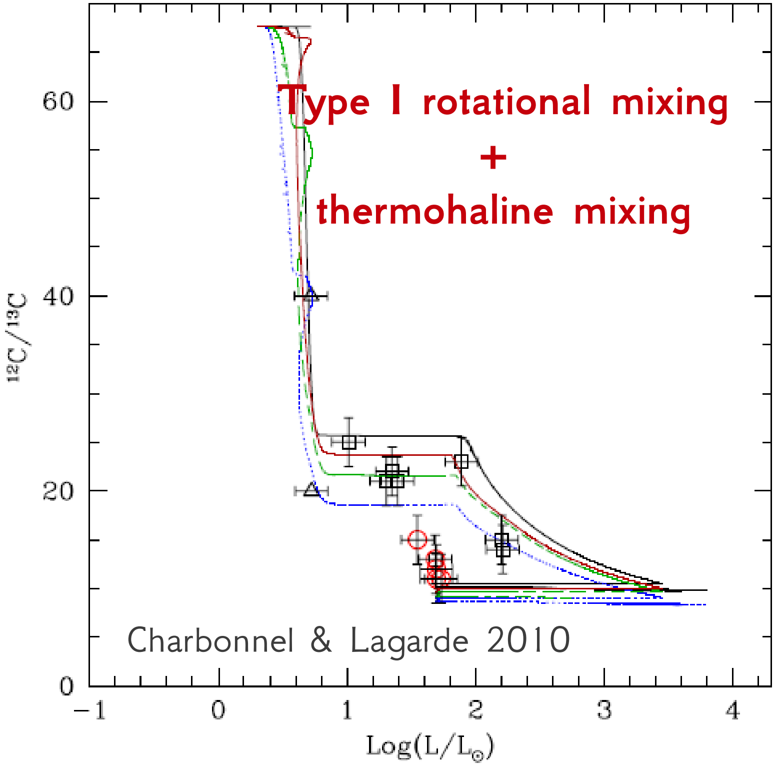
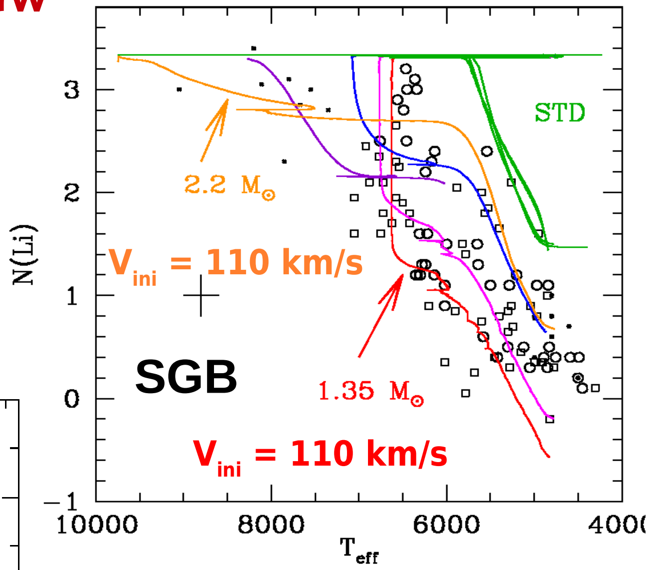




Type I rotational mixing + thermohaline mixing

Charbonnel & Lagarde 2010

Type I rotational mixing

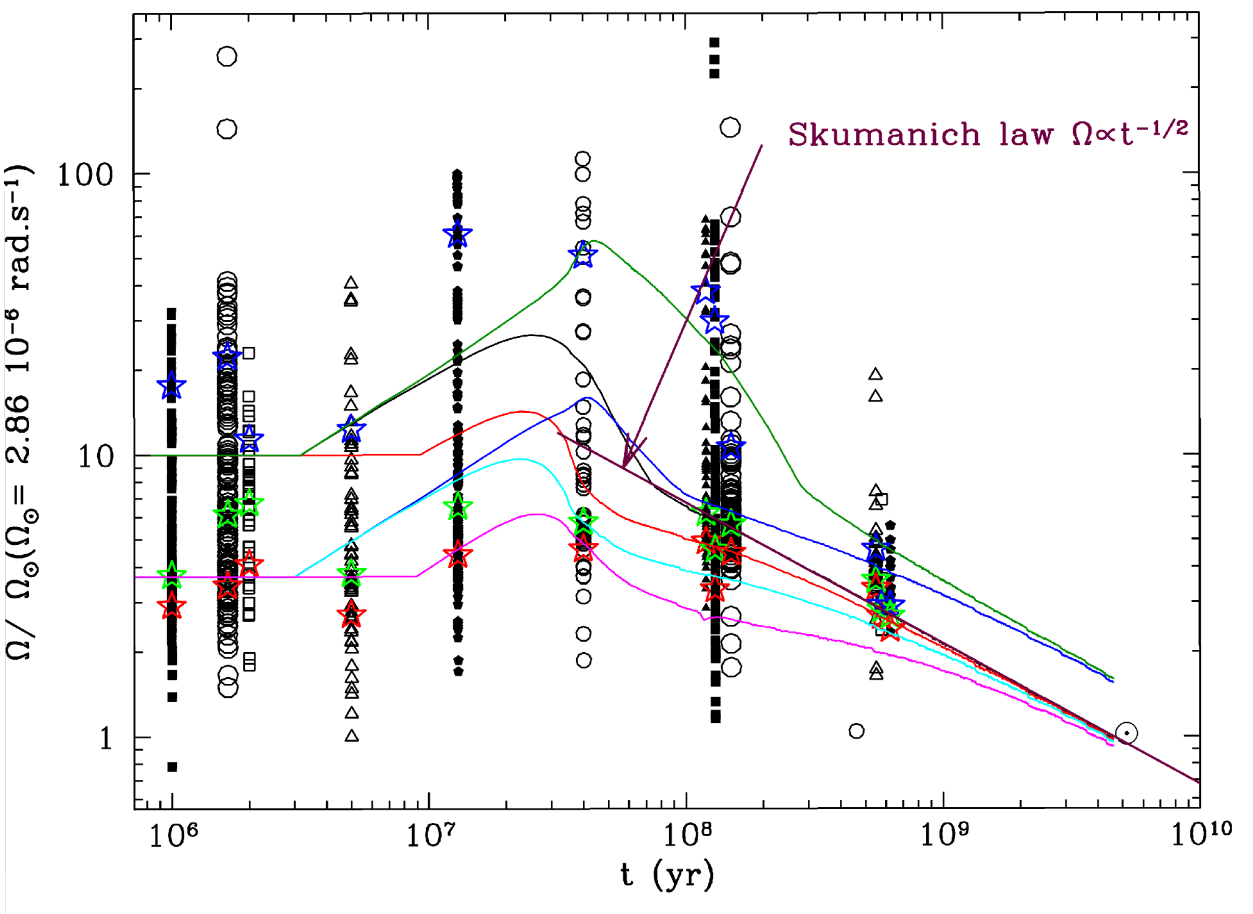


Palacios et al. 2003

Understanding the rotational evolution of low-mass stars

Pre-Main Sequence

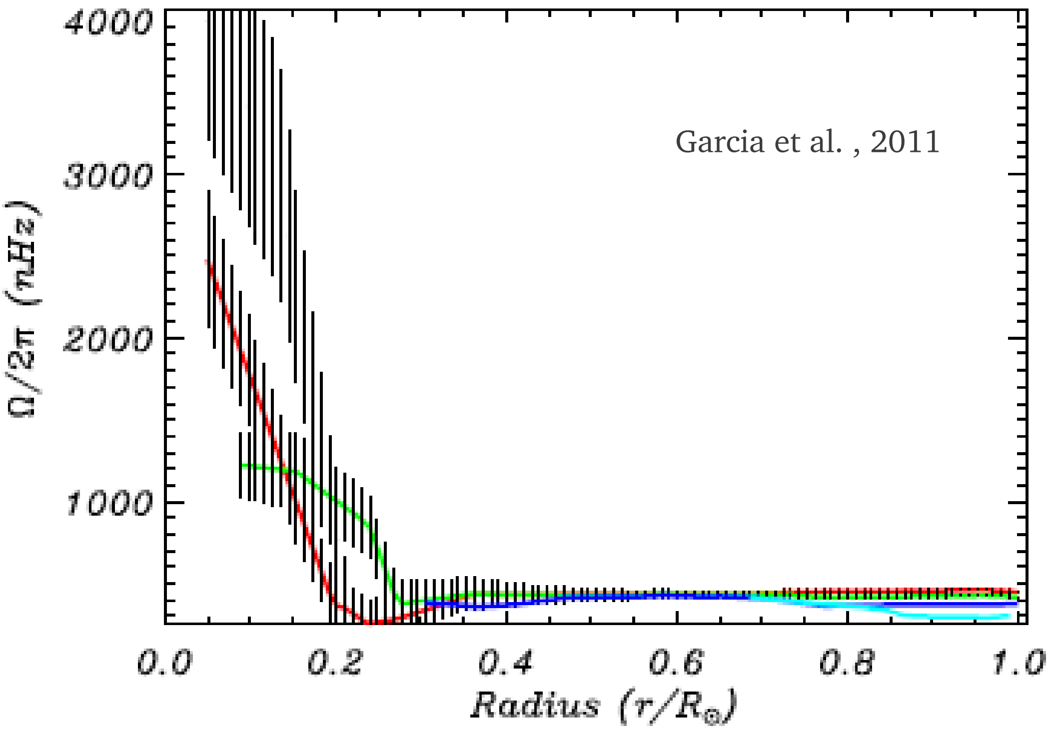
Models with angular momentum transport by meridional circulation, turbulence and wind braking give a satisfactory surface angular velocity evolution of the median rotators



Understanding the rotational evolution of low-mass stars

Main Sequence

Internal solar rotation profile inversion using 4608 days of MDI and GOLF data.

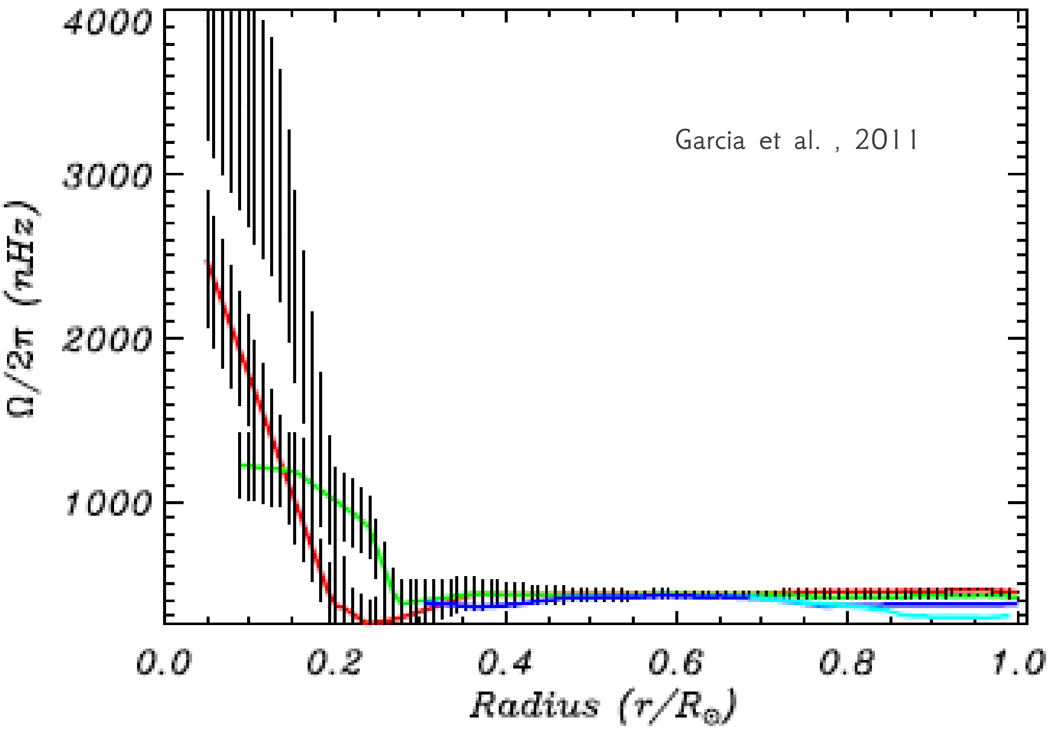


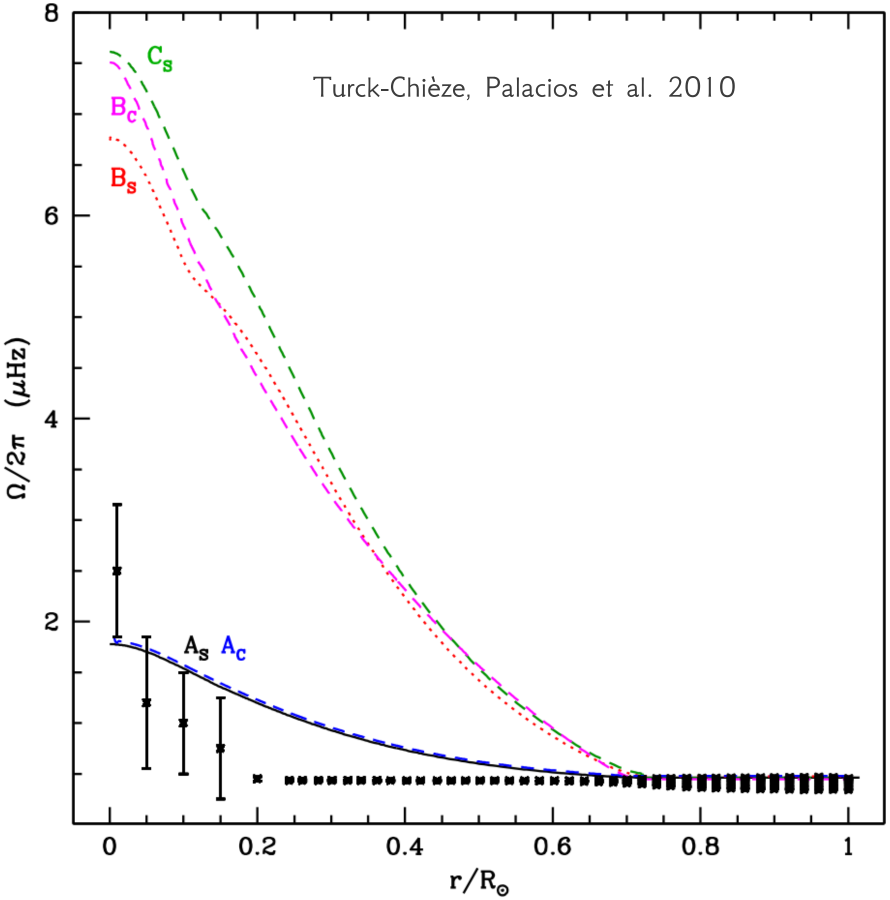
Understanding the rotational evolution of low-mass stars

Main Sequence

Internal solar rotation profile inversion using 4608 days of MDI and GOLF data.

Marginal agreement if the Sun was a slow rotator



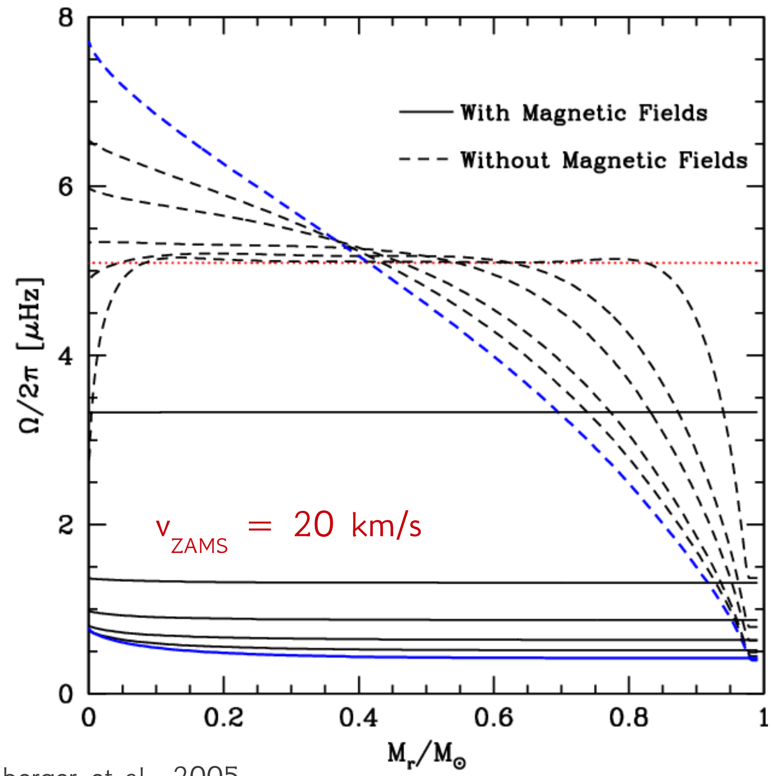


Understanding the rotational evolution of low-mass stars

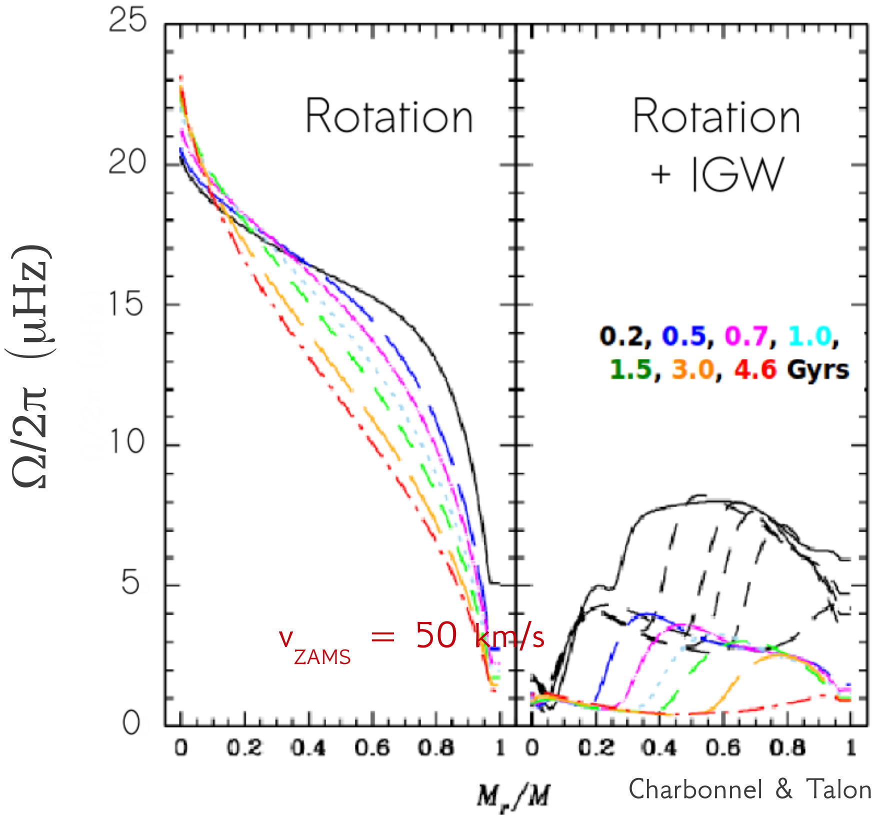
Main Sequence

Meridional circulation and shear fail to efficiently couple the core and the envelope of solar-type stars also during the main sequence evolution.

IGW and magnetic fields are the best candidate processes to generate the coupling



Eggenberger et al., 2005

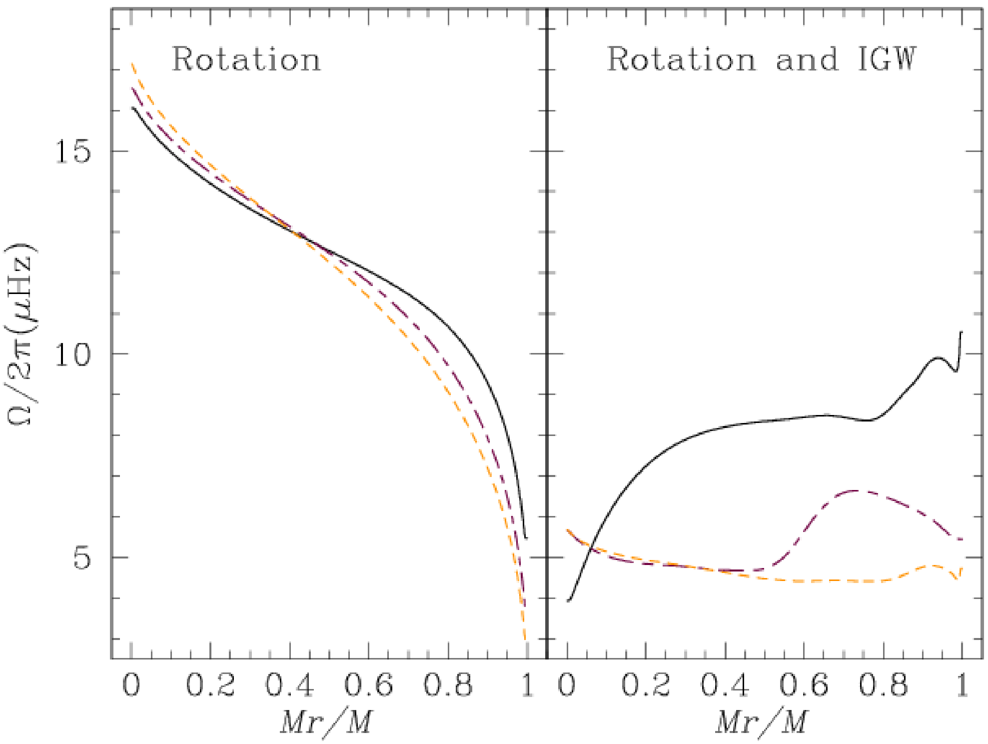


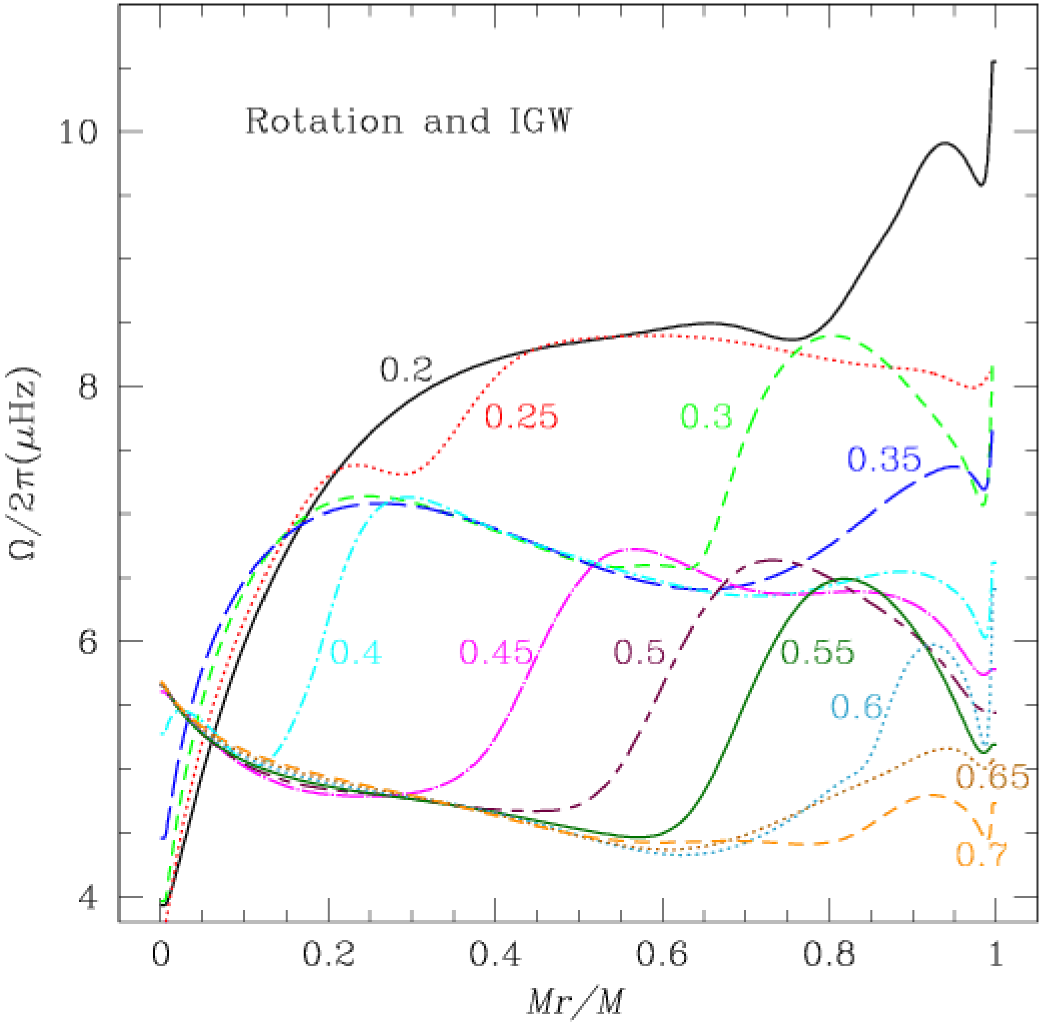
Charbonnel & Talon, 2005

Understanding the rotational evolution of low-mass stars

Main Sequence

1.2 Msun, Z = 0.02, v_{ZAMS} = 50 km/s, Kawaler torque





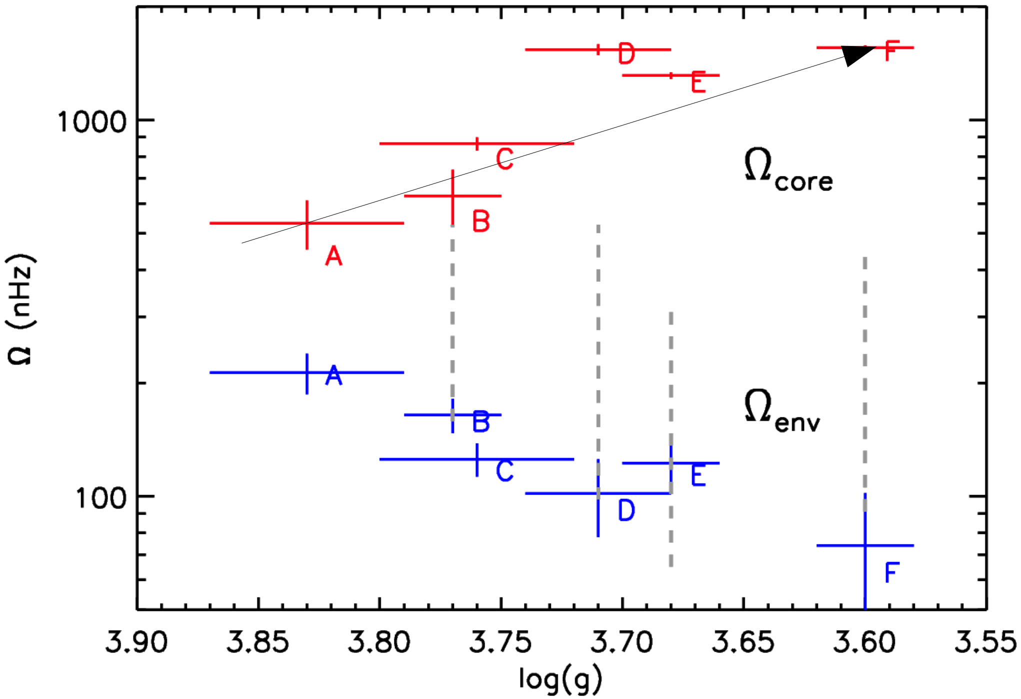
Talon & Charbonnel, 2005, A&A 440, 981

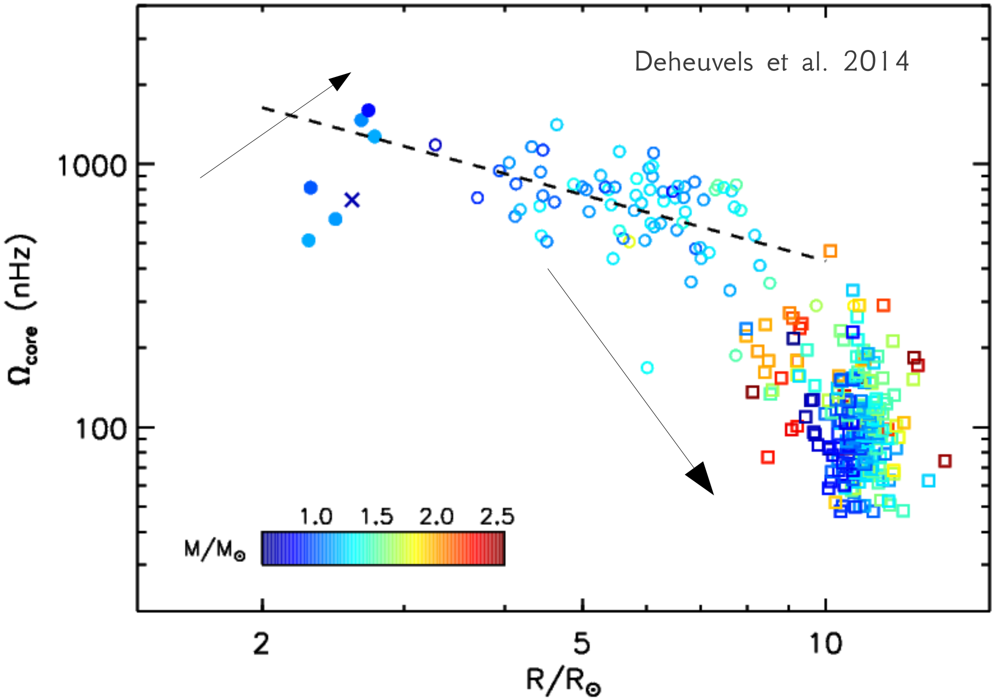
Asteroseismology applied to evolved low-mass stars

SubGiant and Red Giant Branches / Clump

Inversion of the rotation gradient in subgiant branch stars from the Kepler fields

The direction of the arrows indicates increasing age

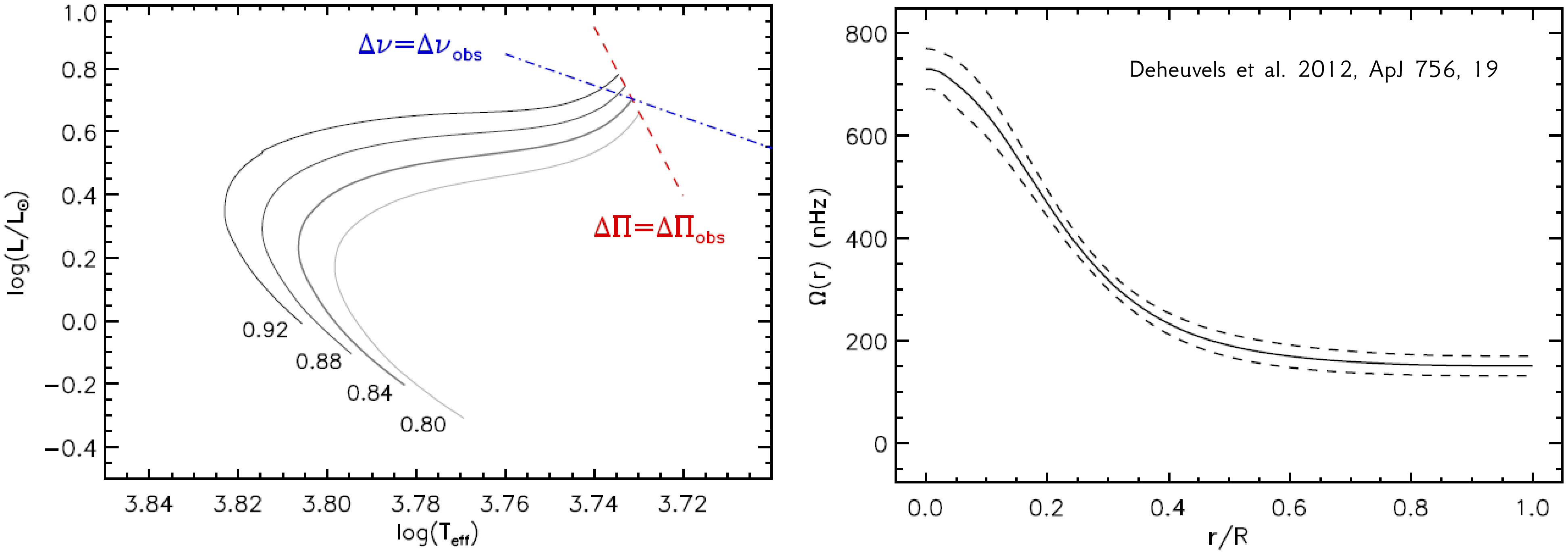




Asteroseismology applied to evolved low-mass stars

SubGiant and Red Giant Branches / Clump

Small amount of differential rotation deduced from low red giant branch stars

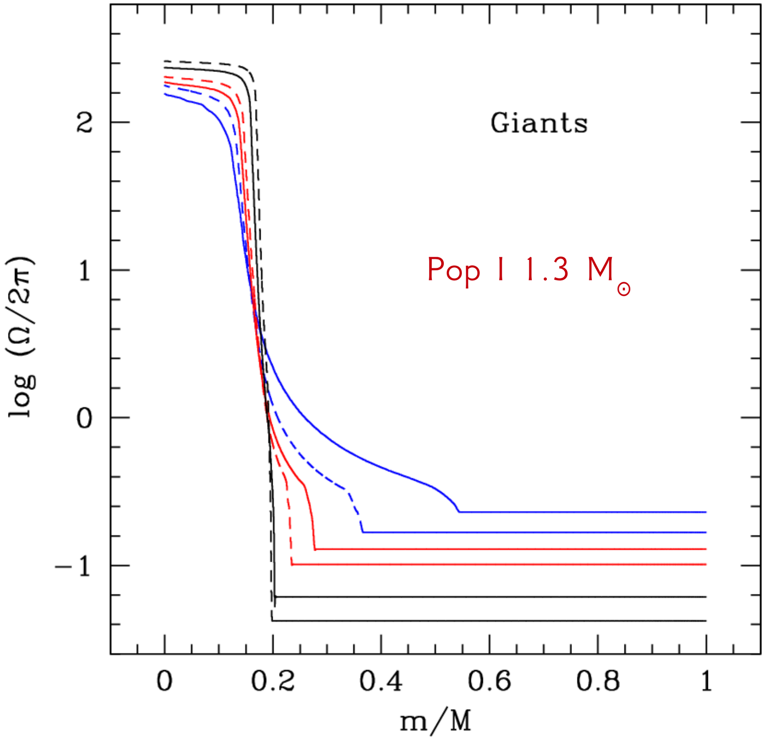


Deheuvels et al. 2012, ApJ 756, 19

Modelling AM transport in evolved low-mass stars

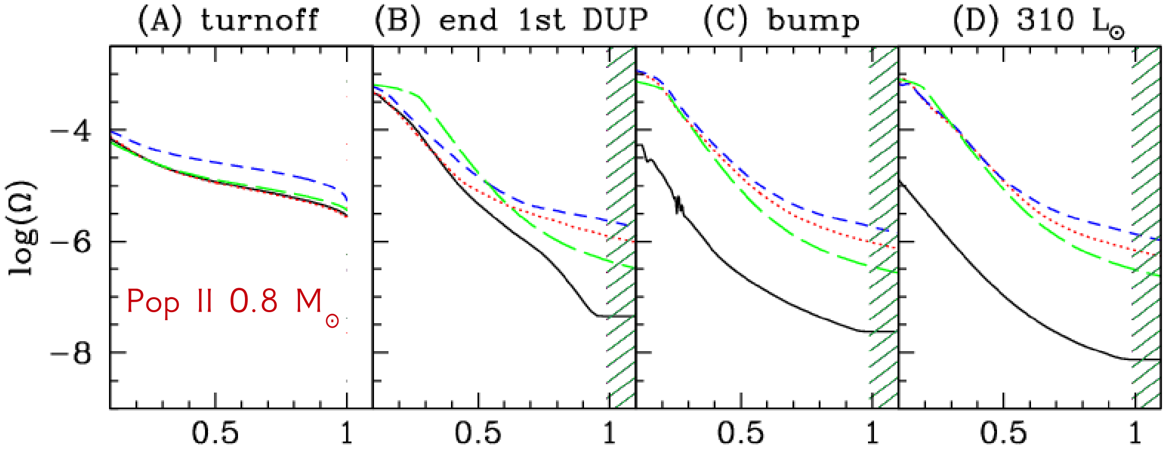
SubGiant and Red Giant Branches / Clump

Rot. transport of type I is inefficient during the post-MS evolution to transport AM



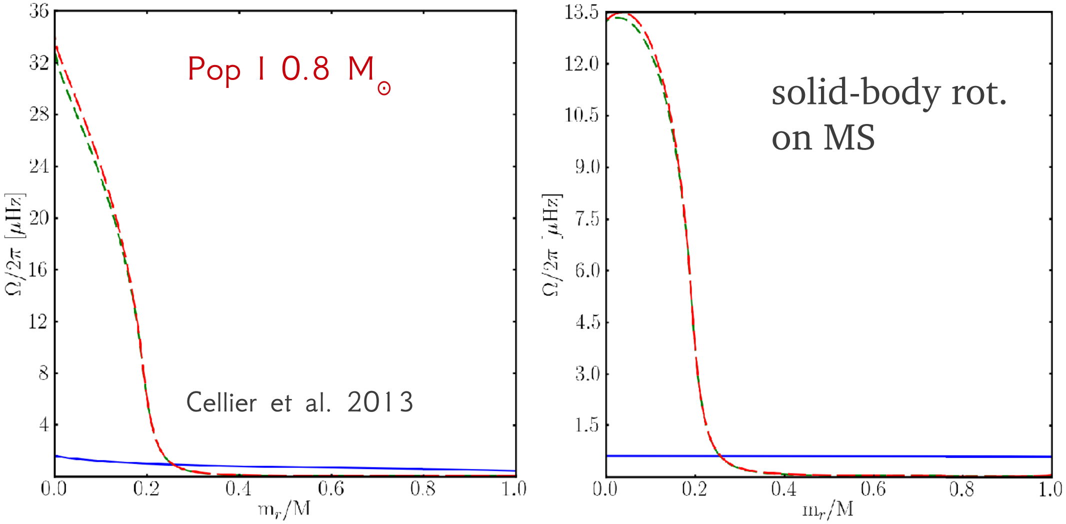
Pop I 1.3 $M_\odot$

Marques et al., 2013



Pop II 0.8 $M_\odot$

Palacios et al. 2006



Pop I 0.8 $M_\odot$

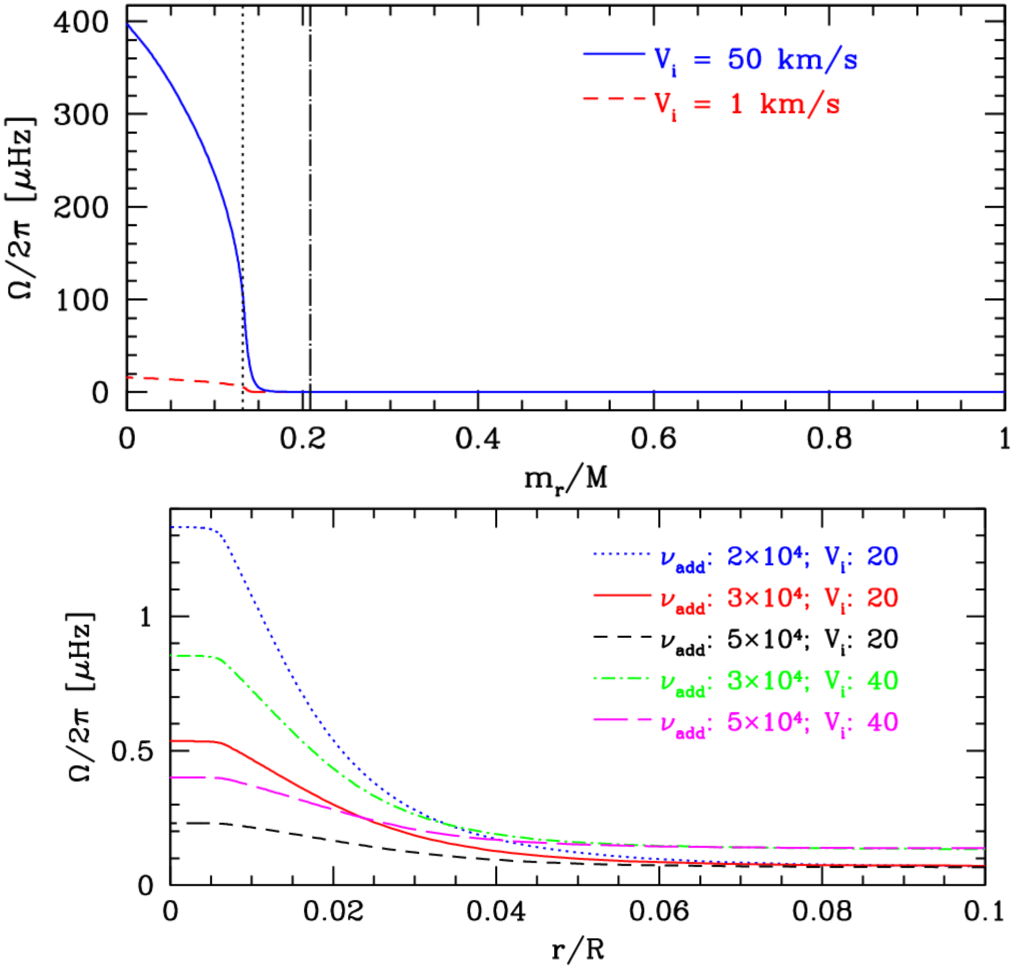
solid-body rot. on MS

Cellier et al. 2013

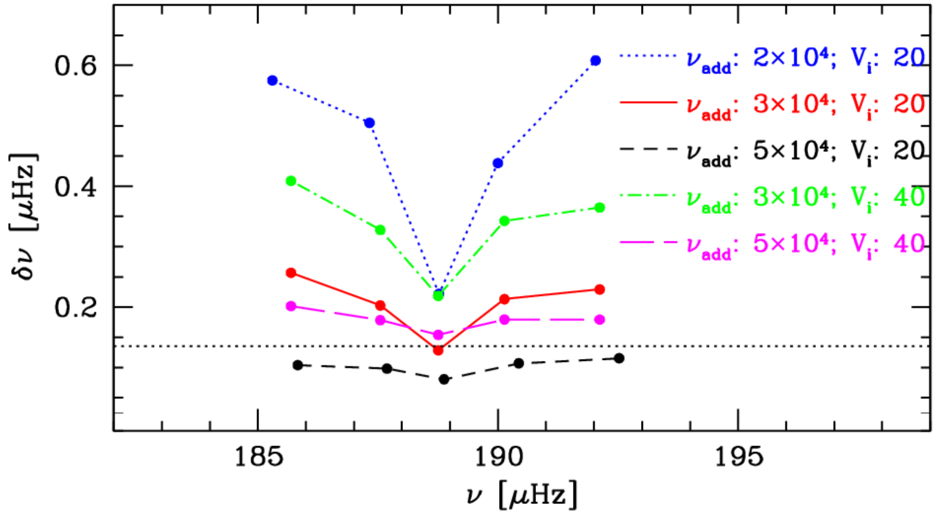
Modelling AM transport in evolved low-mass stars

SubGiant and Red Giant Branches / Clump

Improvement is found when adding a viscous transport process for AM only
→ nature unknown



See also Poster 26 by N. Lagarde et al.



Eggenberger et al. 2012

Conclusions and Open questions

Importance of transport processes in radiative zones on the structure and evolution of stars

Limitations of the modelling (apart from the 1-D limitation)

- What about coupling and interaction between processes?
- What about magnetic fields in the end?
- What is the flux of IGW and how efficient are they for AM transport?
- Efficiency of thermohaline mixing?

Impact of interplay between atomic diffusion and other transport processes on the pulsational behaviour of A-type stars

Asteroseismology points to yet other unidentified AM transport processes in low-mass red giants

→ Need to understand the integrated AM evolution self-consistently