A journey through Bayesian Magic Land

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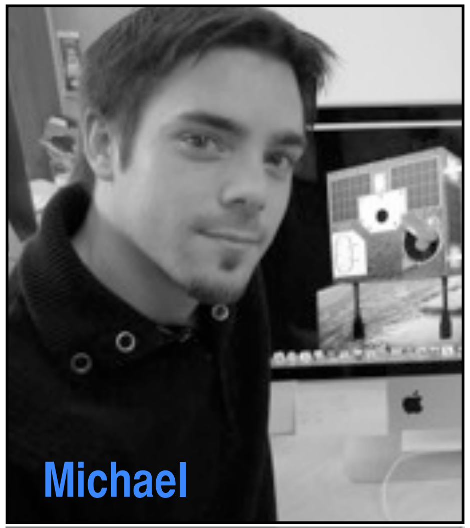
with your tour guides **Thomas Kallinger and Michael Gruberbauer**

why do I stand here?



first of all... because Michael left Astronomy and got a 'real' job (in Meteorology)

I am NOT an expert in 'Bayesian statistics'

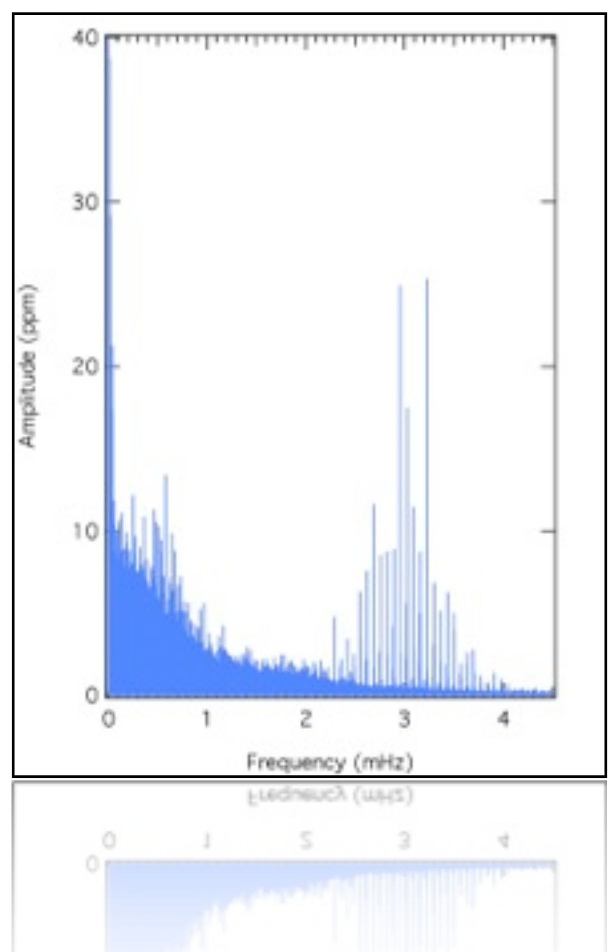
... but I am an experienced user

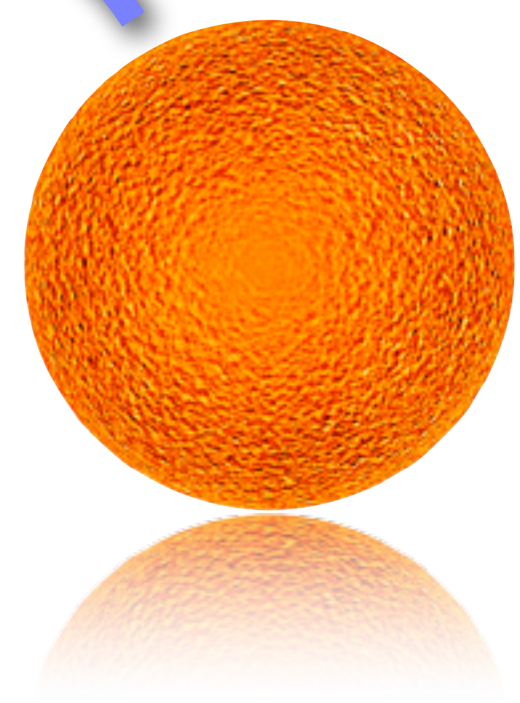
anonymous user

what I am talking about?

Session: Probing stellar structure and evolution with 'asteroseismology'

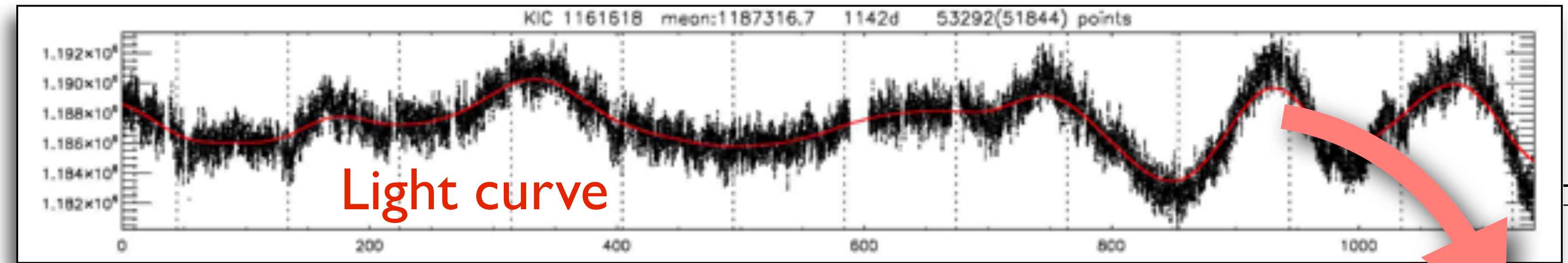
asteroseismology, *n.*
The study of the interior of stars by the observation and analysis of oscillations at their surface. Cf. helioseismology *n.*
[Oxford English Dictionary]

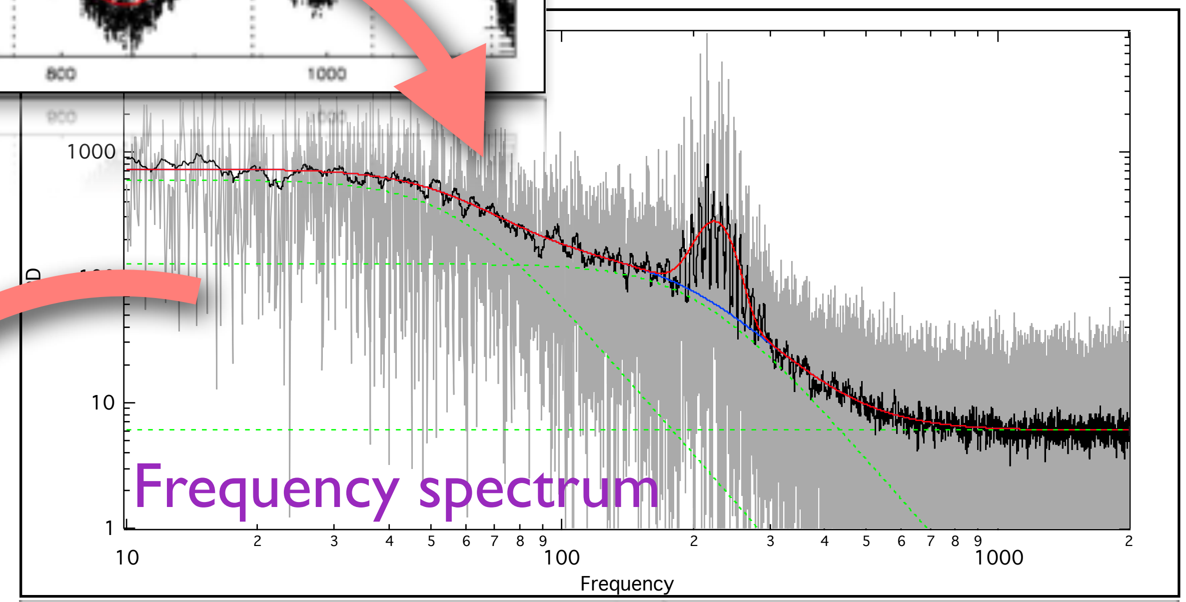


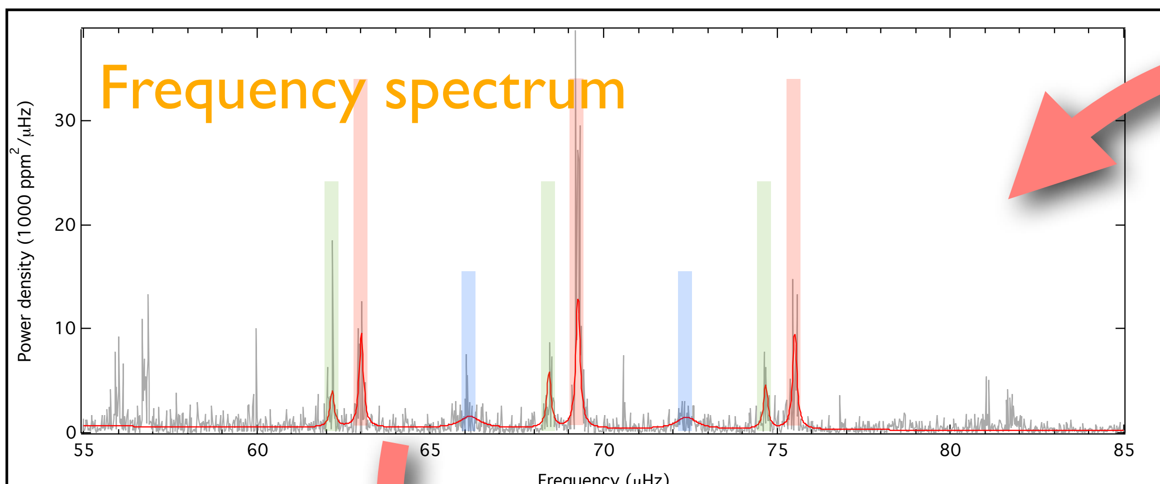


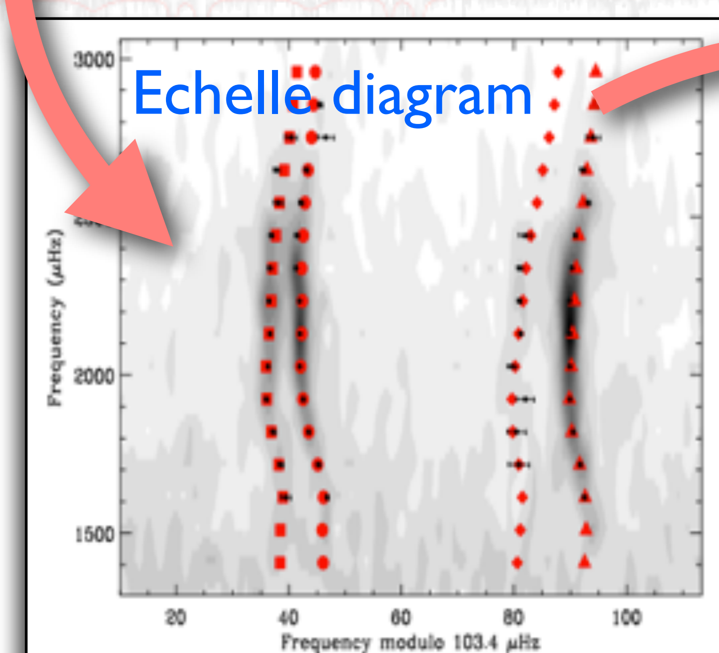
how do we get there?

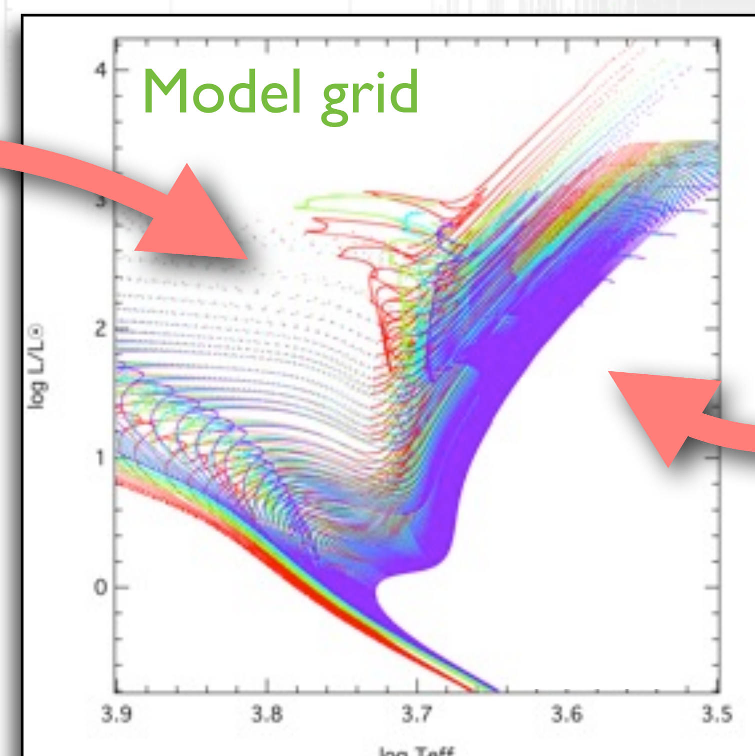
one popular path!!!

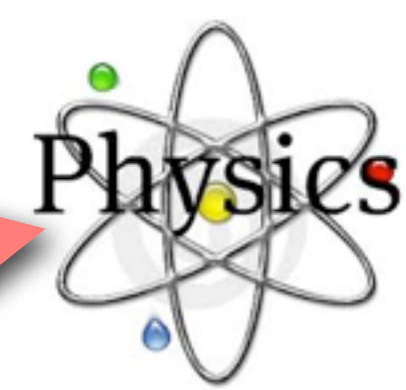






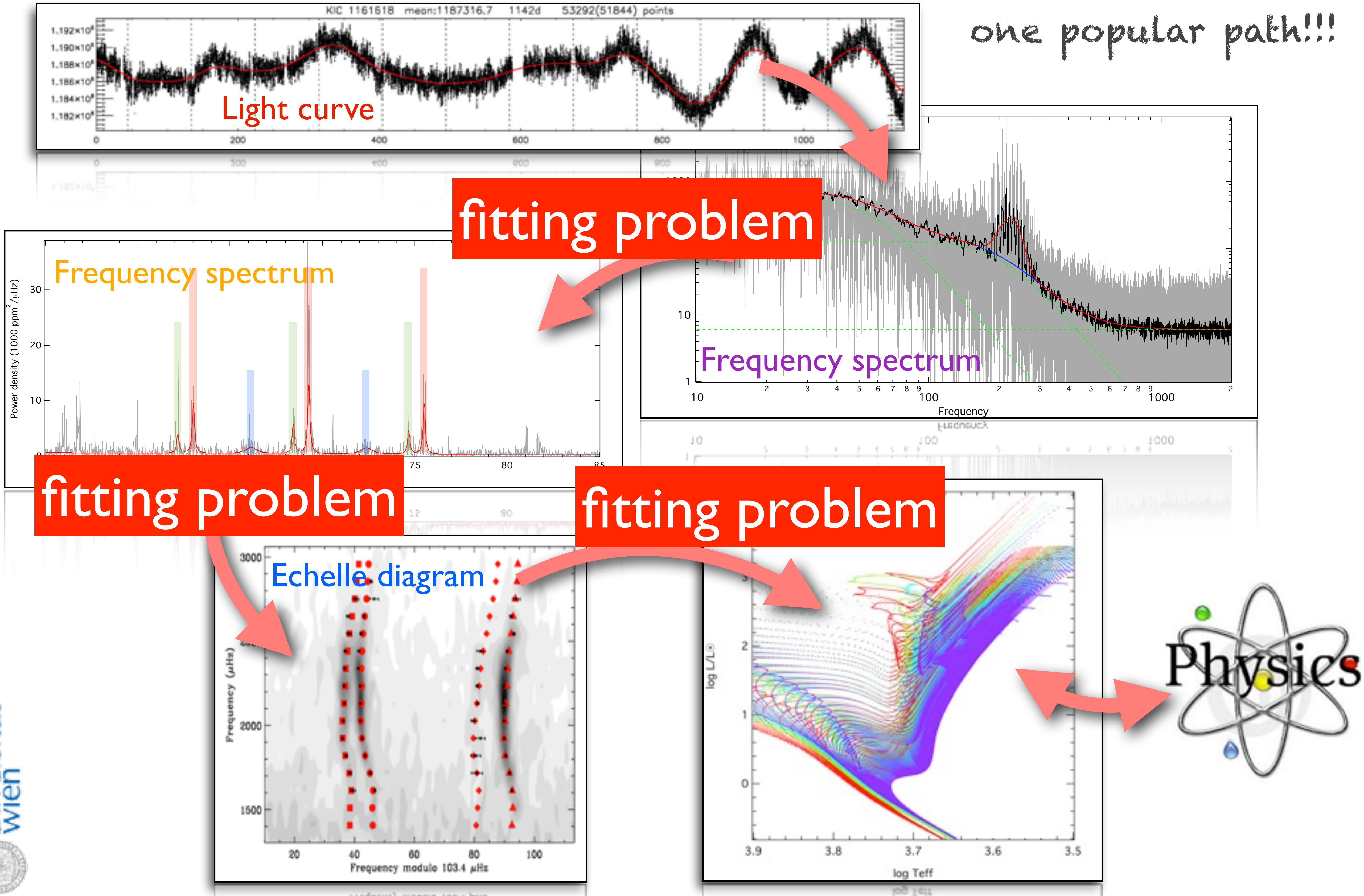




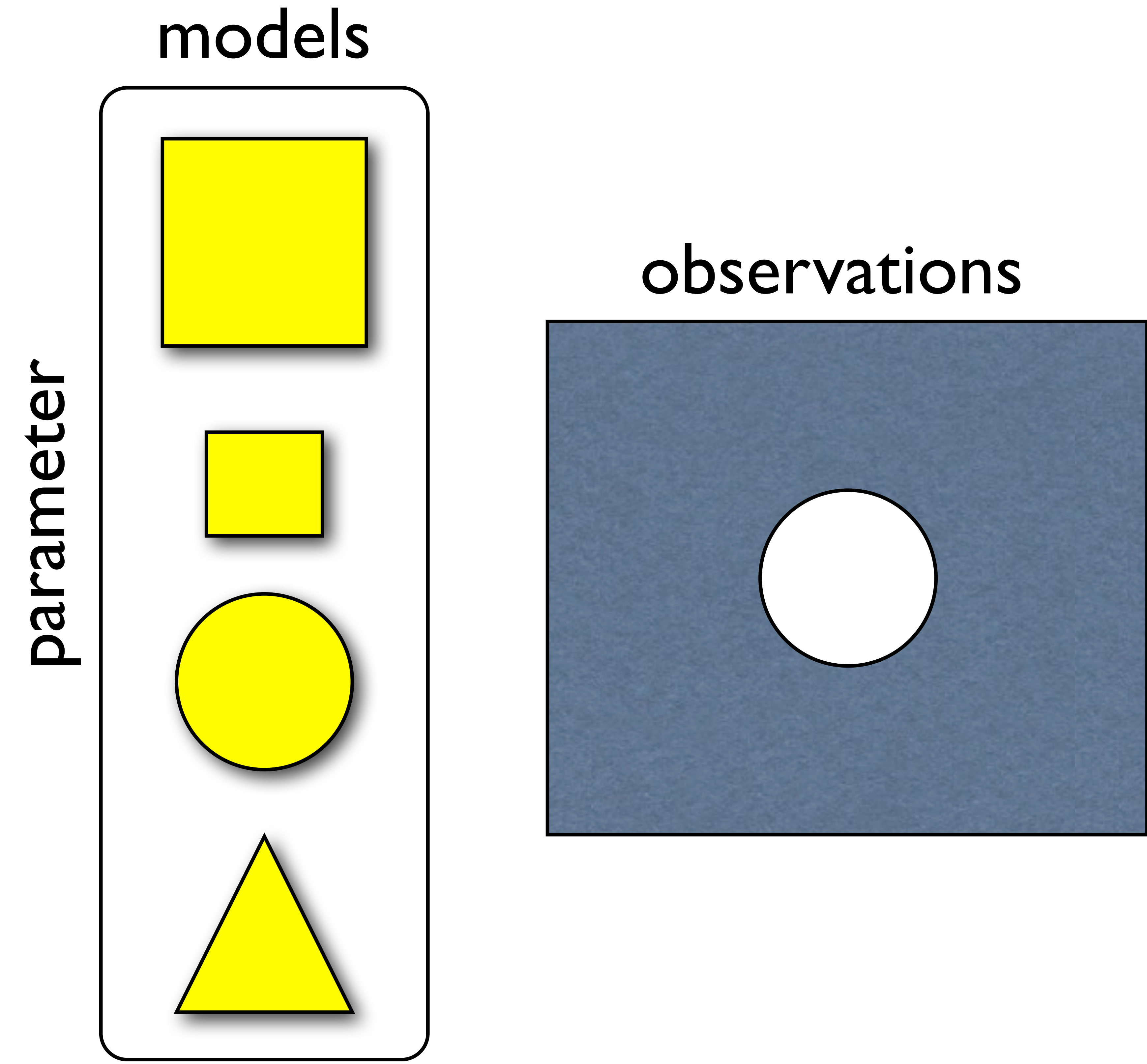


how do we get there?

one popular path!!!



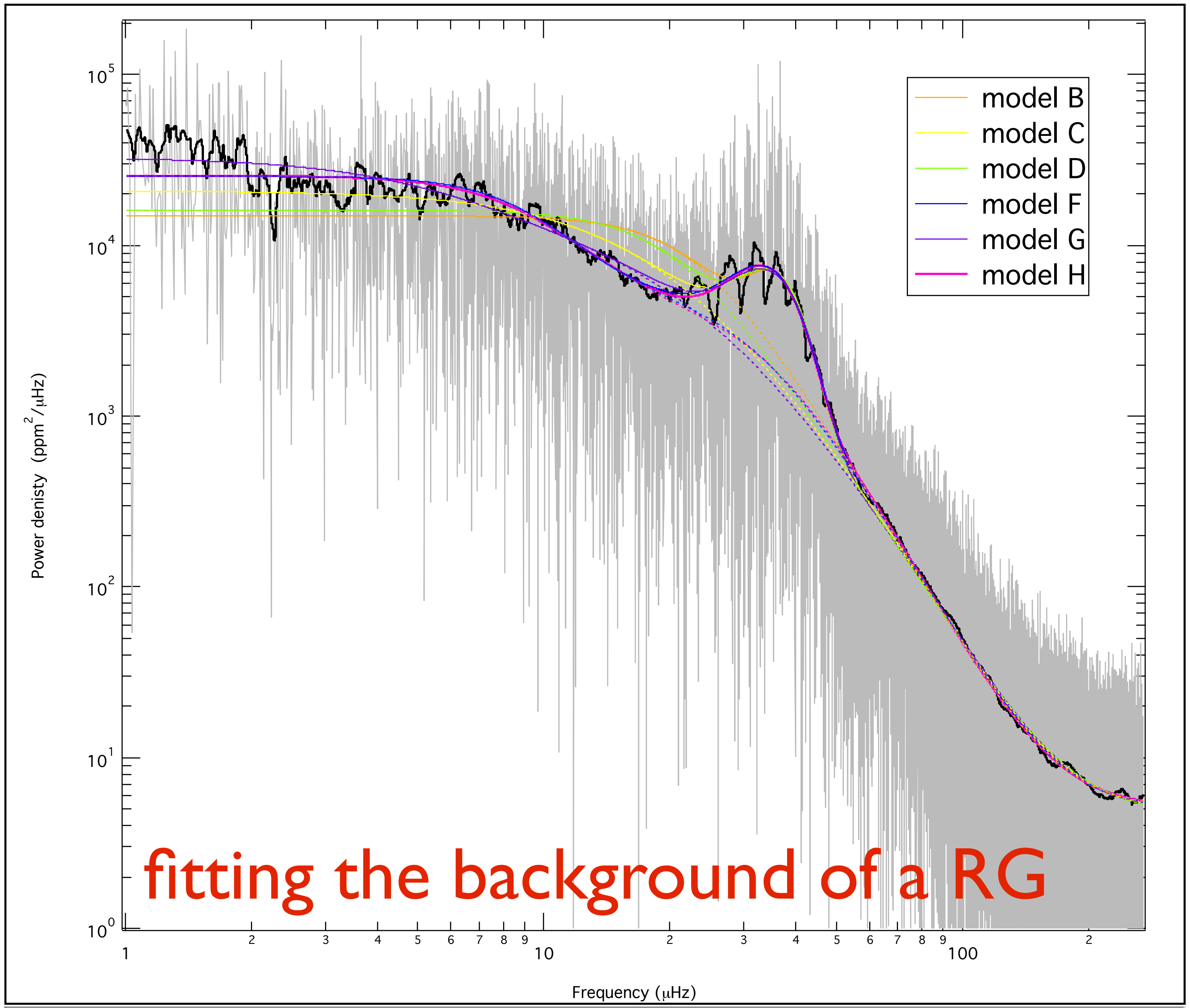
fitting problem



easy!!!!

fitting problem

but what about this...



Probabilistic ("Bayesian") analysis

There are 2 (and only 2) rules needed to spawn all of probability theory

e.g., see E.T. Jaynes 2003 - "Probability Theory"

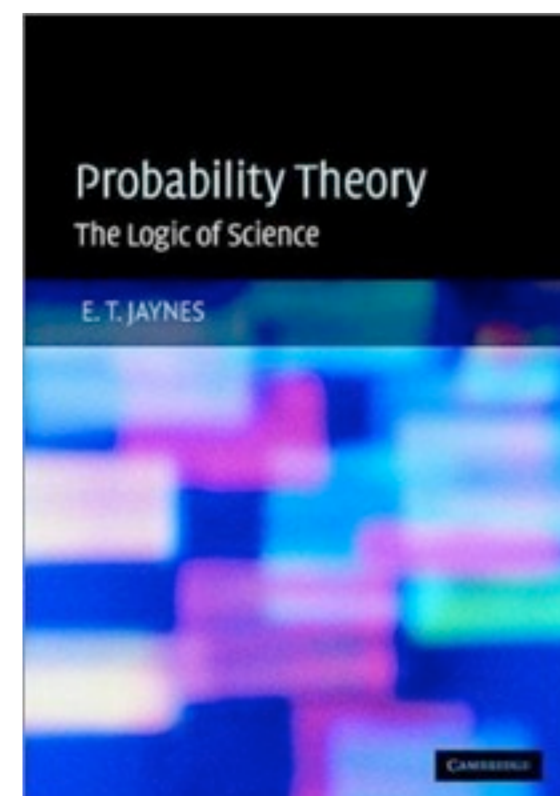
A, B, C... proposition

Product rule: $P\left(A, B|C\right) = P\left(A|C\right) P\left(B|A, C\right)$

"A and B given C"

Sum rule: $P\left(A + B|C\right) = P\left(A|C\right) + P\left(B|C\right) - P\left(A, B|C\right)$

"A or B or both given C"



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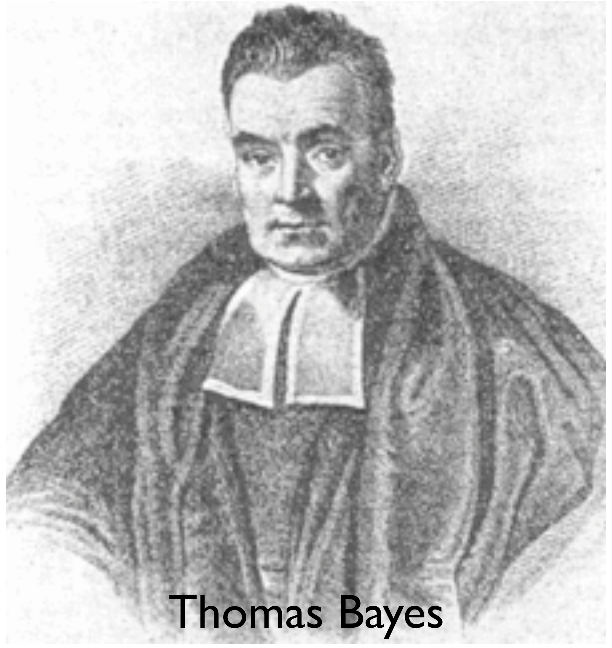
Probabilistic ("Bayesian") analysis

$$P(A, B \mid C) = P(A \mid C)\, P(B \mid A, C) = P(B \mid C)\, P(A \mid B, C)$$

from the Product rule follows

Bayes' Theorem

$$p(A|B,C) = \frac{p(A|C)p(B|A,C)}{p(B|C)}$$



facts behind the buzzword

Probabilistic analysis ("Bayesian analysis") simply uses the theorems of probability theory to determine the probabilities of propositions (i.e., parameter values, models, hypotheses)

- quantitative approach to "scientific inference"

determine parameter values and their uncertainties

- consistency & correct normalisation guaranteed

evaluate models (and physics)

- marginalisation - a consequence of the sum rule

get rid of 'unwanted' parameters

$$P(\theta_0, ..., \theta_{n-1}|M, D, I) = \int P(\theta_0, ..., \theta_n|M, D, I)\, \mathrm{d}\theta_n$$

GRANULATION

background

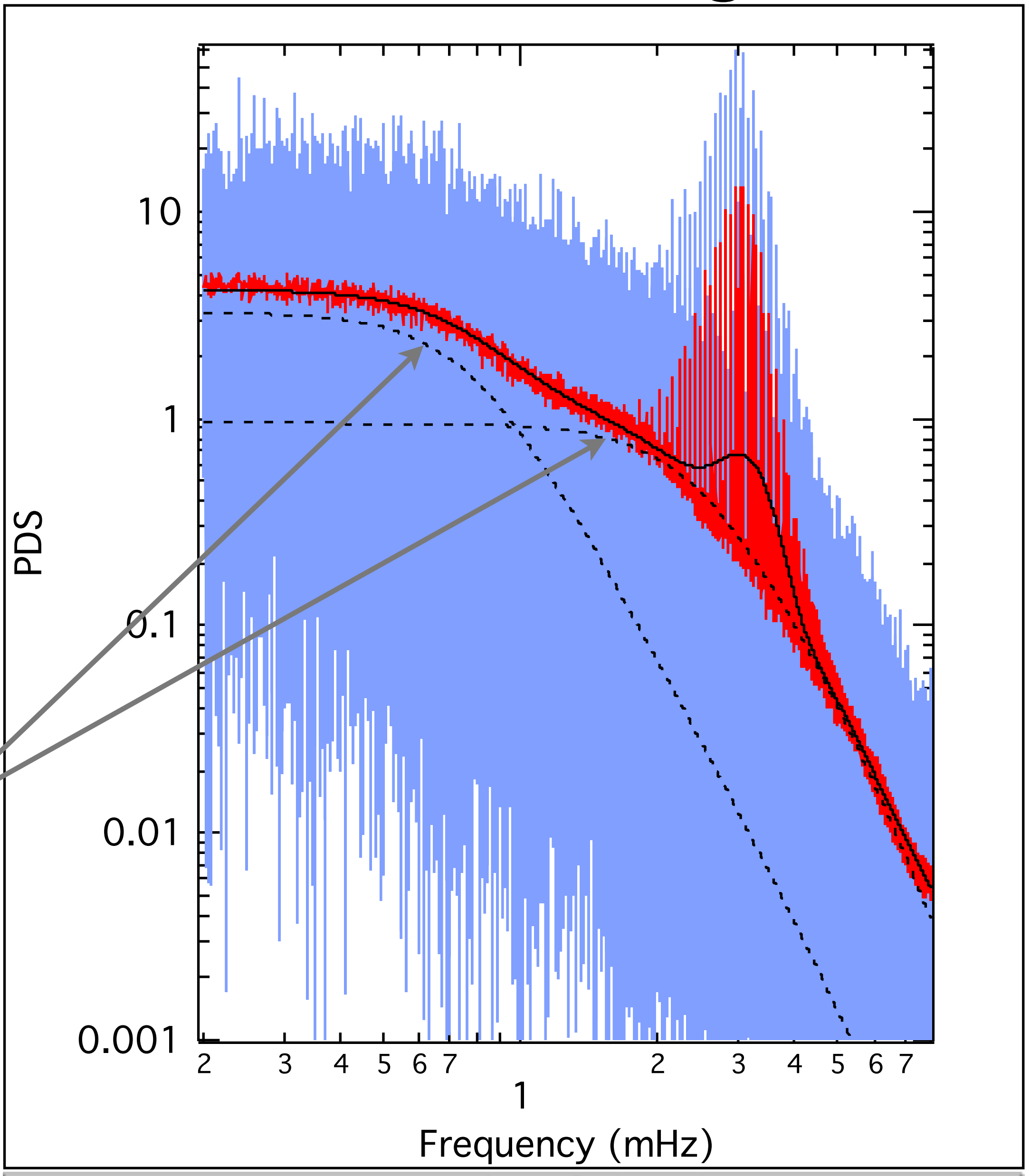
granulation background signal

Harvey model

$$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1 + (2\pi\nu\tau_i)^2}$$

background signal

Sun@SOHO VIRGO



Harvey model zoo...

e.g.

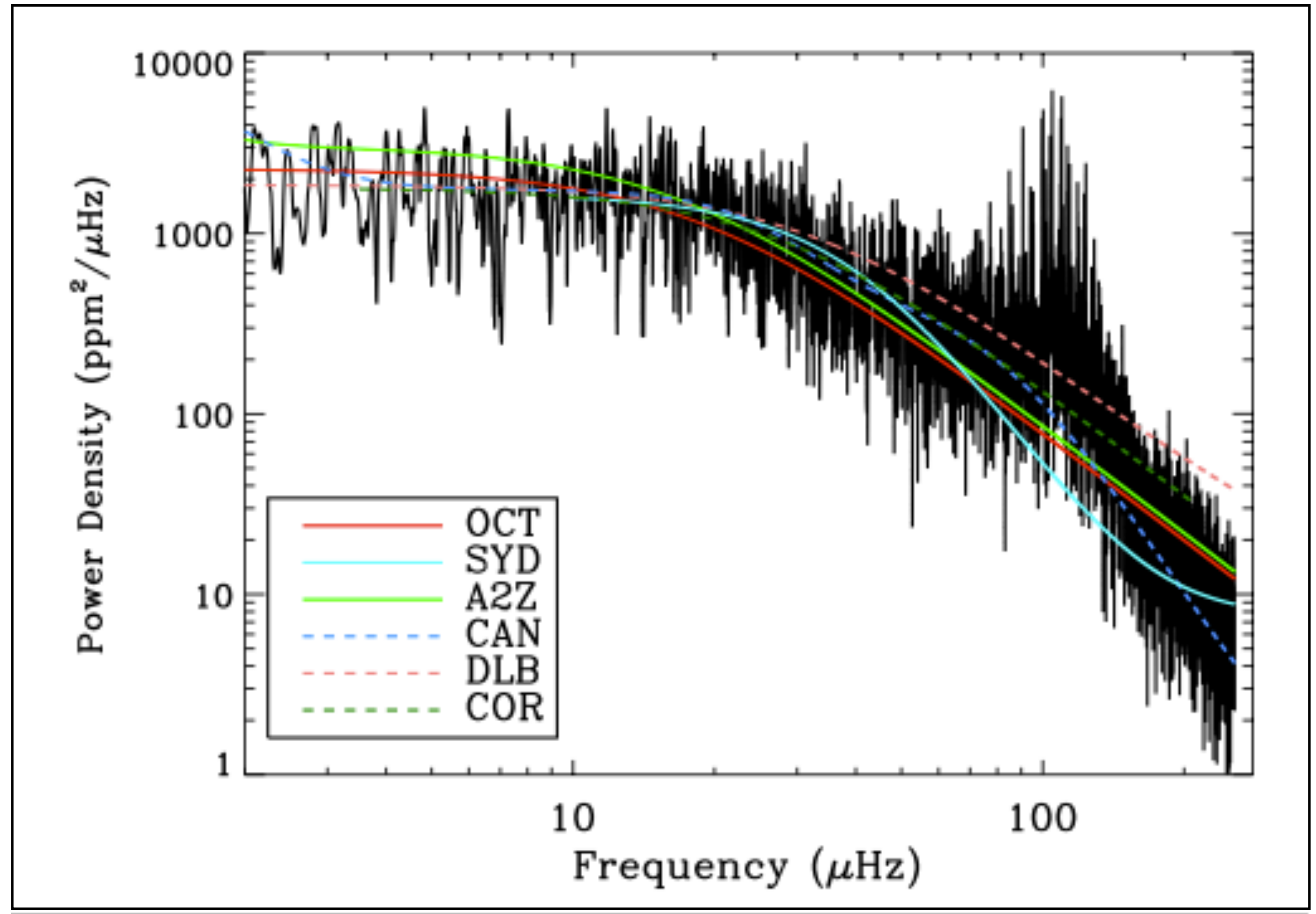
$$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1 + (2\pi\nu\tau_i)^2}$$

$$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{[1 + (2\pi\nu\tau_i)^2]^2}$$

$$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1 + (2\pi\nu\tau_i)^4}$$

$$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1 + (2\pi\nu\tau_i)^{\alpha_i}}$$

Mathur et al. 2011



$\Rightarrow$ Bayesian model comparison

picking the right model

the tool ... **MultiNest** Feroz et al. 2009

... *Bayesian Nested Sampling* Algorithm
- probability distributions for the parameters
- global evidence for the fit

A/E $$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1+(2\pi\nu\tau_i)^2}$$

B/F $$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1+(2\pi\nu\tau_i)^4}$$

C/G $$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{[1+(2\pi\nu\tau_i)^2]^2}$$

D/H $$\sum_i \frac{\zeta \sigma_i^2 \tau_i}{1+(2\pi\nu\tau_i)^{\alpha_i}}$$

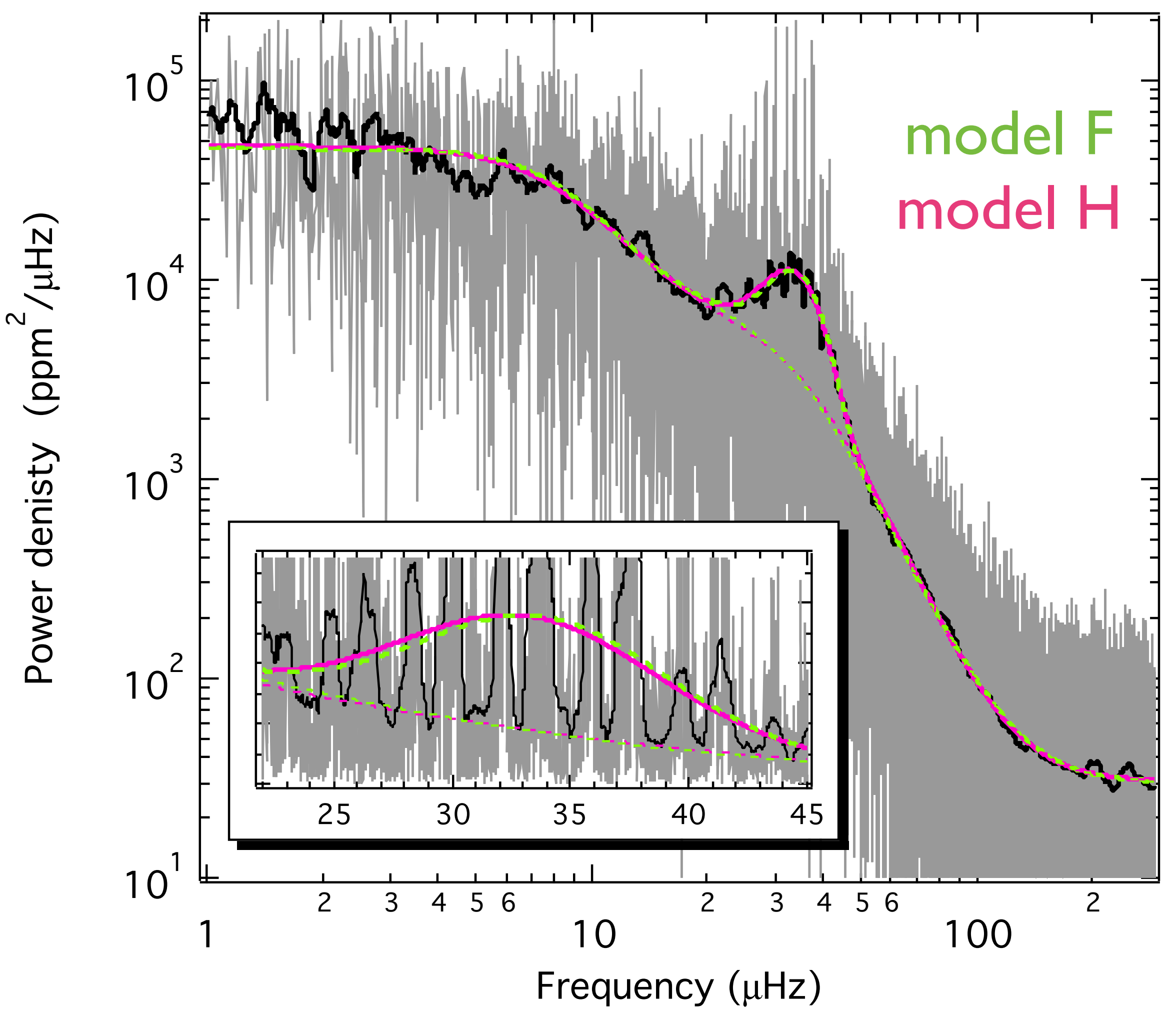
i=1,2 ... 1 or 2 components

	$\ln(z/z_0)$	p	Gaussian P_g	Gaussian ν_{max}	Gaussian σ	1st component a_1	1st component b_1	1st component c_1	2nd component a_2	2nd component b_2	2nd component c_2
A	-1587.7	$< 10^{-200}$	5.4(2)	30.38(02)	13.1(2)	560(12)	2.3(1)	2*			
B	-255.7	$\sim 10^{-111}$	4.8(3)	35.7(3)	5.1(2)	624(6)	23.7(2)	4*			
C	-75.8	$\sim 10^{-33}$	5.5(3)	34.5(2)	6.0(1)	606(6)	22.5(2)	2/4*			
D	-243.4	$\sim 10^{-102}$	5.1(3)	35.2(2)	5.7(2)	601(28)	20.8(4)	3.7(1)			
E	-1592.4	$< 10^{-200}$	5.4(2)	30.42(02)	13.2(2)	571(15)	2.3(2)	2*	31(4)	34.1(6)	2*
F	**-1.7**	**0.166**	**5.5(2)**	**33.8(4)**	**6.1(2)**	**466(14)**	**9.4(5)**	**4***	**399(19)**	**31.9(1)**	**4***
G	-36.6	$\sim 10^{-16}$	5.7(2)	33.9(2)	6.4(2)	352(26)	8.5(9)	2/4*	502(18)	25.7(6)	2/4*
H	**-0.1**	**0.833**	**5.6(3)**	**33.5(5)**	**6.1(3)**	**470(35)**	**9.7(6)**	**3.6(3)**	**365(59)**	**35.8(3)**	**4.2(2)**

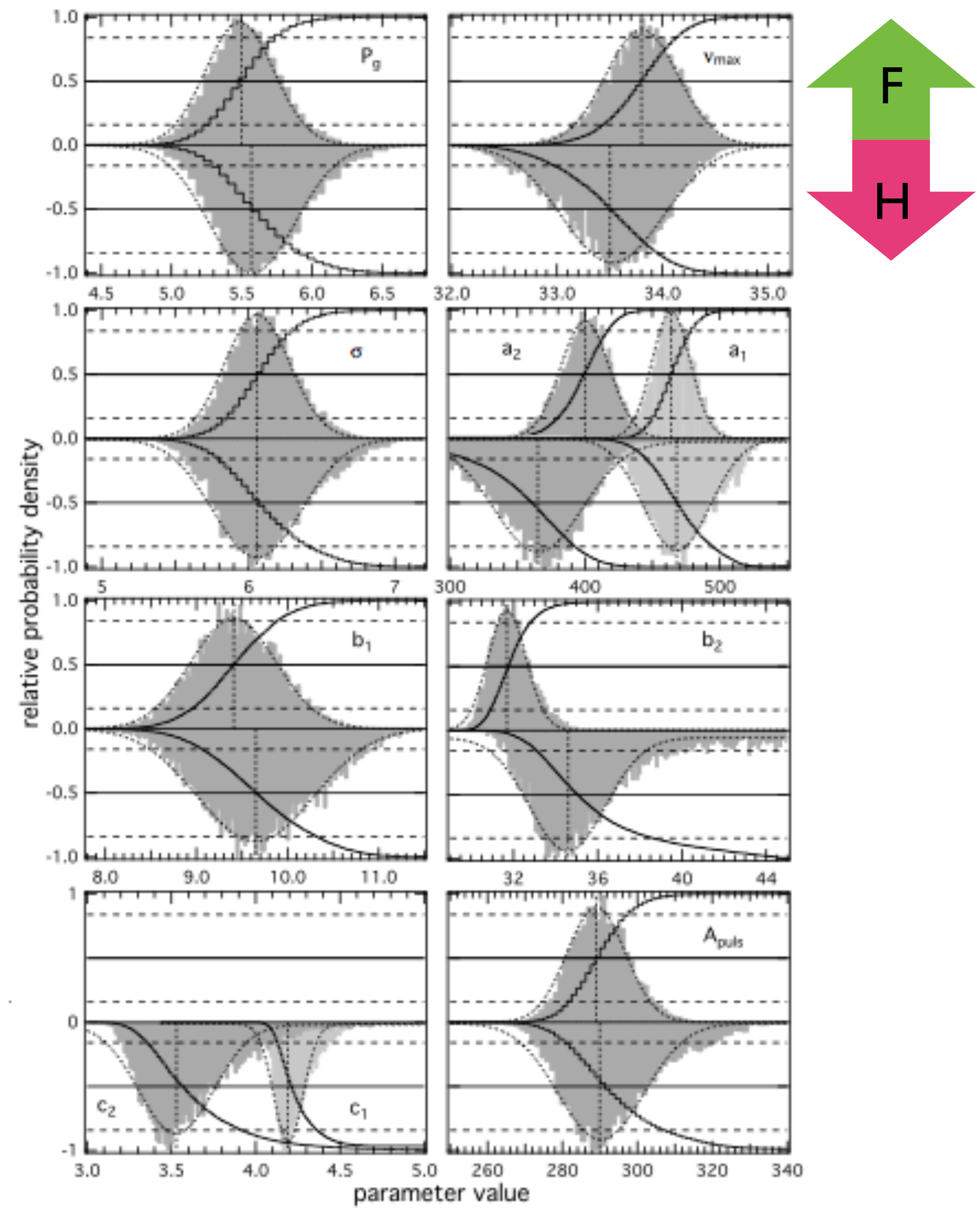
Kallinger et al. (submitted)

picking the right model

the result ...



posterior distributions



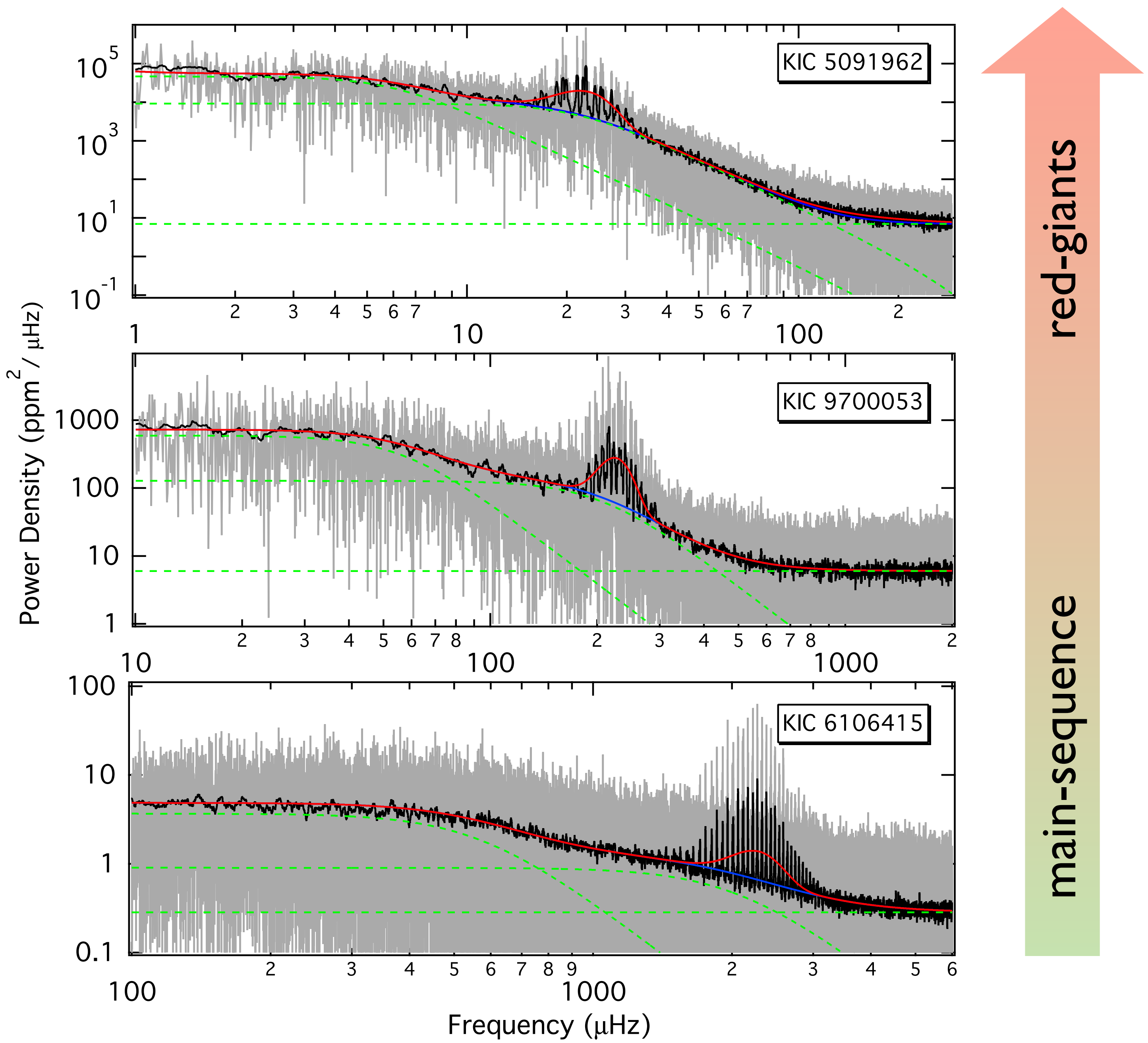
picking the right model

Bayesian analysis tells us...

- the original Harvey model is obsolete

- reliably fitting α is difficult (even with the long Kepler time series)

- a simple super-Lorentzian works for ALL stars and gives reliable parameters



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a little bit of 'Science'

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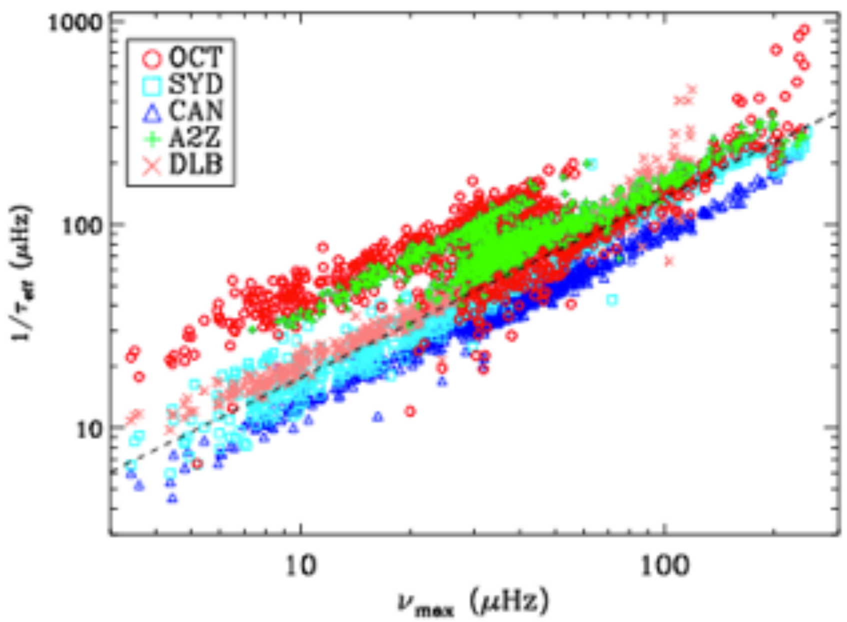
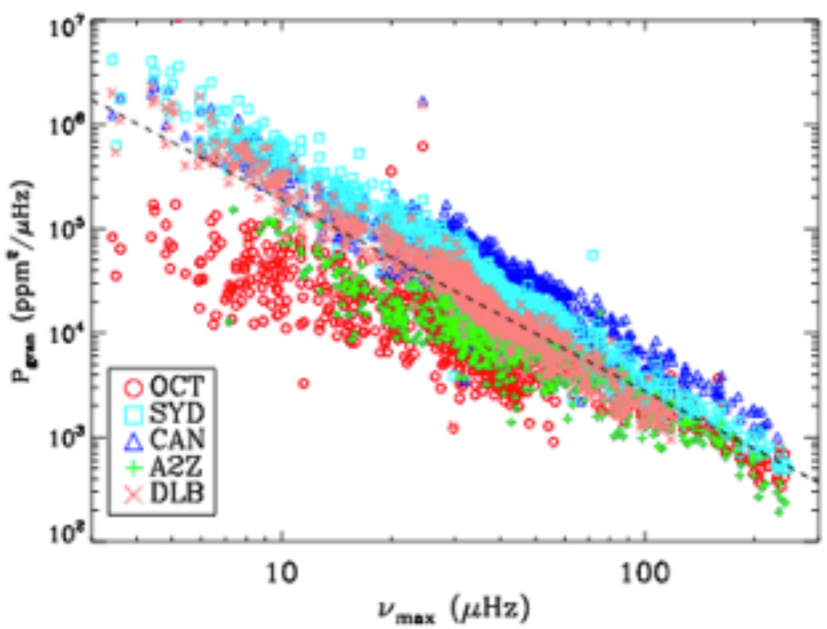
... granulation parameter

timescales

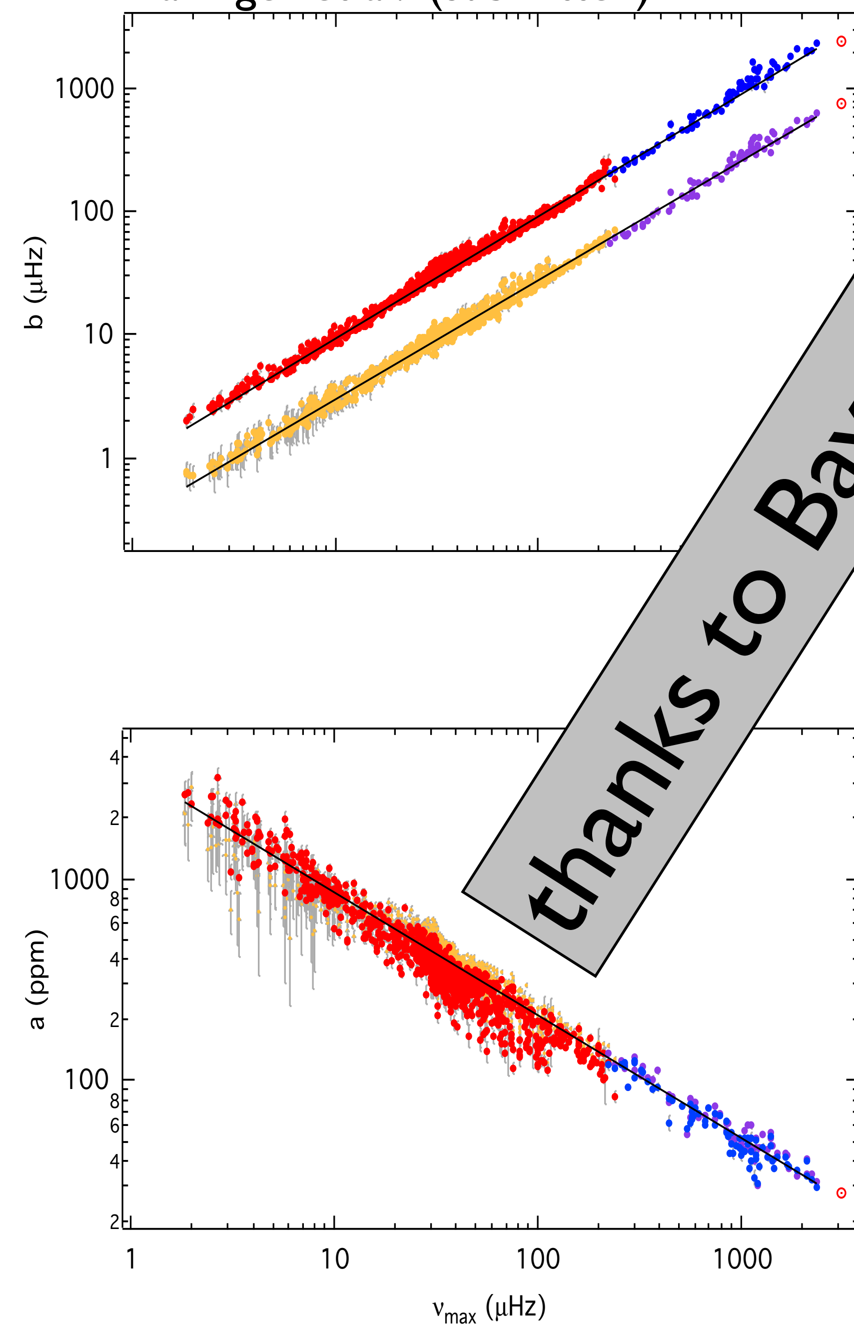
Mathur et al. 2009



amplitudes



Kallinger et al. (submitted)



thanks to Bayesian statistics

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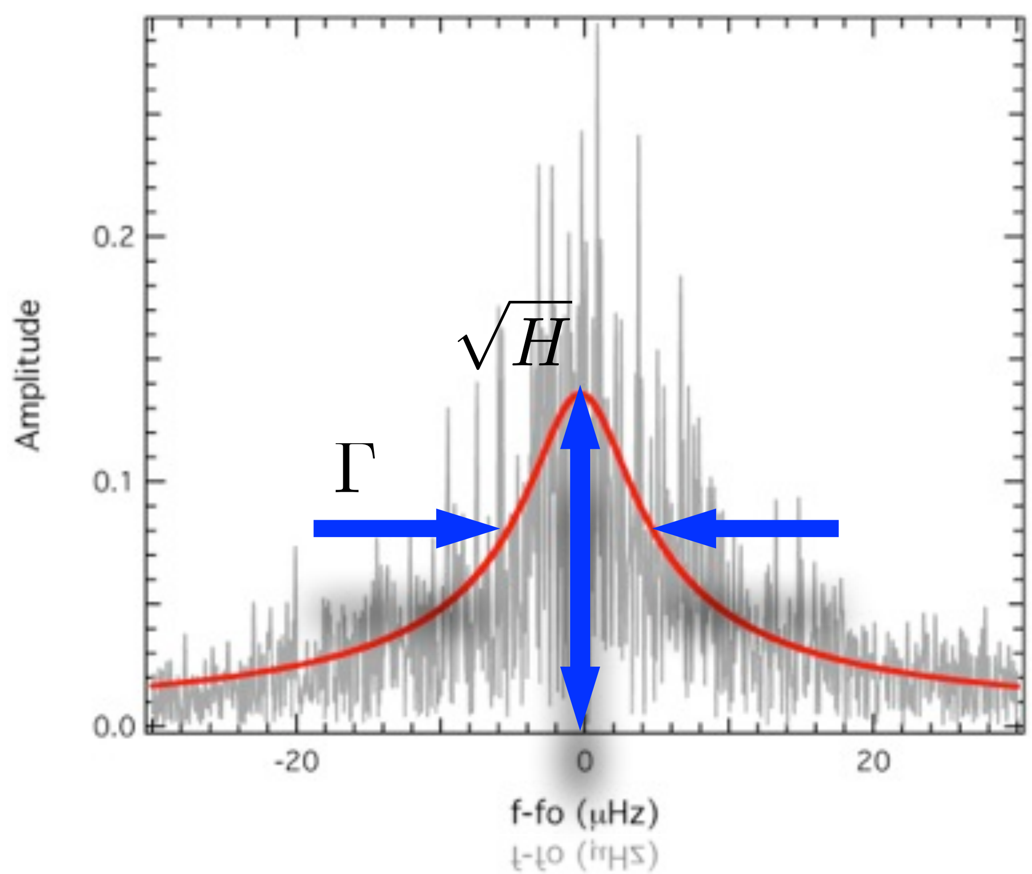
P E A K B A G G I N G
(frequency extraction)

the task ...

... extract mode parameters from the data



Lorentzian profile

$$P(f) = \frac{H}{1 + 4 \cdot (\frac{f - f_o}{\Gamma})^2}$$

f_o ... mode frequency
H ... mode height
Γ ... line width

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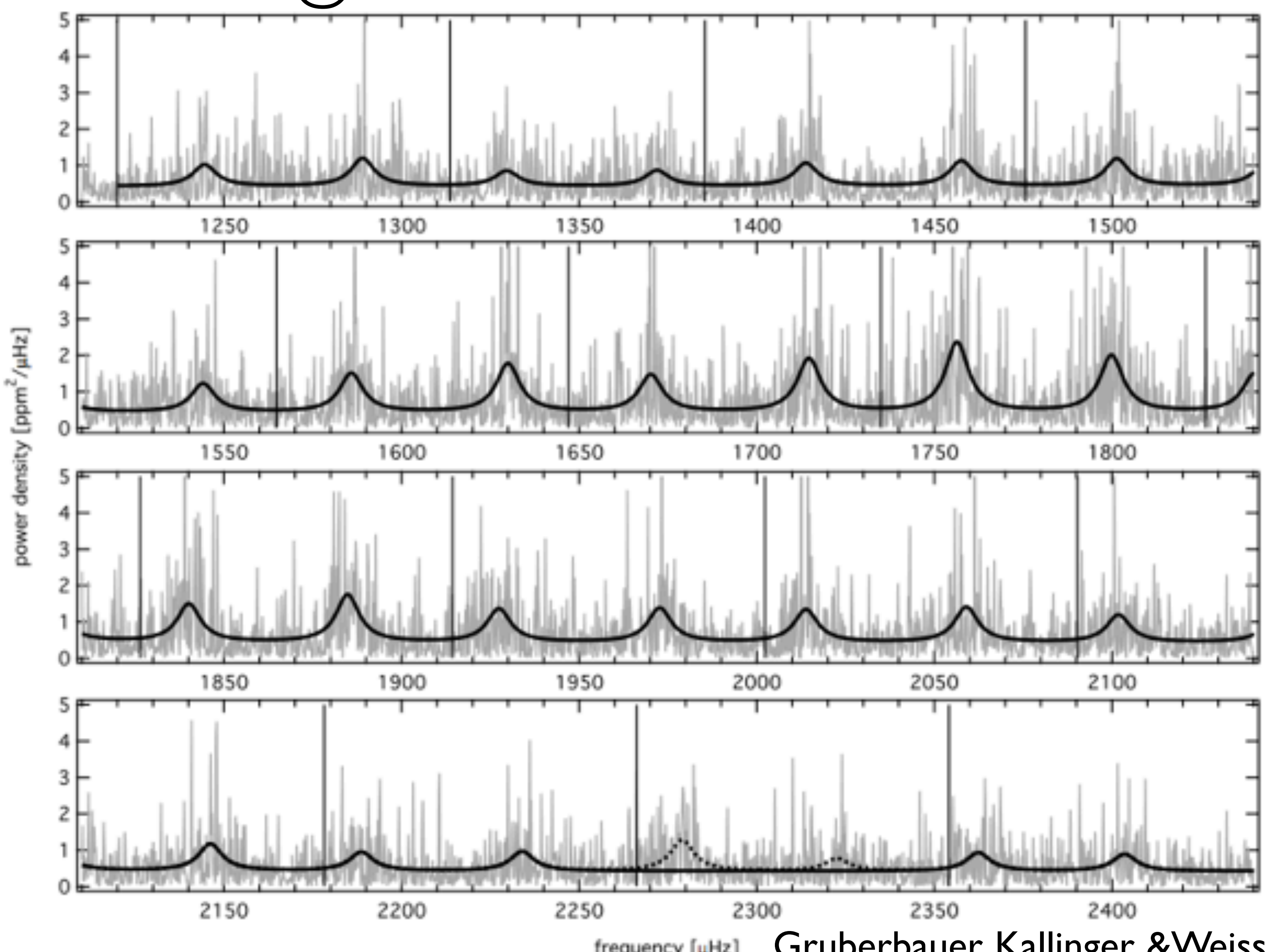
can get tremendously difficult

~28 modes (á 3 parameters)

mode identification, rotation, visibility of l=2 modes, mode lifetimes, ...
???

HD49933@CoRoT



Gruberbauer, Kallinger, &Weiss 2008

Appourchaux et al. 2008

(MLE)

Gruberbauer et al. 2008

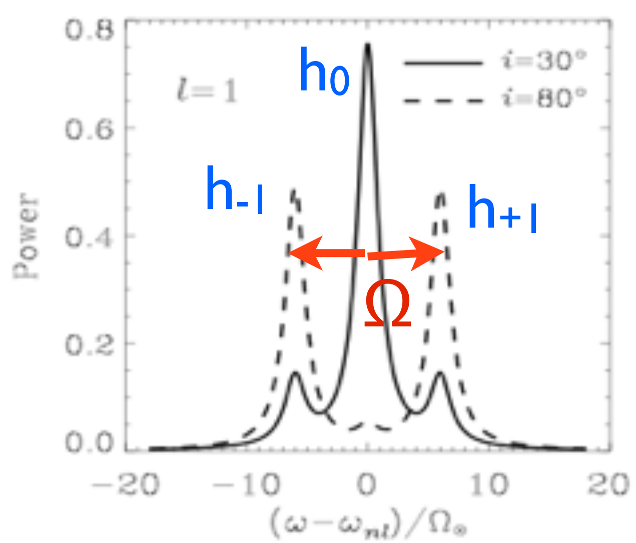
(Bayesian MCMC)

Benomar et al. 2008

(Bayesian MCMC)

"Bayesian evolution" in peakbagging

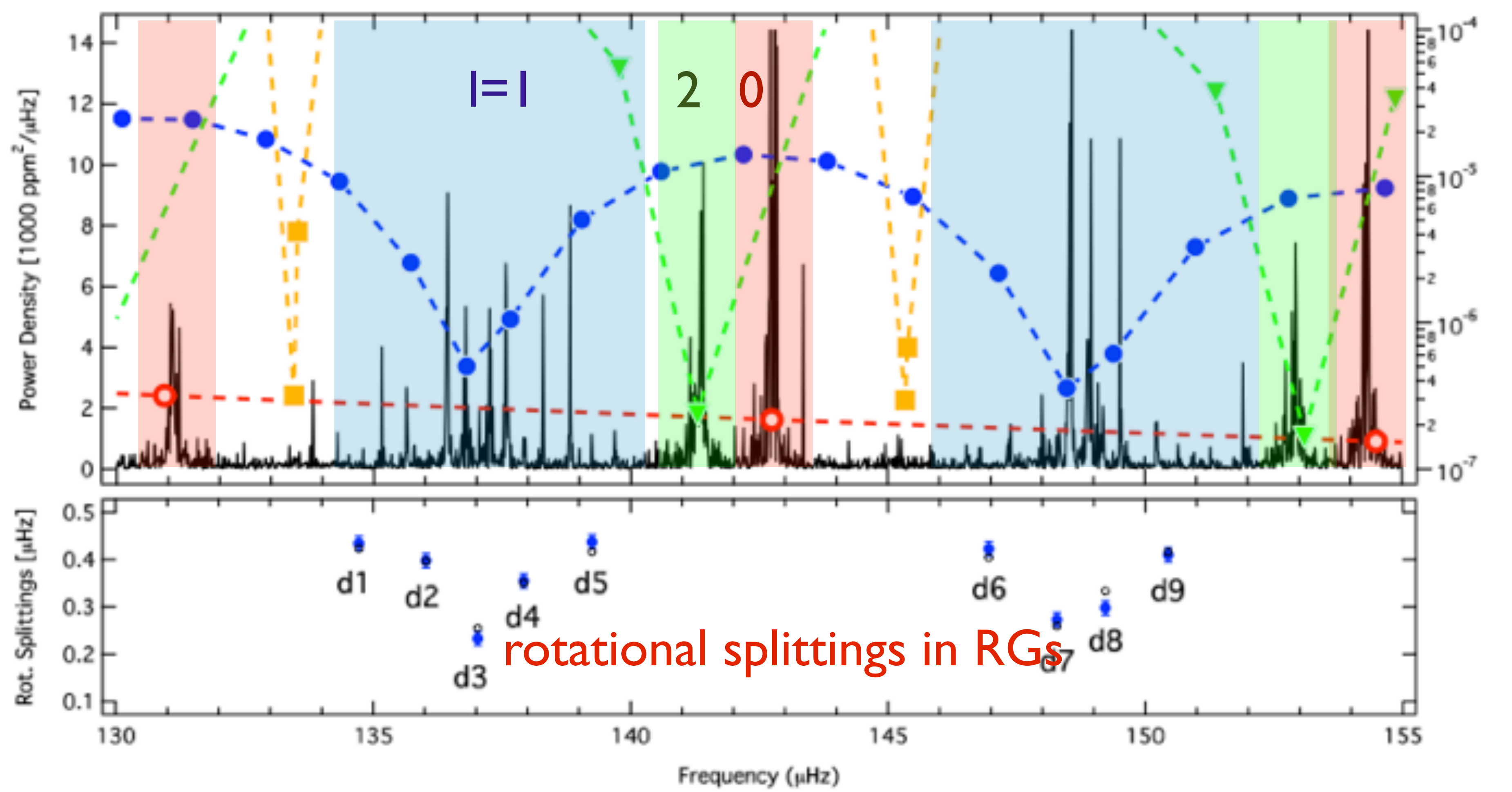
including rotation



$h_0 = h\ cos^2(i)$
$h_{\pm 1} = h/2\ sin^2(i)$

rotation adds 2 parameters per split mode

Beck et al. 2013



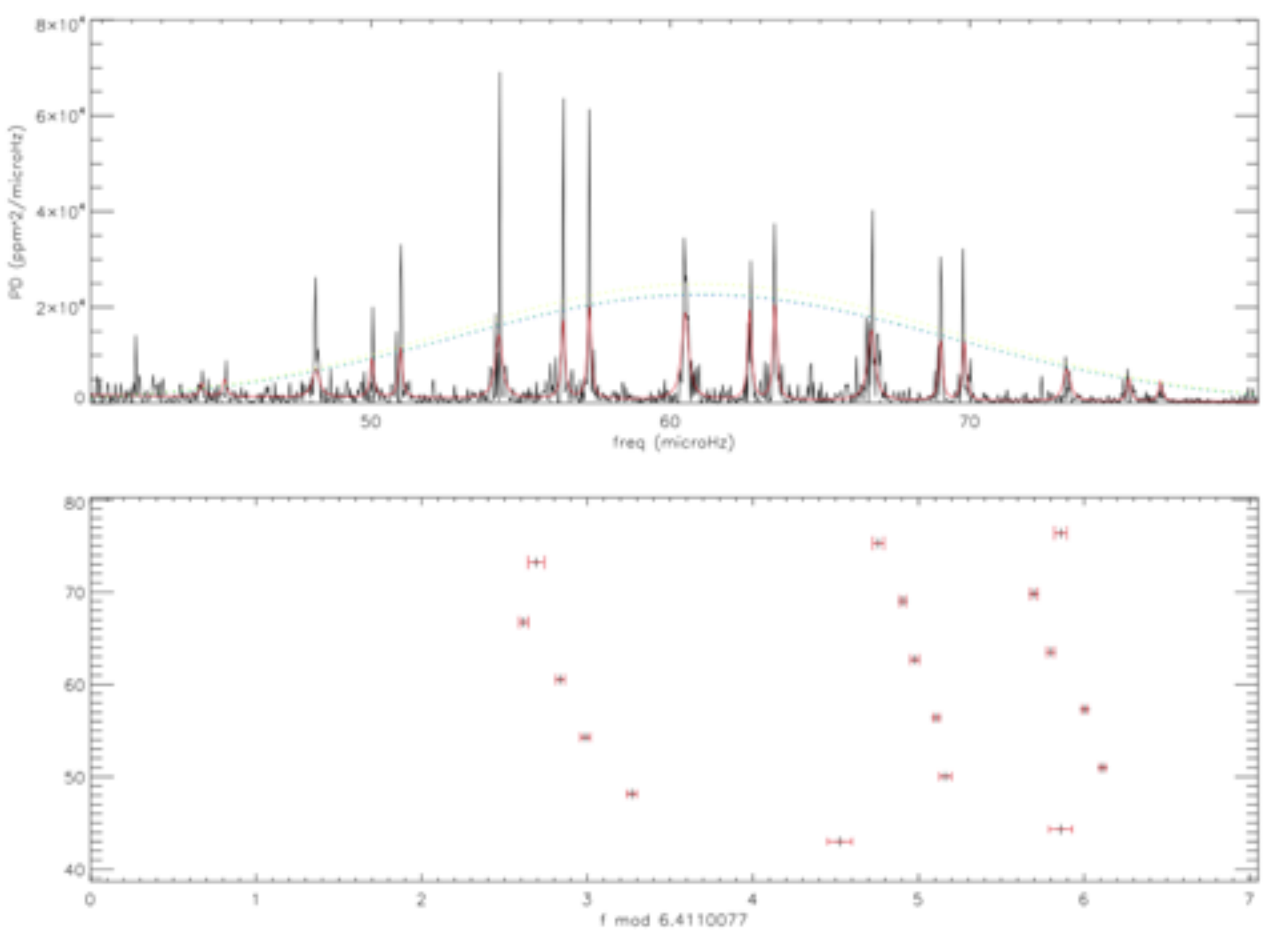
MultiNest again

based on ...

MultiNest
Feroz et al. 2009

... designed to:

- (semi) automatically peakbag many many stars
- as flexible as possible
  (turn on/off rotation, combine parameters, ...)
- multi-model analysis
  (compare evidence of different hypotheses)



work in progress!

red giant peakbagging

see next talk:

“*Peak Bagging of red giant stars observed by Kepler*”
(Enrico Corsaro)

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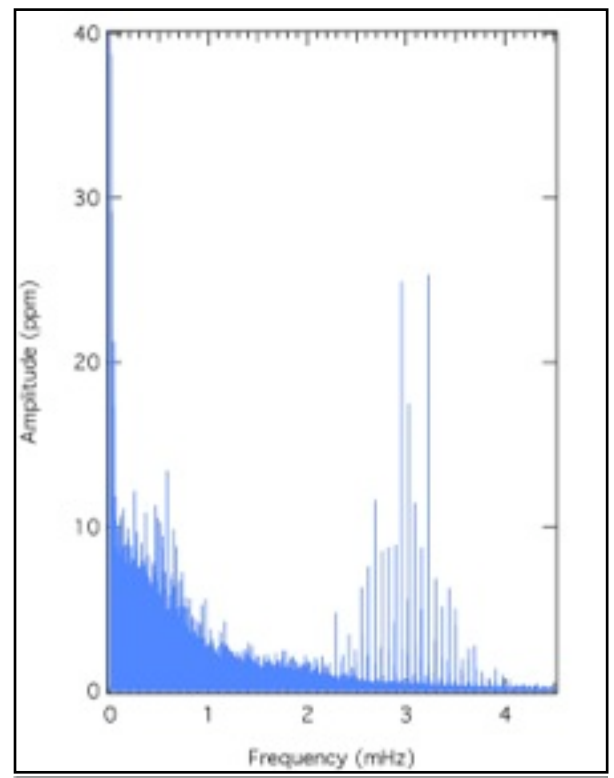
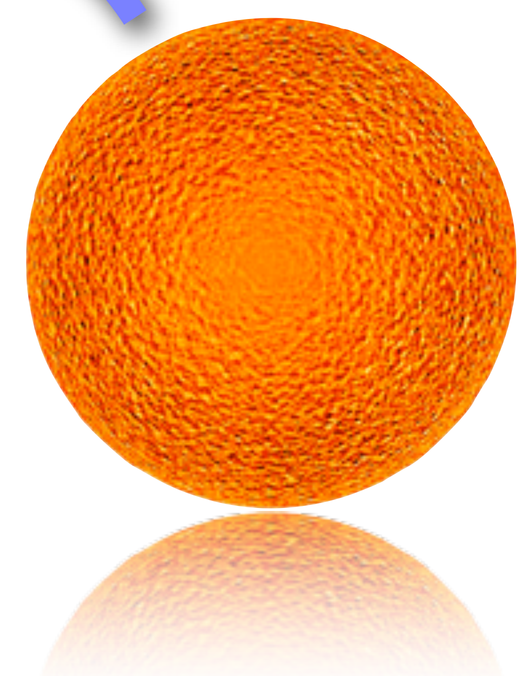
GRID MODELING

asteroseismic model fitting

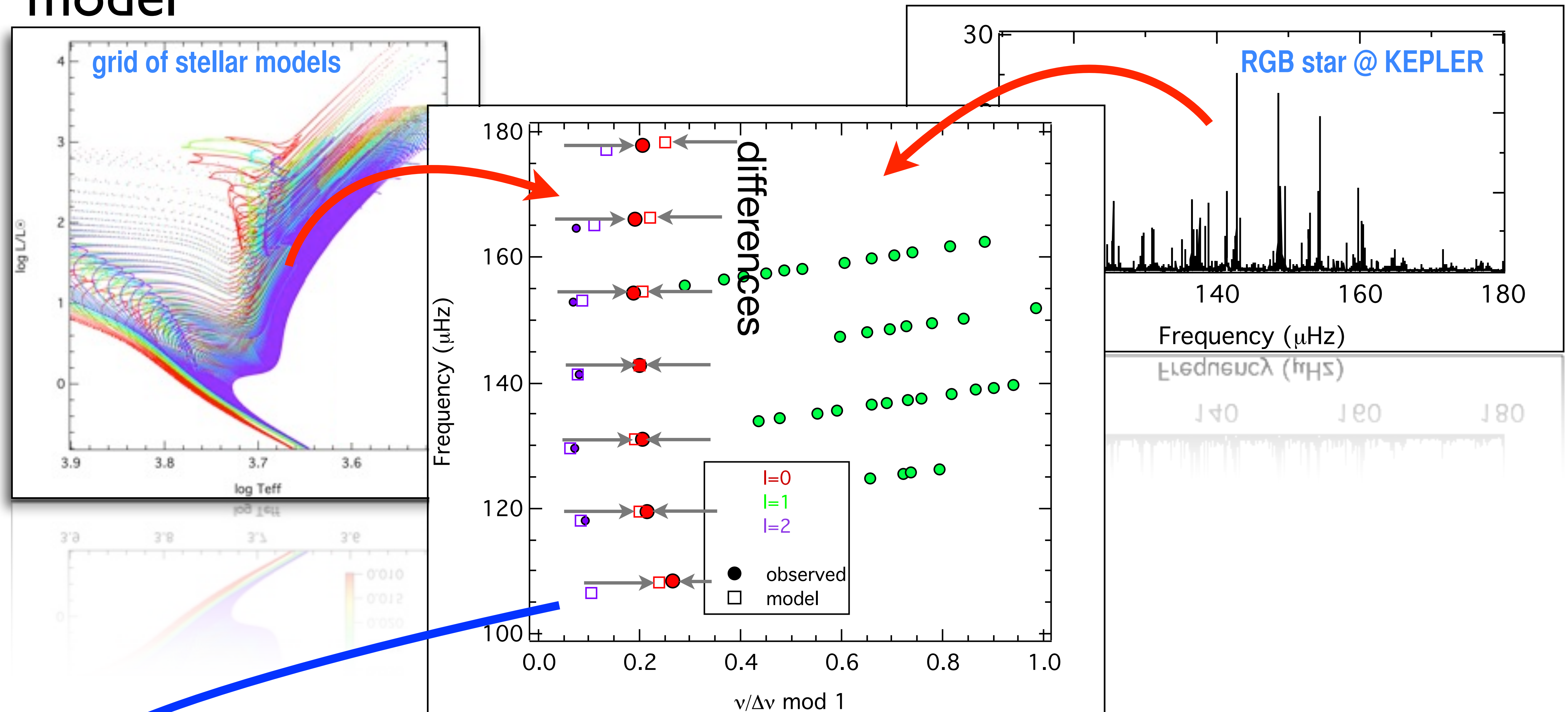




the classical χ^2-approach

model

observations



$$\chi^2 = \frac{1}{N_{obs}} \sum_{i=1}^{N_{obs}} \frac{(\nu_{i,o} - \nu_{i,m})^2}{\sigma^2_{i,o} + \sigma^2_{i,m}}$$

e.g., Guenther & Brown 2004

best-fit model

f(R, M, L, T_{eff}, ..., input physics)

classical χ^2-approach - problems

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- (ambiguous) mode identification
- rotationally split mode
  (especially for fast rotators)
- finite grid resolution
  (deadly for bumped modes)
- systematic errors in the models
  (e.g.,"surface effect")

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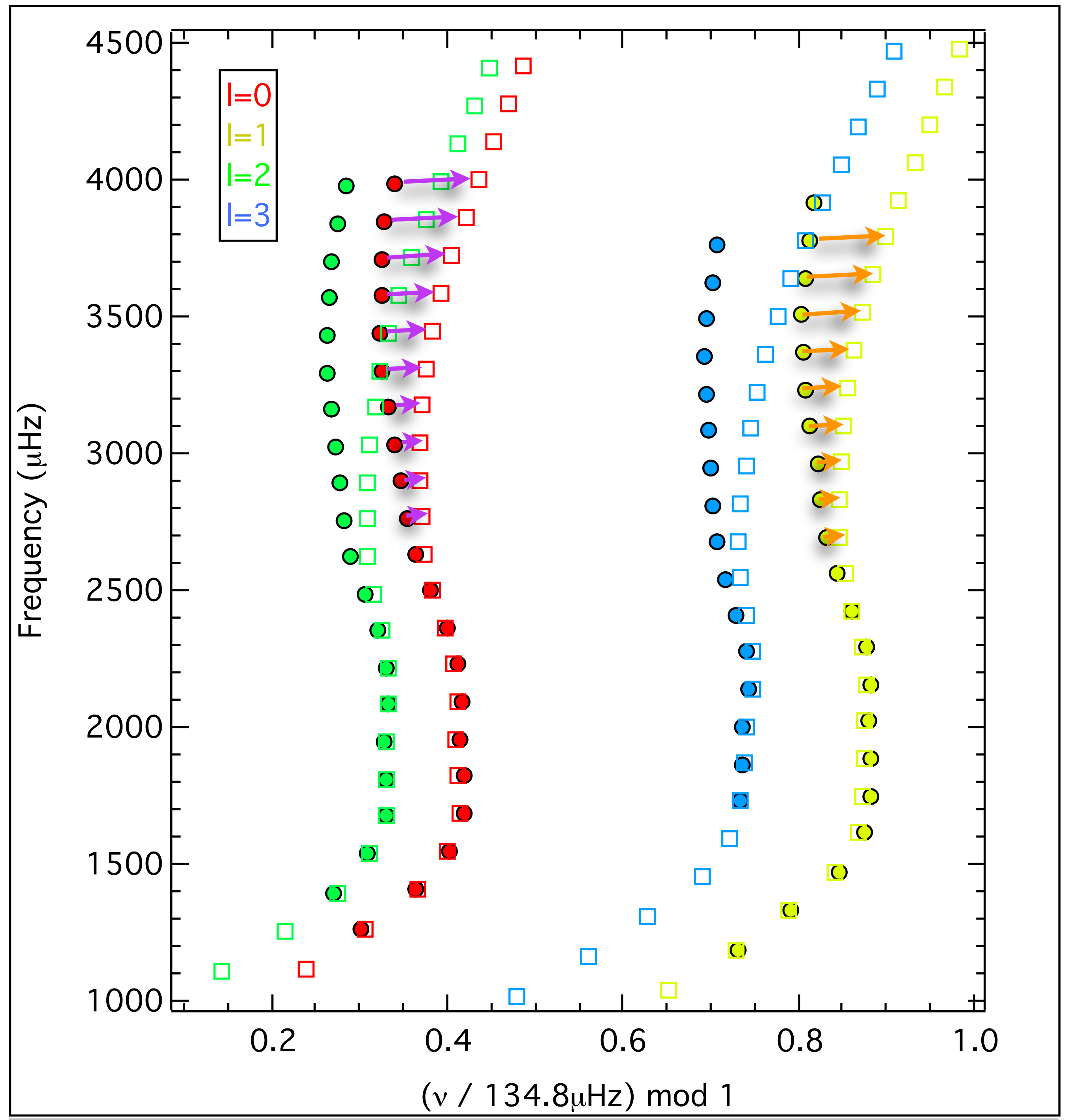
the "surface effect"

incorrect modelling of the outer layers of cool stars (like the Sun)

↓

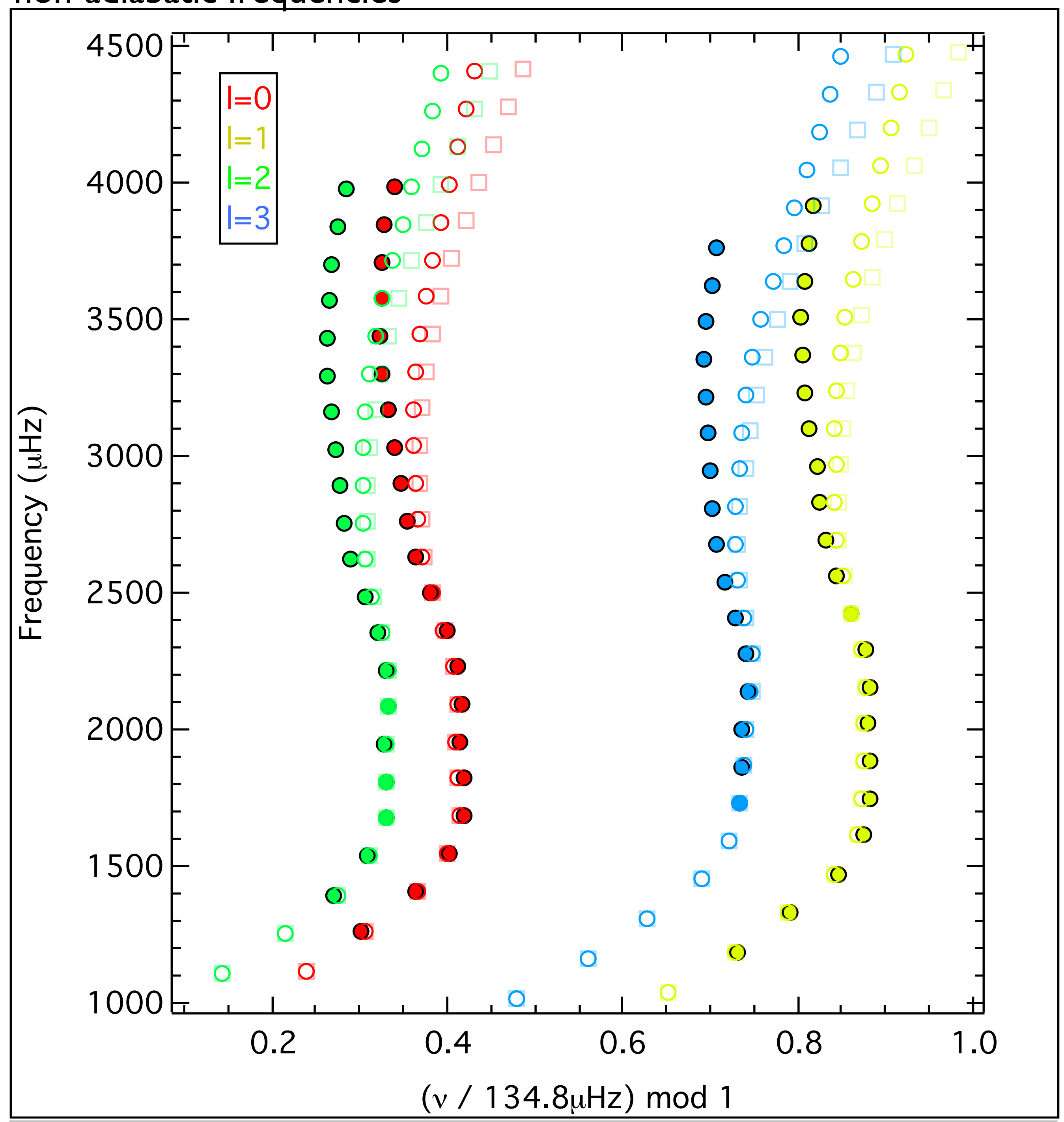
systematic deviations at high radial orders



solutions...

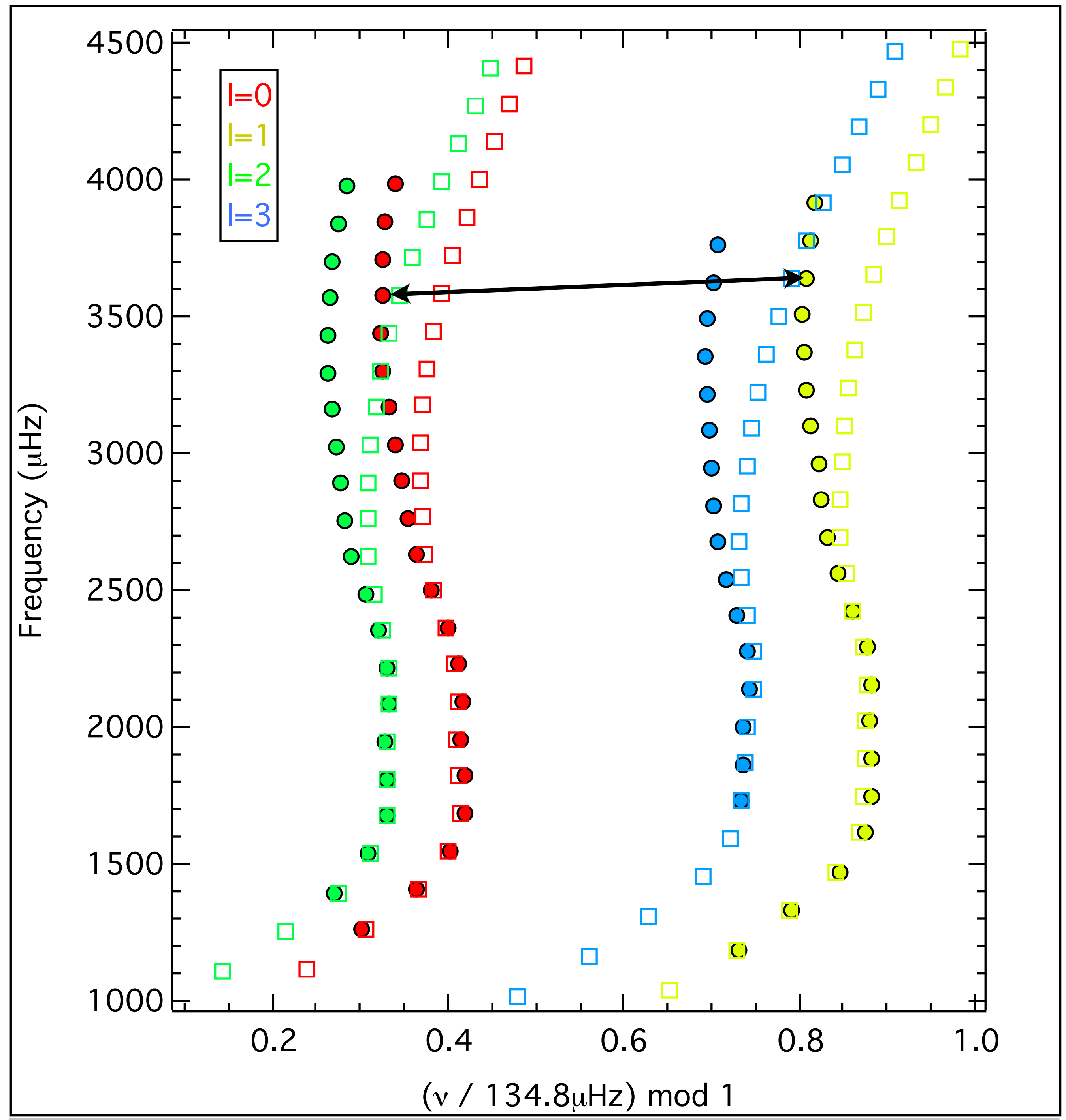
including (radiative) non-adiabatic effects alleviates the problem

non-adiabatic frequencies



solutions...

look at frequency differences (e.g. Roxburgh 2005)



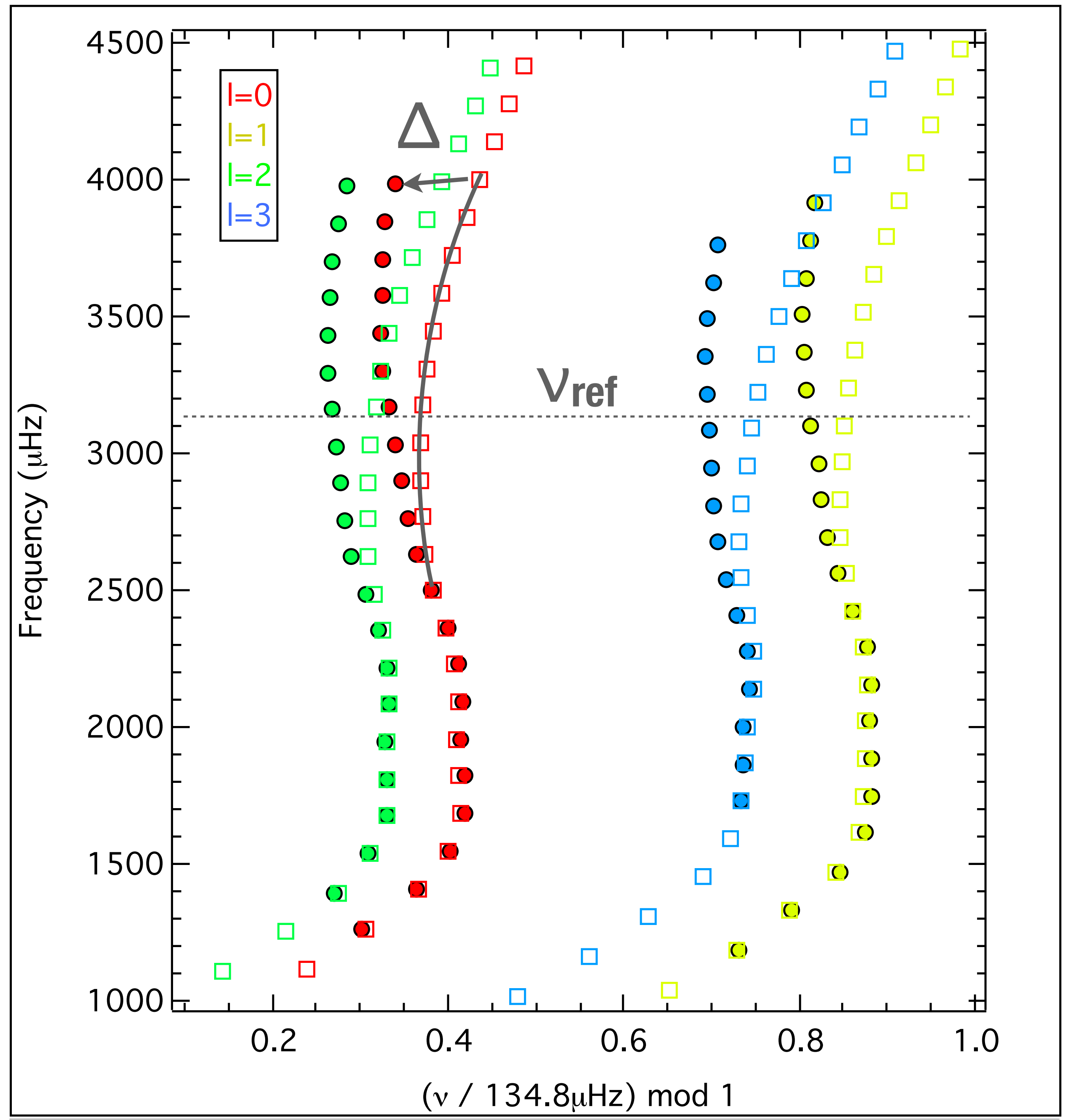
solutions...

correct with a solar-calibrated model (Kjeldsen et al. 2008)

$$\Delta_i \approx a \left(\frac{\nu_i}{\nu_{ref}} \right)^b$$

for stars≠Sun
b fixed to 4.9
(i.e. solar value)

downside: assume that Δ is always like the solar case



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solutions... Bayesian approach

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probability to match a single frequency

$$P(\nu_i|M_j,I) \sim \exp\left[\frac{-(\nu_{i,obs}-\nu_{i,model})^2}{2\sigma_i^2}\right]$$

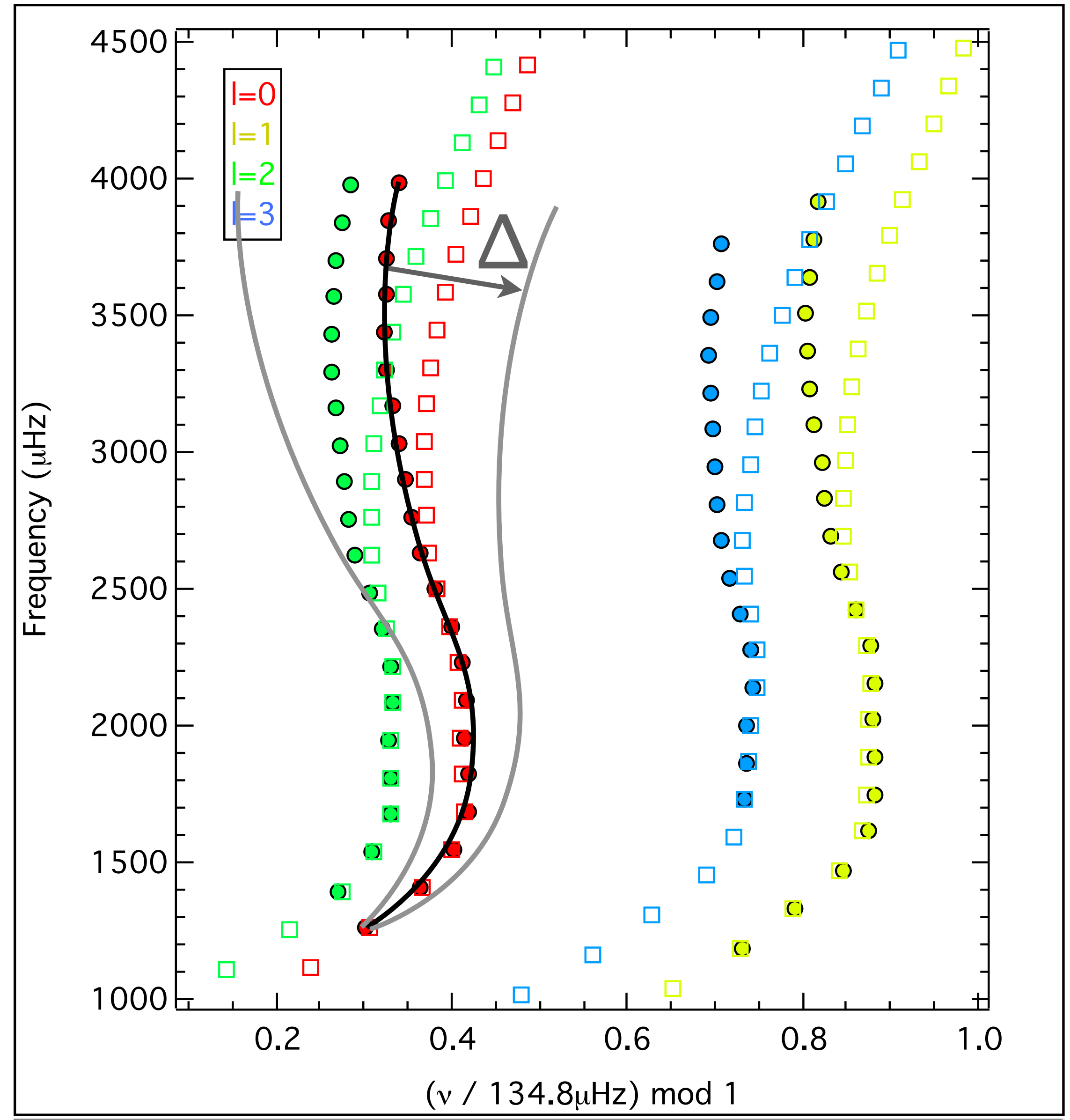
model probability to match all observed frequencies

$$P(D|M_j,I) = \prod_{i=1}^{n_{obs}} P(\nu_i|M_j,I)$$

allow for systematic frequency difference

[-1,1]

$$P(\nu_i|M_j^{\Delta},I) \sim \exp\left[\frac{-(\nu_{i,obs}-\nu_{i,model}-\gamma\Delta_i)^2}{2\sigma_i^2}\right]$$



"Bayesian magic" ⇒ $\int_{\Delta_{i,min}}^{\Delta_{i,max}} P(\nu_i|M_j^{\Delta},I)\,d\Delta_i$ integrate Δ_i out

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Gruberbauer, Guenther & Kallinger, 2012

solutions... Bayesian approach

- systematic errors in the models ✔

- (ambiguous) mode identification

- finite grid resolution

solved in a similar way

(see Gruberbauer et al. 2012 for more details)

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first application... the Sun

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5 million models that cover

- 3 different chemical compositions

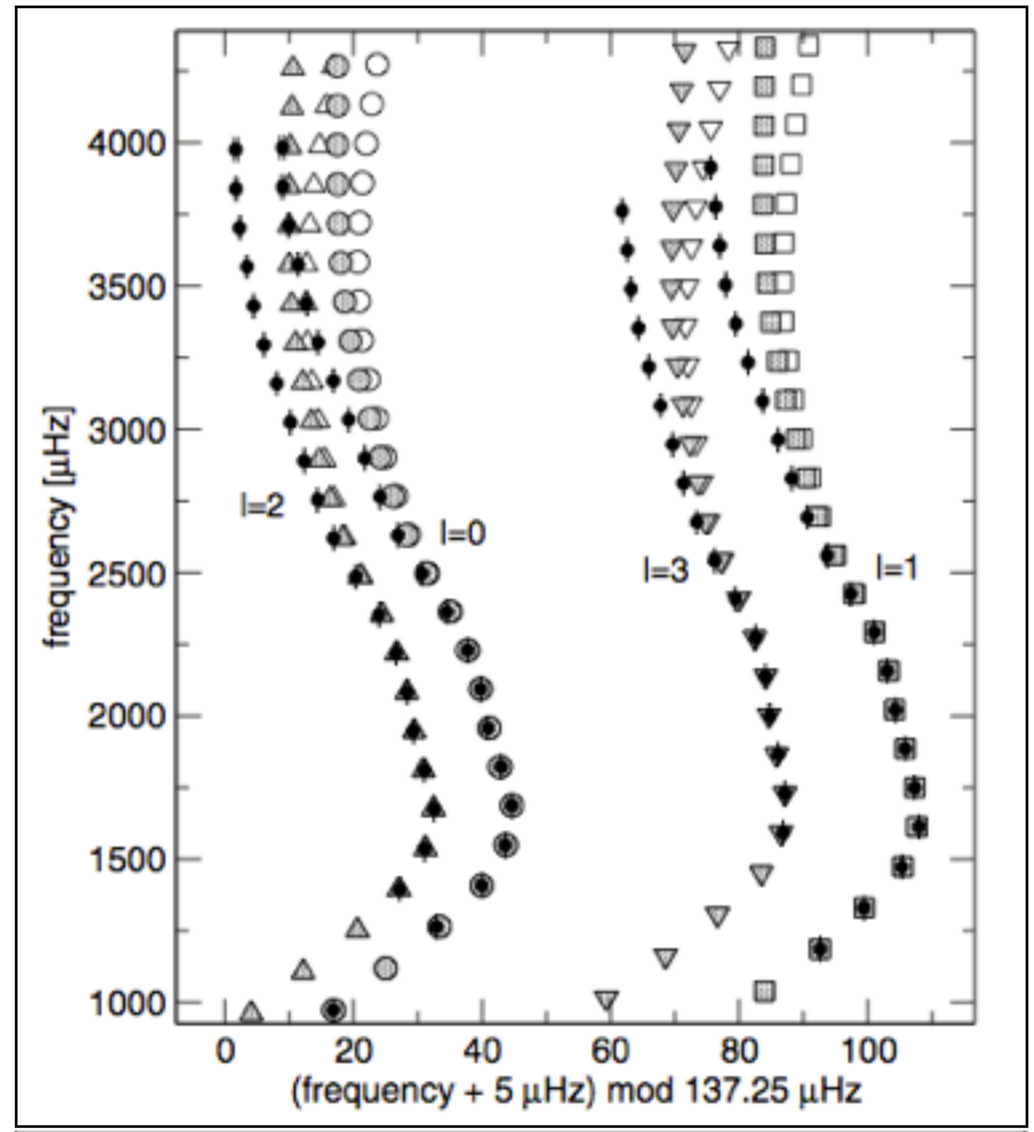
GS98	AGS05	AGSS09
(Grevesse & Sauval 1998)	(Asplund et al. 2005)	(Asplund et al. 2009)

- 2 different nuclear reaction rates

Standard	NACRE
(Bahcall & Ulrich 1988)	(Angulo et al. 1999)

- large range of fundamental parameters for each grid

best fit to BiSON frequencies



Gruberbauer & Guenther, 2013

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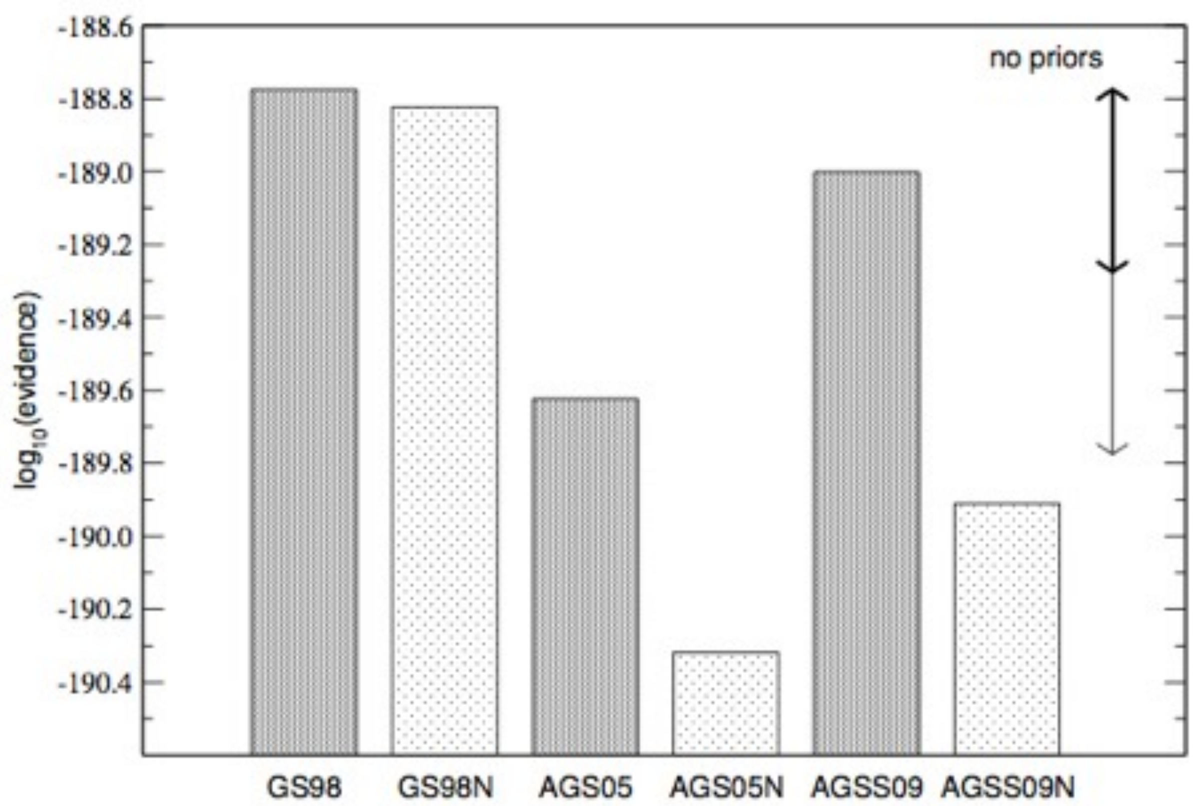
first application... the Sun

results ...

using only the frequencies

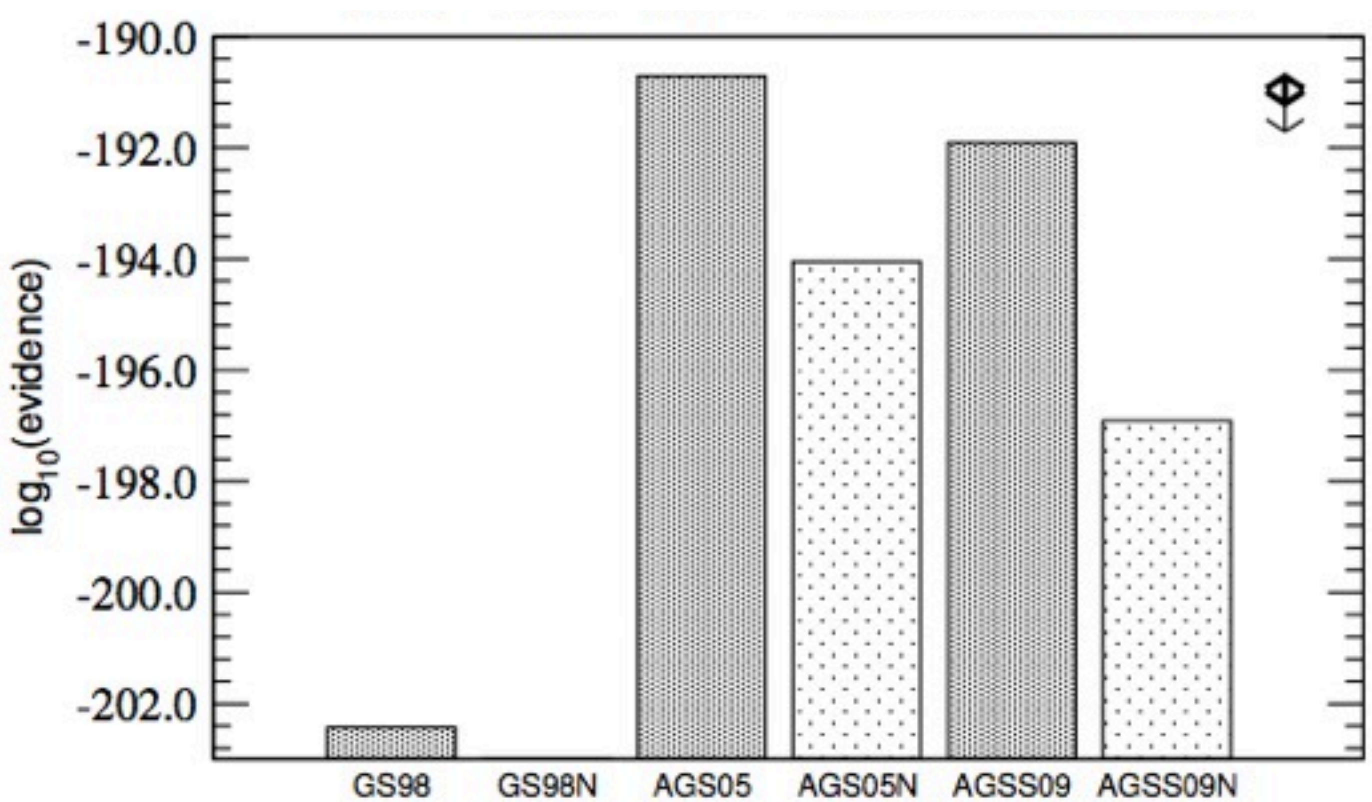


No strong evidence for any model in particular

first application... the Sun

results ...

implying prior information on L, T_{eff}, age



frequencies select revised abundances

first application... the Sun

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conclusions ...

- no definite solar model because best fits are too old
- something is fundamentally wrong with the models (physics)

but

- contrary to the literature, helioseismology does not favour traditional abundances!
- revised abundances are strongly favoured when our prior knowledge about the Sun is employed
- surface effect is not the problem

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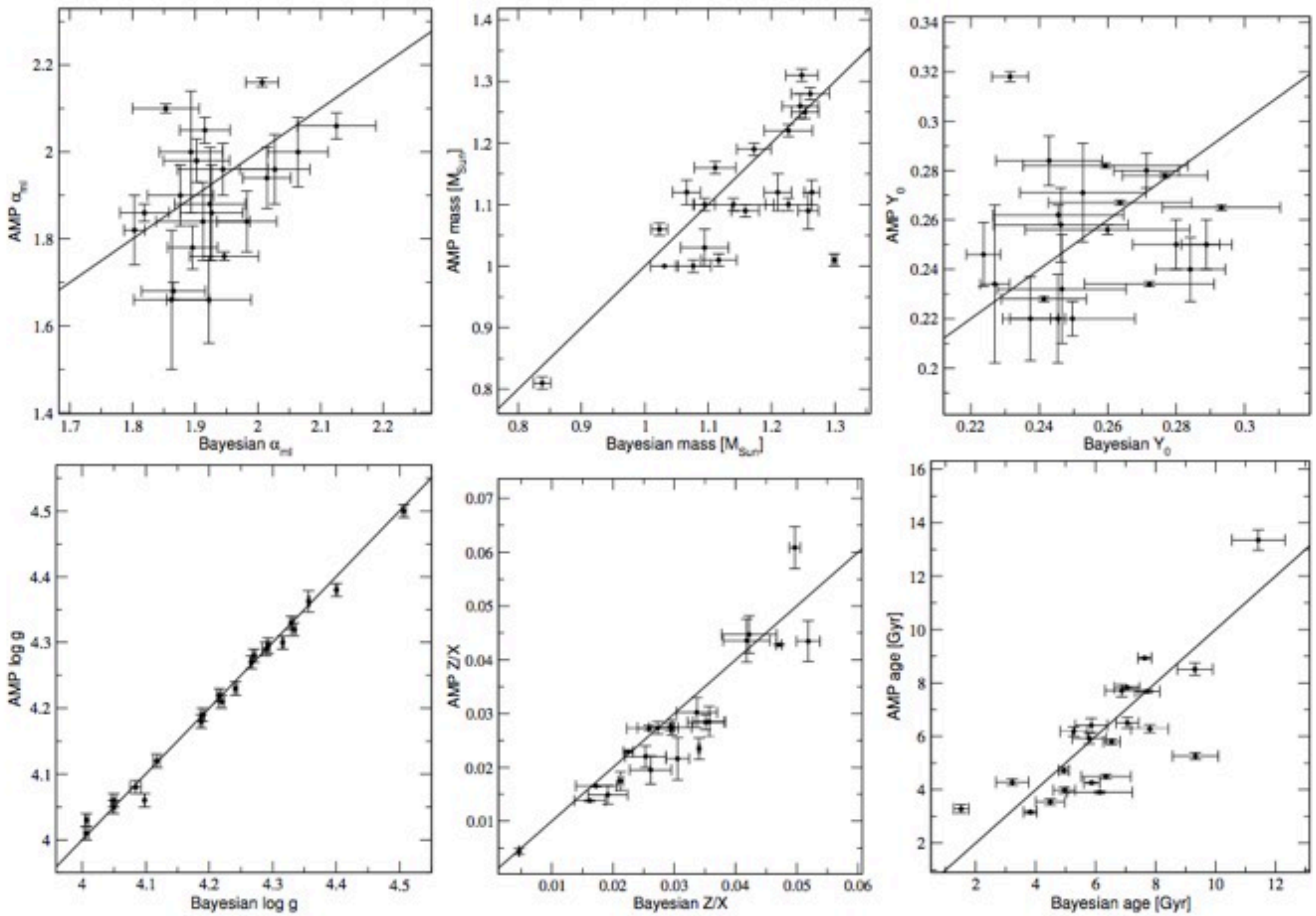
application to Kepler targets

detailed modelling of 23 Kepler targets ...



AMP = Asteroseismic Modelling Portal (Metcalfe et al. 2009)

Asteroseismic comparison results from Mathur et al. (2012) & Metcalfe et al. (2012)

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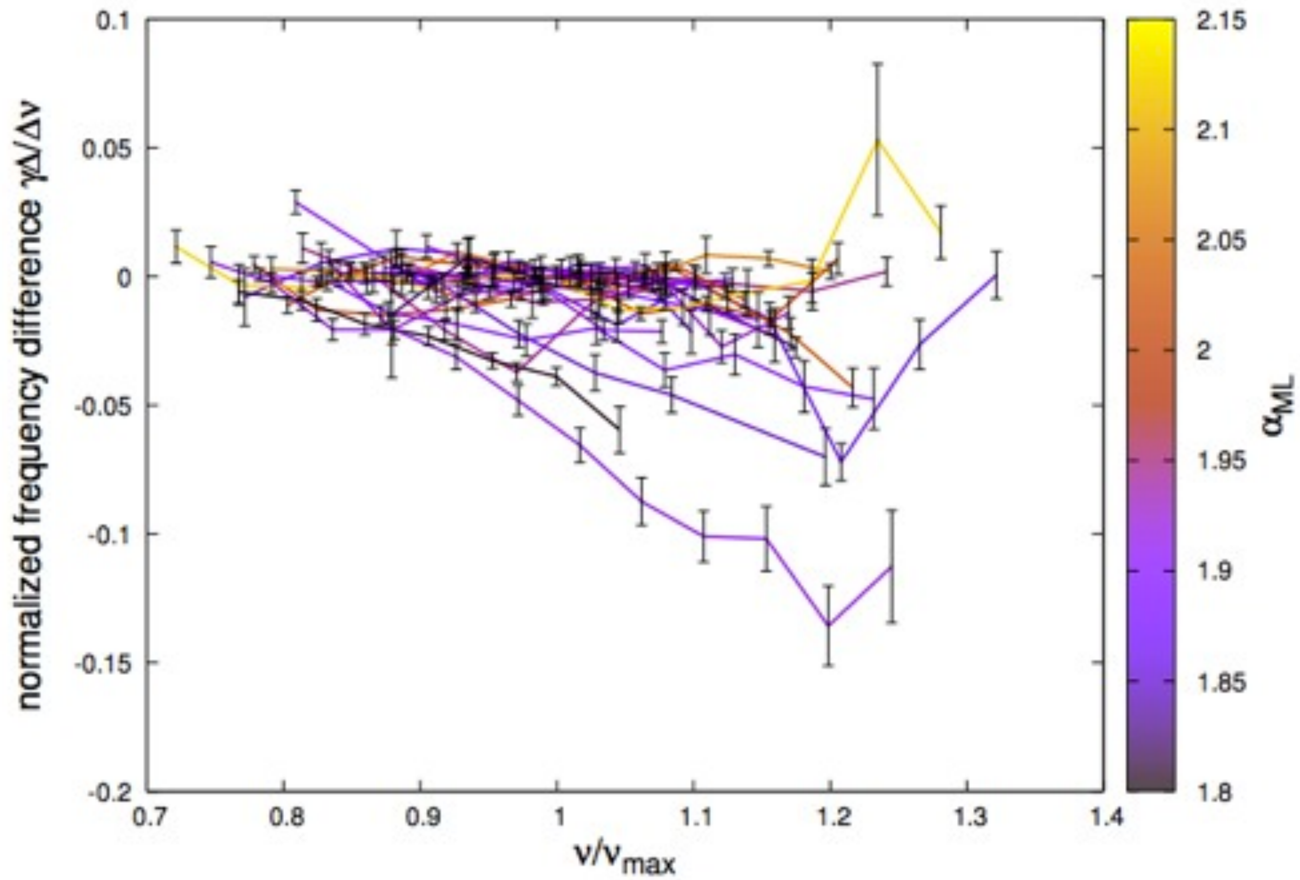
application to Kepler targets

Gruberbauer et al. 2013



- first probabilistic measurements of stellar surface effects
- correlation with mixing length parameter?

Bayesian advertising

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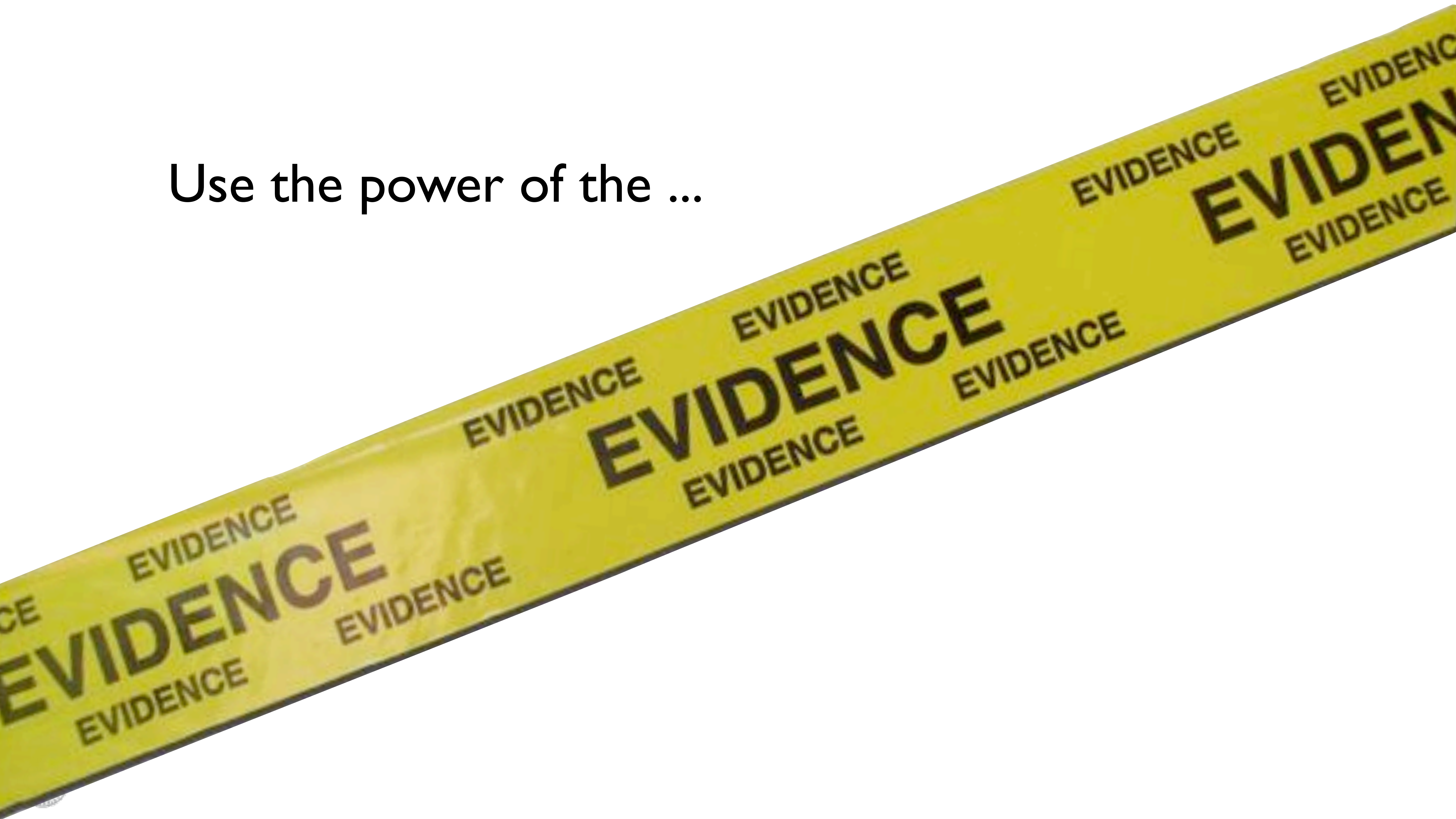
Use the power of the ...



sound speed profile of the Sun