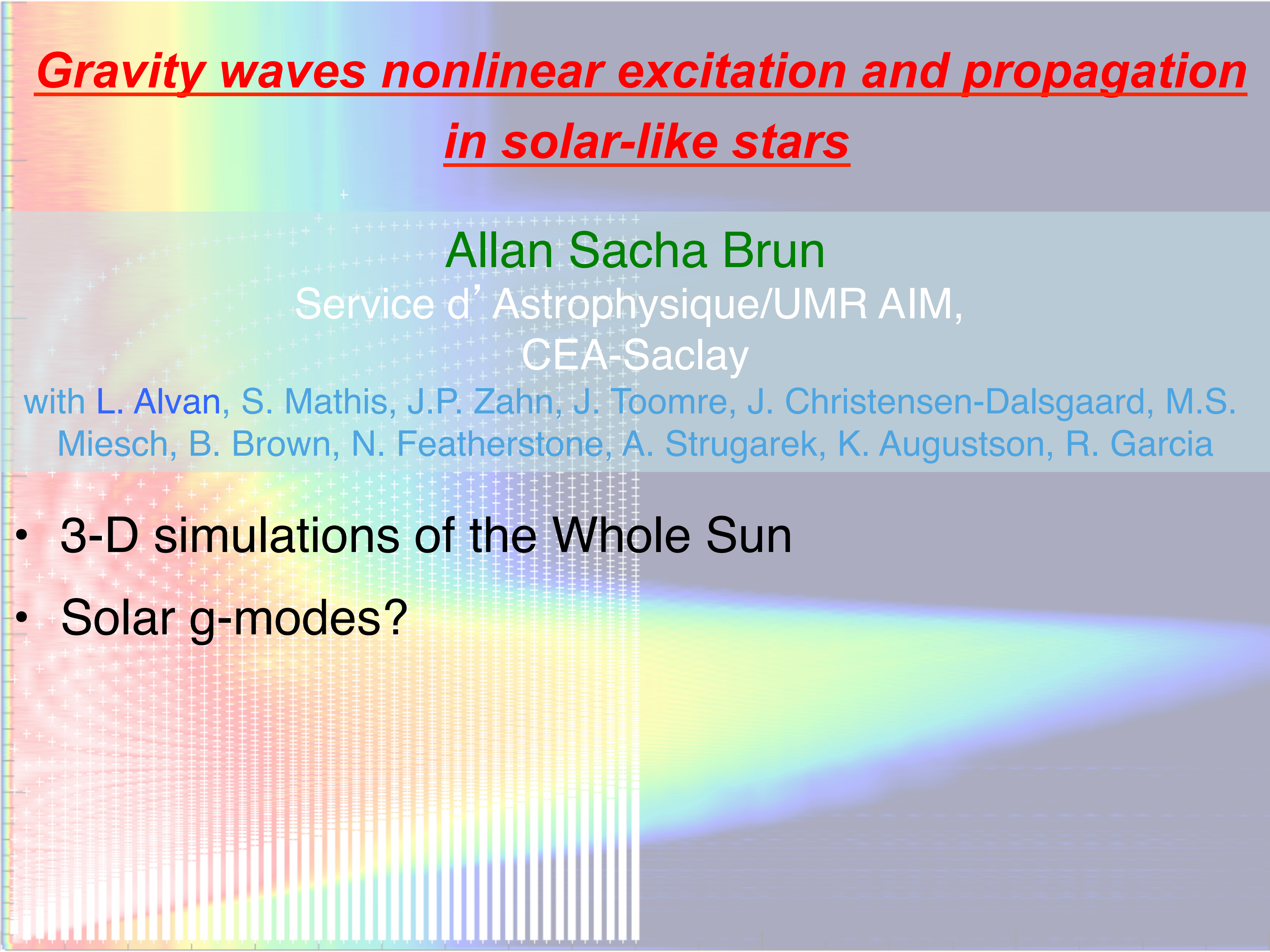


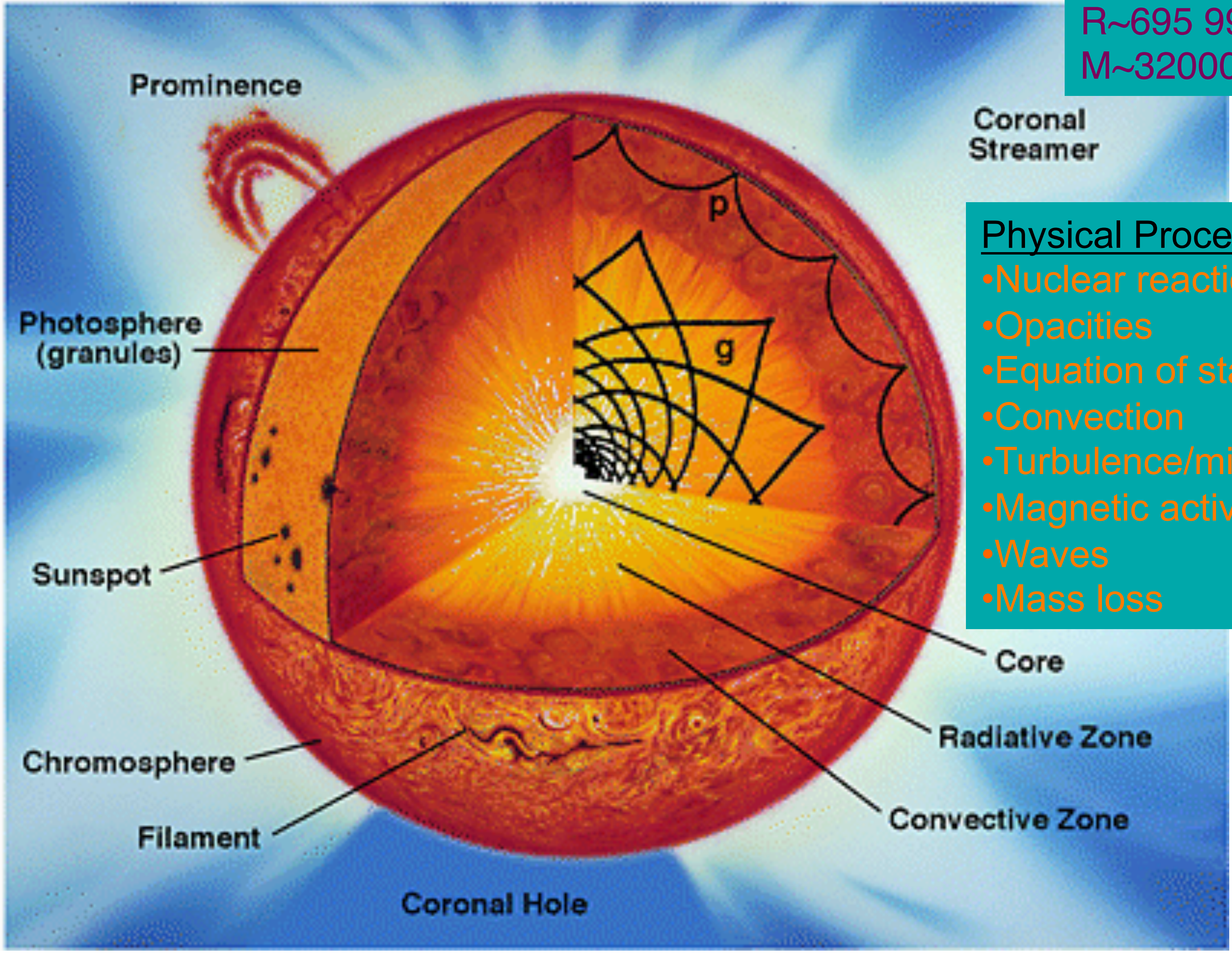
Gravity waves nonlinear excitation and propagation in solar-like stars

Allan Sacha Brun
Service d'Astrophysique/UMR AIM,
CEA-Saclay
with L. Alvan, S. Mathis, J.P. Zahn, J. Toomre, J. Christensen-Dalsgaard, M.S. Miesch, B. Brown, N. Featherstone, A. Strugarek, K. Augustson, R. Garcia

- 3-D simulations of the Whole Sun
- Solar g-modes?

Solar Interior:a cartoon view

$T_c \sim 15.5\ 10^6$ K
$\rho_c \sim 155$ g/cm3
R~695 990km
$M \sim 320000\ M_{terre}$



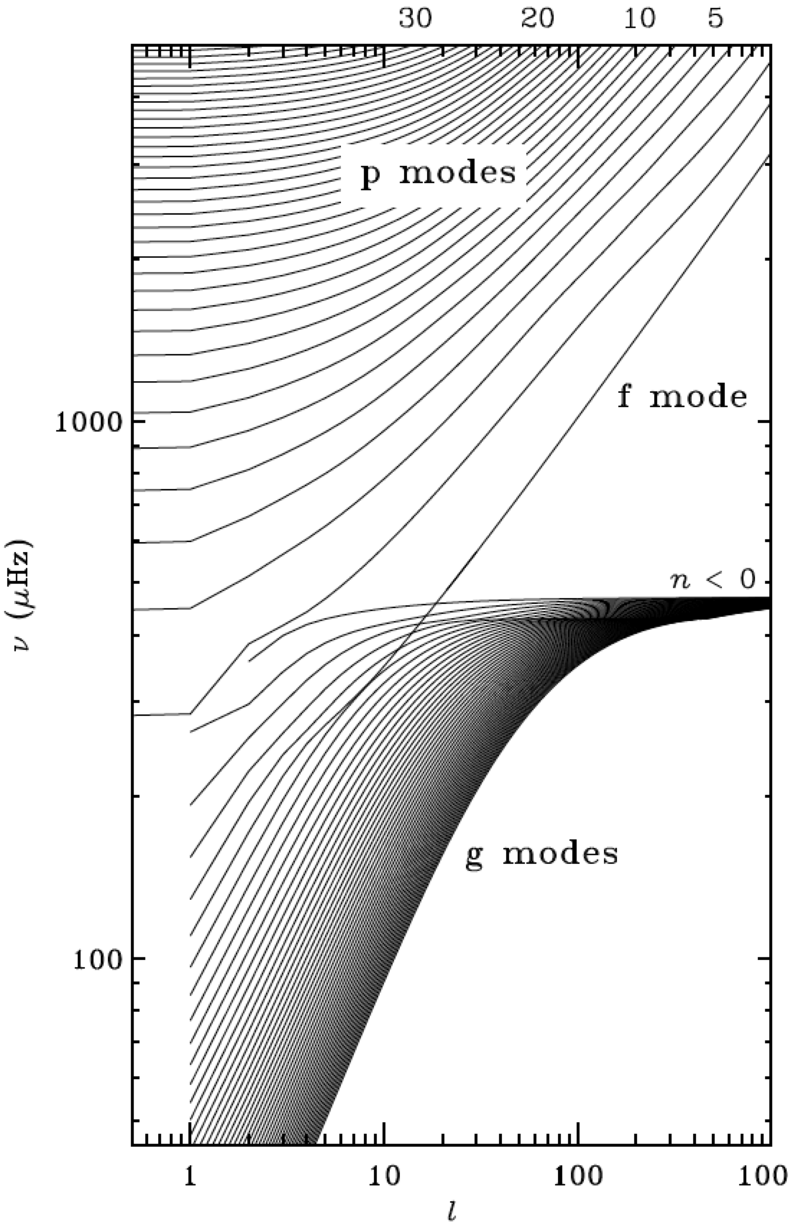
Physical Processes:

- Nuclear reaction
- Opacities
- Equation of state
- Convection
- Turbulence/mixing
- Magnetic activity
- Waves
- Mass loss

general web site: http://science.nasa.gov/ssl/pad/solar/default.htm

A Quick Reader Digest on Waves inside the Sun

JCD's Lecture Notes

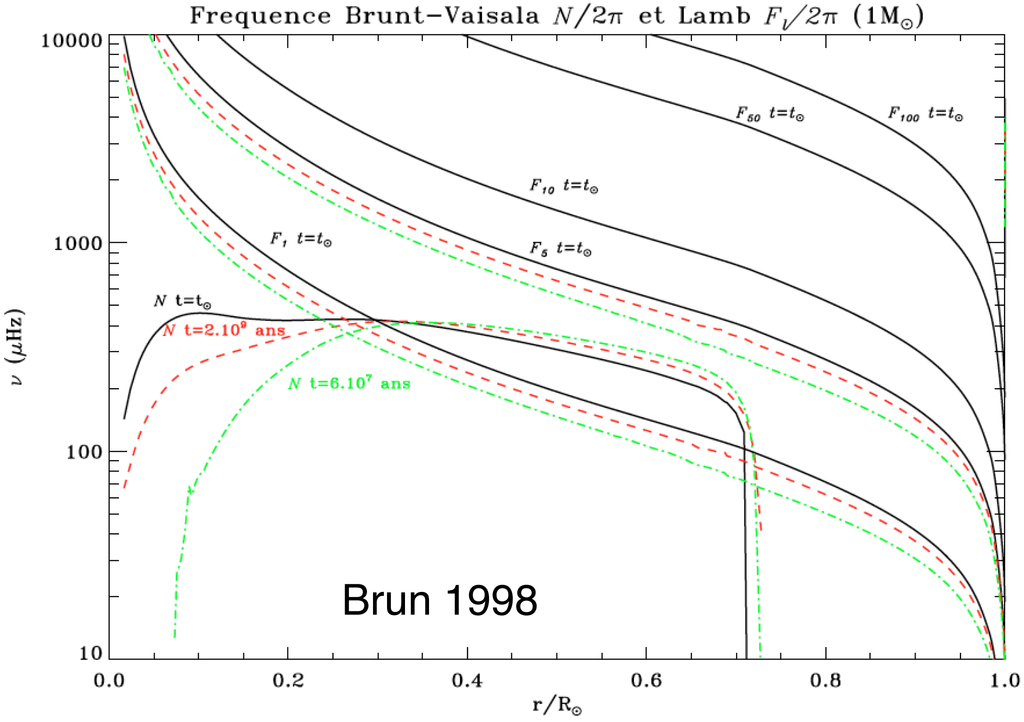


Acoustic and Internal waves are excited inside the Sun

$$\frac{d^2X}{dr^2} + \frac{1}{c^2}\left[S_l^2\left(\frac{N^2}{\omega^2}-1\right)+\omega^2-\omega_c^2\right]X = 0\,.$$

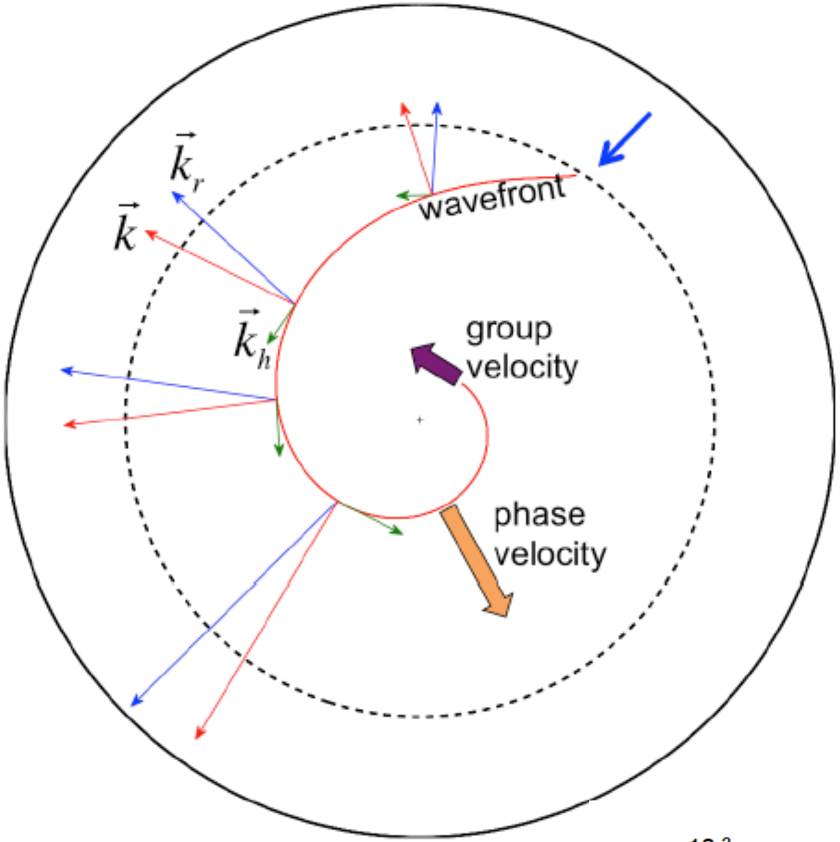
$$N^2 = g_0\left(\frac{1}{\Gamma_{1,0}}\frac{d\ln p_0}{dr} - \frac{d\ln\rho_0}{dr}\right) \qquad S_l = l(l+1)c^2/r^2$$

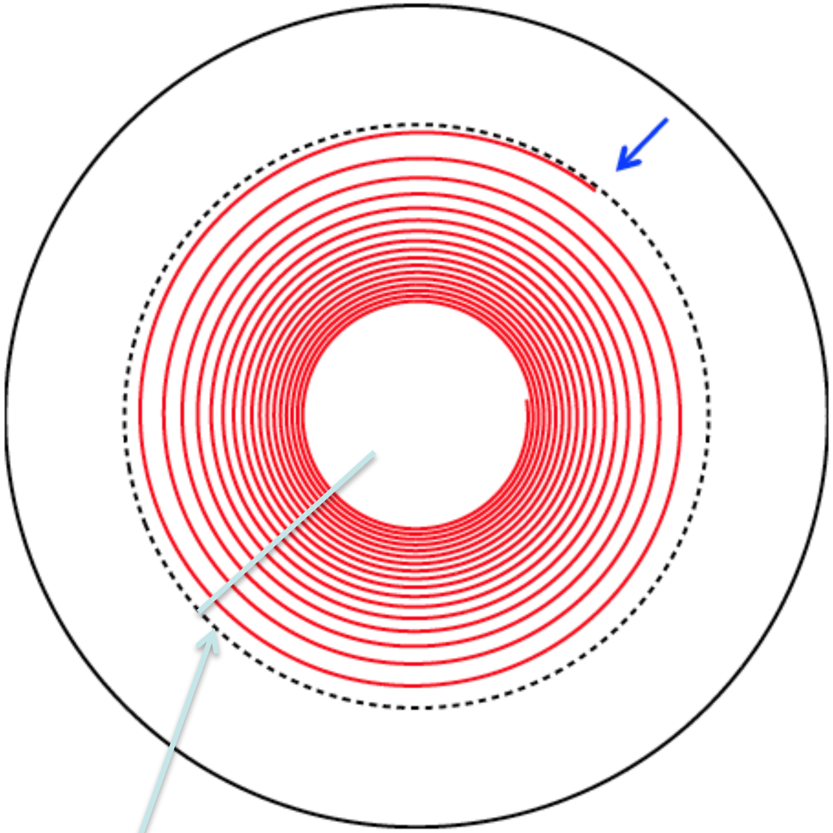
Brunt-Vaisala & Lamb Frequencies

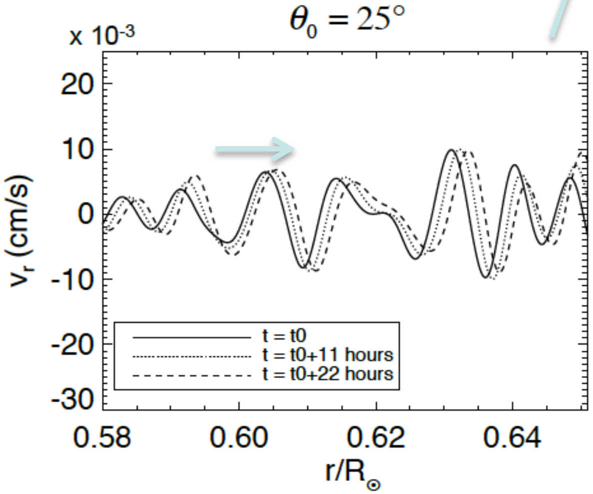


Brun 1998

Basic Properties of internal waves







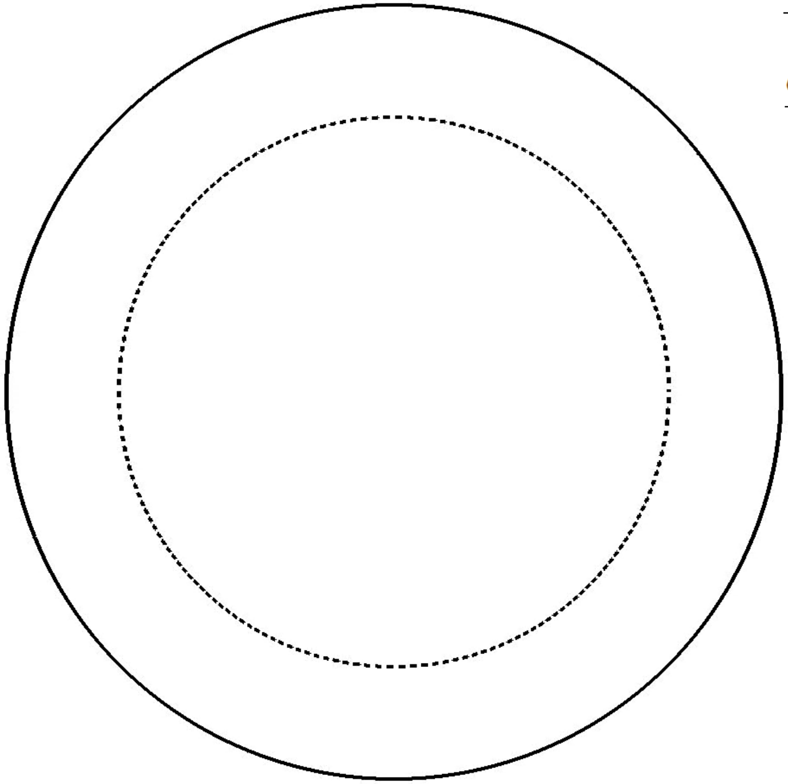
Radial cut in ASH simulation

Phase velocity
clearly outward

G vs P-modes: simple ray paths

$$\begin{aligned}\frac{dx_i}{dt} &= \frac{\partial W}{\partial k_i}\\ \frac{dk_i}{dt} &= -\frac{\partial W}{\partial x_i}\end{aligned}$$

P-modes

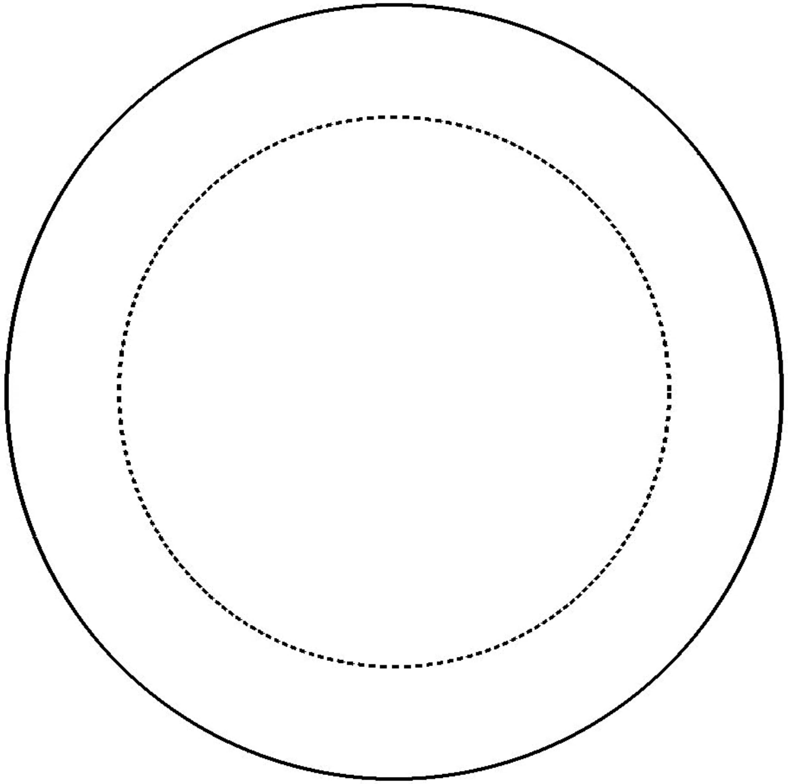


$$\begin{aligned}k_r^2 &= \frac{w^2}{c_s^2} - \frac{l(l+1)}{r^2}\\ k_h^2 &= \frac{l(l+1)}{r^2}\end{aligned}$$

$$w = c_s\sqrt{k_r^2 + k_h^2} = c_s k$$

$$\begin{aligned}dr &= \frac{k_r}{k} c_s dt\\ d\theta &= \frac{k_h}{k} c_s dt \frac{1}{r}\end{aligned}$$

G-modes



$$\begin{aligned}k_r^2 &= \frac{l(l+1)}{r^2}\left(\frac{N^2}{w^2} - 1\right)\\ k_h^2 &= \frac{l(l+1)}{r^2}\end{aligned}$$

$$w = \frac{k_h}{\sqrt{k_r^2 + k_h^2}} N = \frac{k_h}{k} N$$

$$\begin{aligned}dr &= -\frac{k_r k_h}{k^2}\frac{N dt}{k}\\ d\theta &= \left(1 - \frac{k_h^2}{k^2}\right)\frac{N dt}{k}\frac{1}{r}\end{aligned}$$

Note: the Eikonal equation allowing to compute the ray paths
are indept of l for g-modes, hence changing the order l does not change
the ray path (does change the wave speed). Only changing the frequency does.

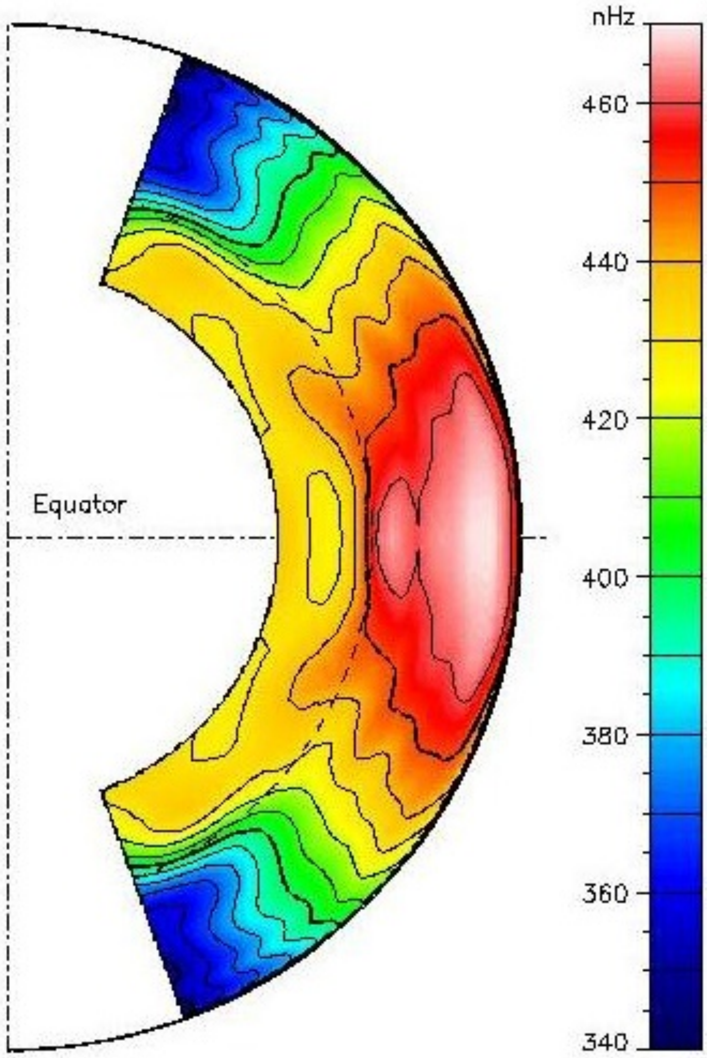
St Andrew’s Cross

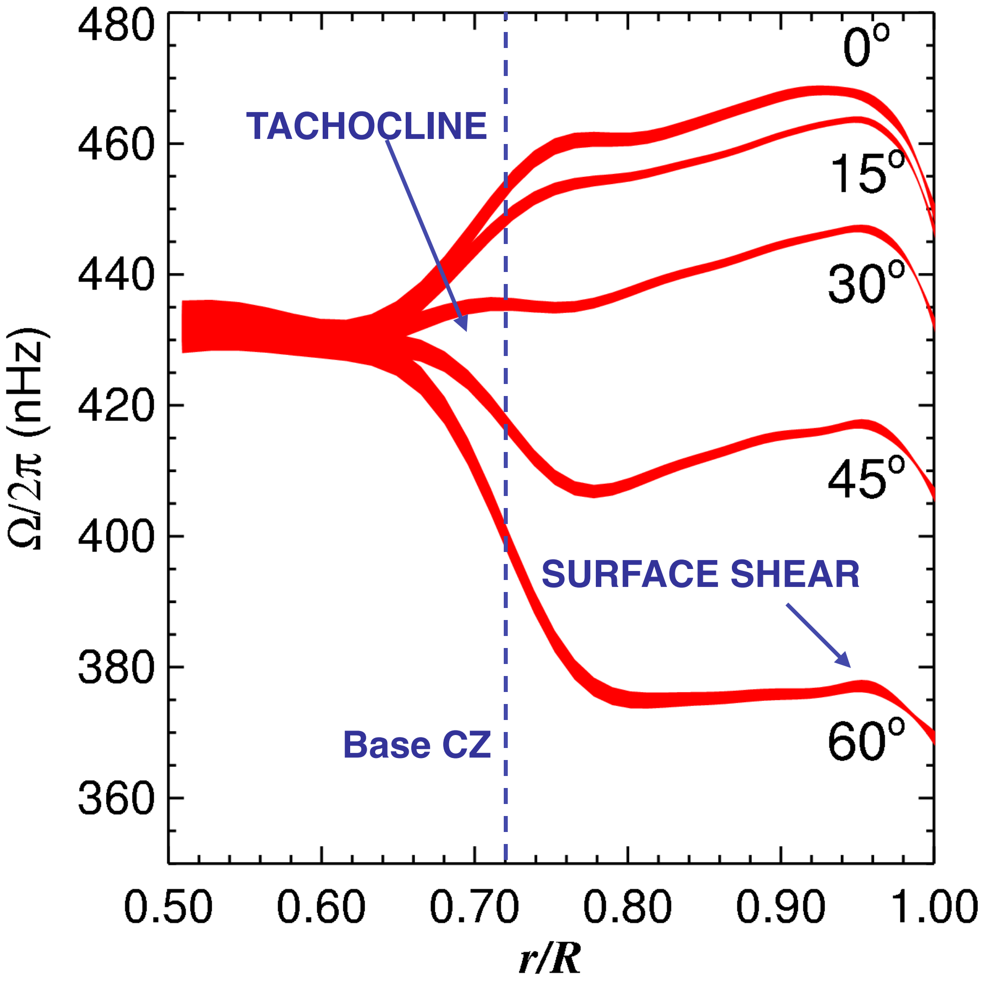
$\cos \alpha = \omega / N$

Solar Internal Rotation

(GONG, MDI data)

Helioseismology Results





ASH models of the Whole Sun

⇒ MHD anelastic equations
⇒ 3-D global spherical Models
⇒ Realistic stratification up to 0.97 Rsol

ASH code: Clune et al. 1999, Brun et al. 2004
New ASH-FD version (20,000+ cores): Featherstone et al. 2013

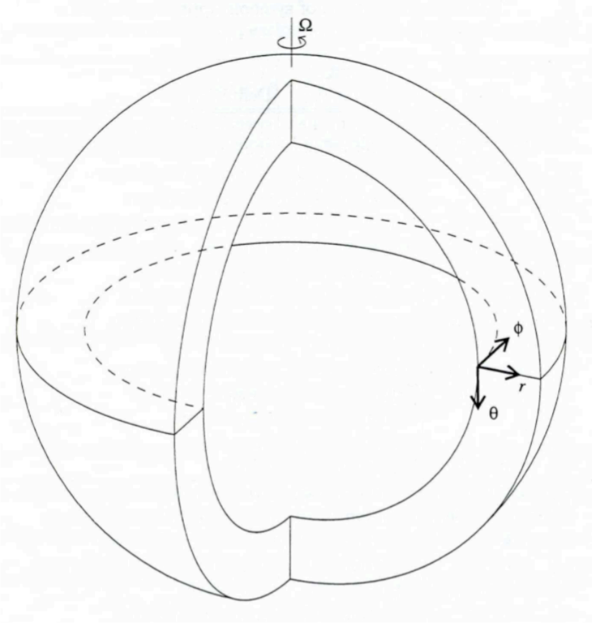
$$\nabla\cdot(\bar{\rho}\boldsymbol{v}) = 0, \tag{1}$$

$$\nabla\cdot\boldsymbol{B} = 0, \tag{2}$$

$$\begin{aligned}\bar{\rho}\left[\frac{\partial\boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + 2\Omega_0\times\boldsymbol{v}\right] = &-\nabla P + \rho\boldsymbol{g} + \frac{1}{4\pi}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}\\ &-\nabla\cdot\boldsymbol{\mathcal{D}} - \left(\nabla\bar{P} - \bar{\rho}\boldsymbol{g}\right),\end{aligned} \tag{3}$$

$$\begin{aligned}\bar{\rho}\bar{T}\frac{\partial S}{\partial t} + \bar{\rho}\bar{T}\boldsymbol{v}\cdot\nabla\left(\bar{S}+S\right) = &\nabla\cdot\left[\kappa_r\bar{\rho}c_p\nabla\left(\bar{T}+T\right)\right.\\ &\left.+\kappa\bar{\rho}\bar{T}\nabla\left(\bar{S}+S\right)\right] + \frac{4\pi\eta}{c^2}\boldsymbol{j}^2 + 2\bar{\rho}\nu\left[e_{ij}e_{ij} - \frac{1}{3}(\nabla\cdot\boldsymbol{v})^2\right] + \bar{\rho}\epsilon,\end{aligned} \tag{4}$$

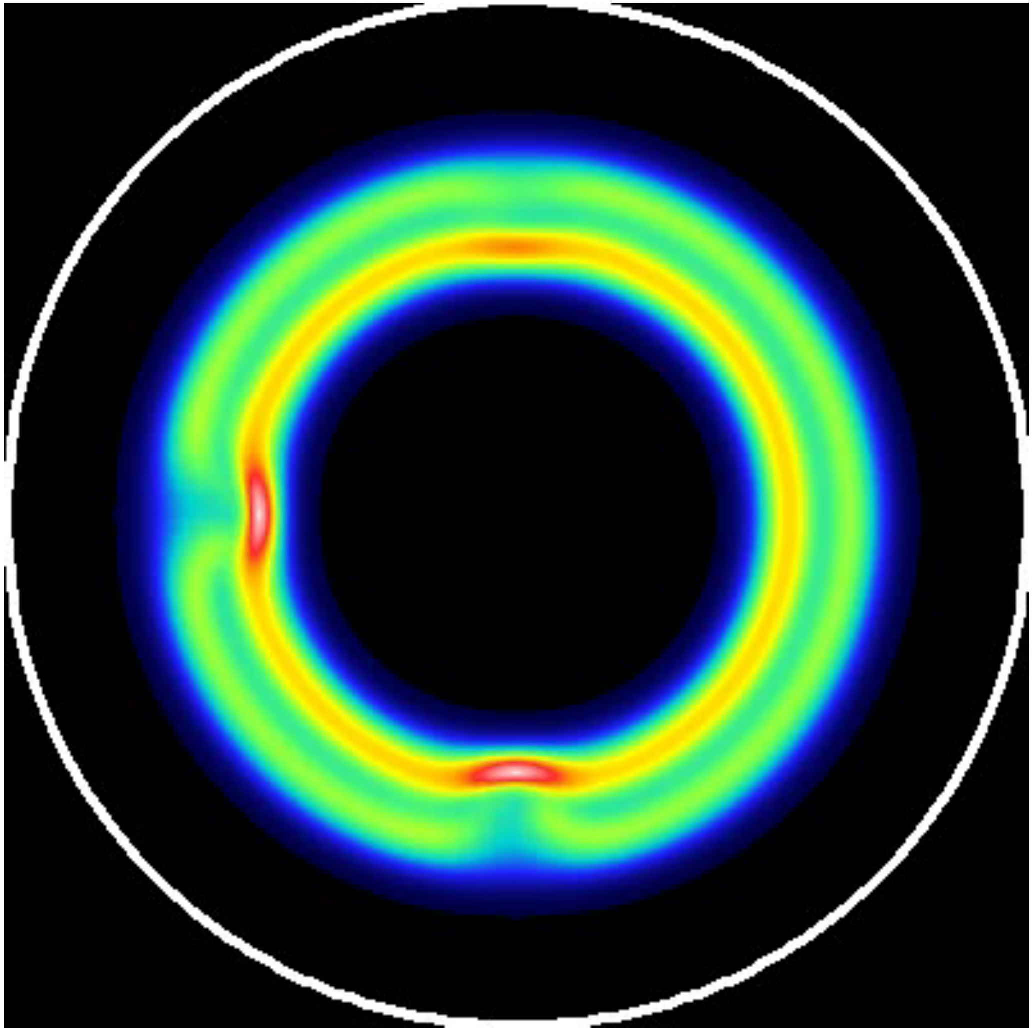
$$\frac{\partial\boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) - \nabla\times(\eta\nabla\times\boldsymbol{B}),$$



Full Sphere Deep Sun Models

ASH Full Sphere: regularization of solution at r=0 and implementation in the code operational (done jointly with N. Featherstone).

Test case: 3 cold entropy blobs and a magnetic torus encounter!



+ Anelastic Formulation

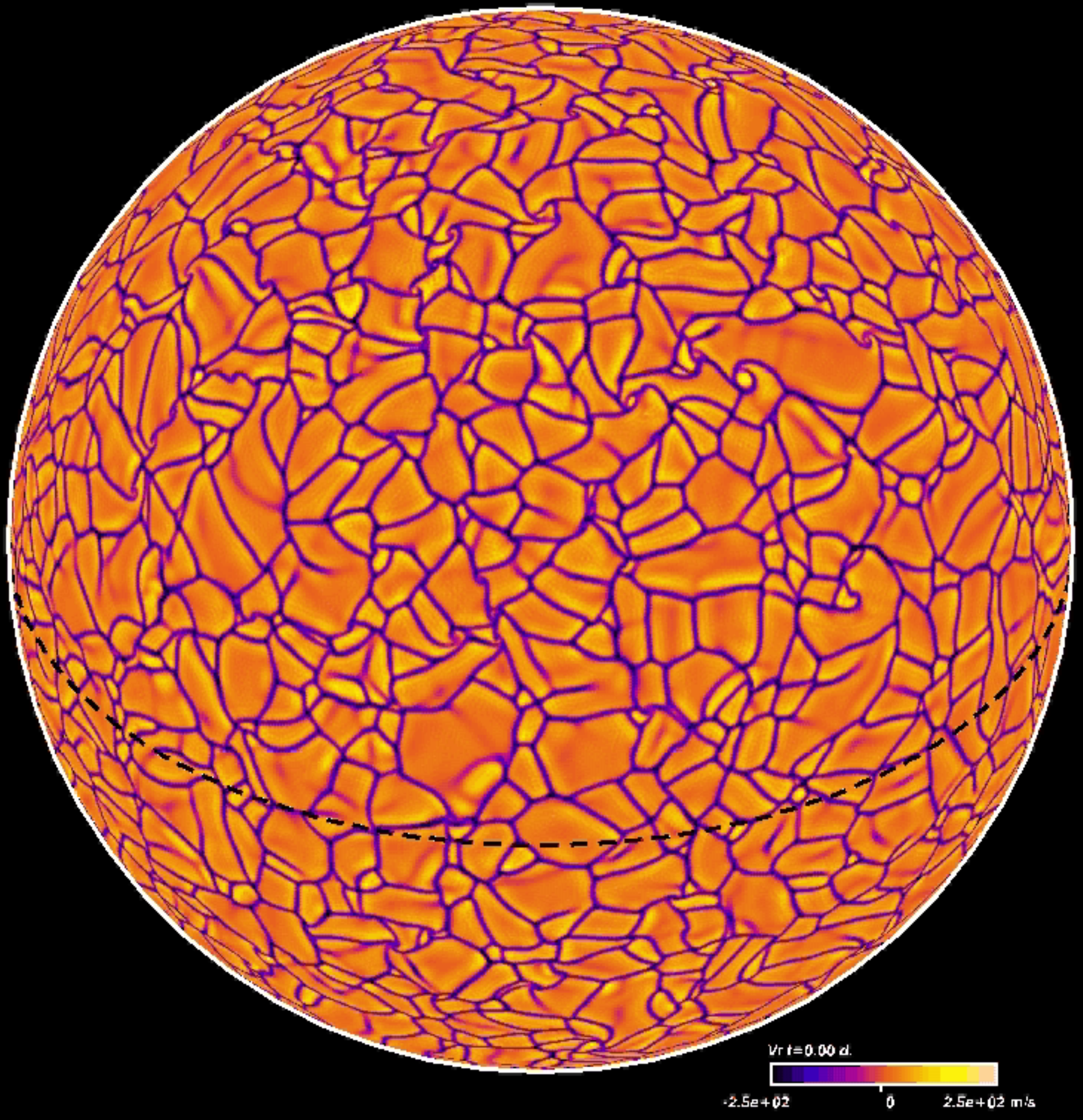
Brown, Vasil & Zweibel 2012 have shown that some formulations are more accurate. Recent tests in ASH do confirm their findings

Convective Motions (radial velocity v_r)

Resolution~ 1500^3
$Re=V_{rms}D/\nu \sim 1000$
Pr=0.25

Simulation a
6000 cpus
(BlueGene/p)
Or 2000 BullX

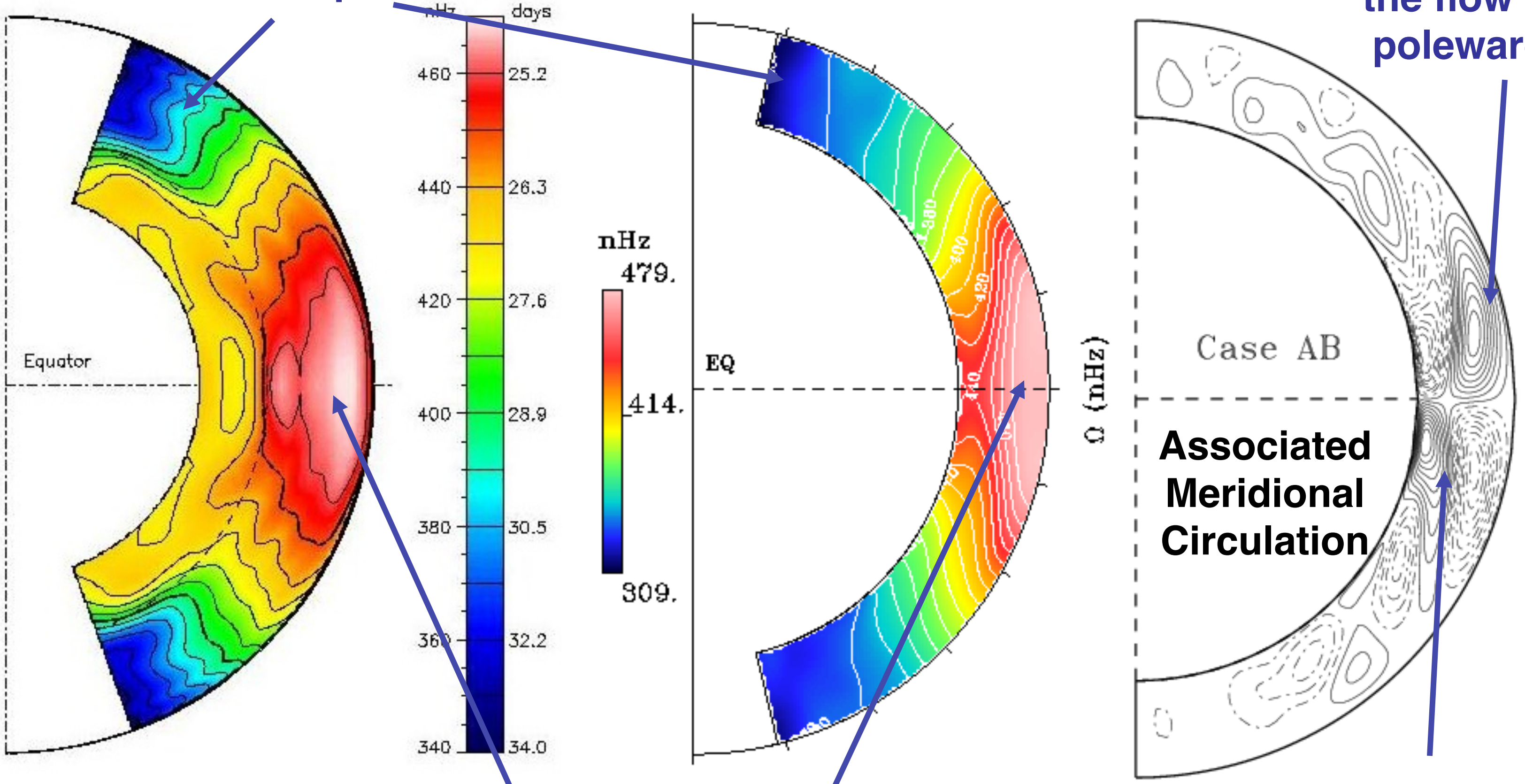
depth=0.96 R



Brun 2011

Mean Angular Velocity Ω

(Brun & Toomre 2002, ApJ 570, 865)



Slow poles

At the equator the flow is poleward

Associated Meridional Circulation

Multi cells flow

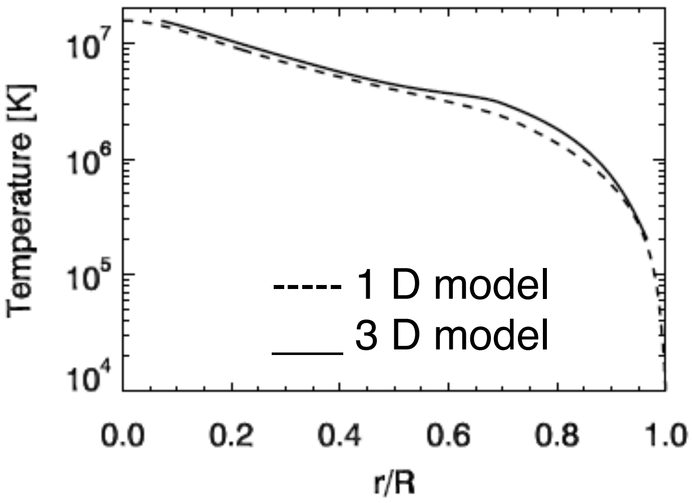
Fast equator

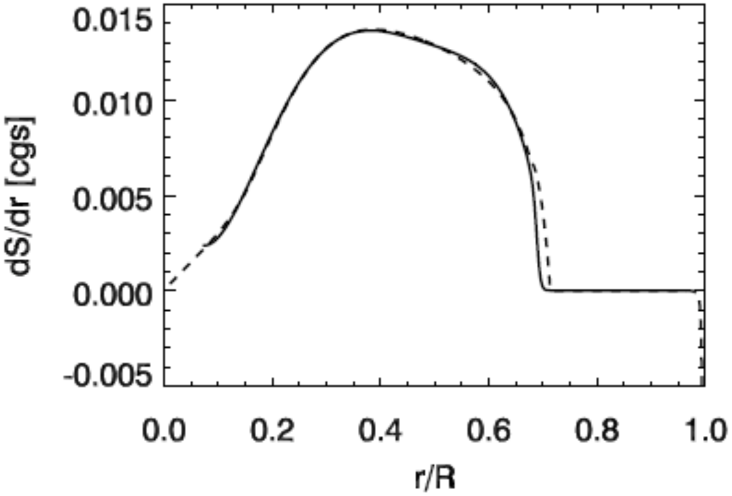
GONG DATA

SIMULATION

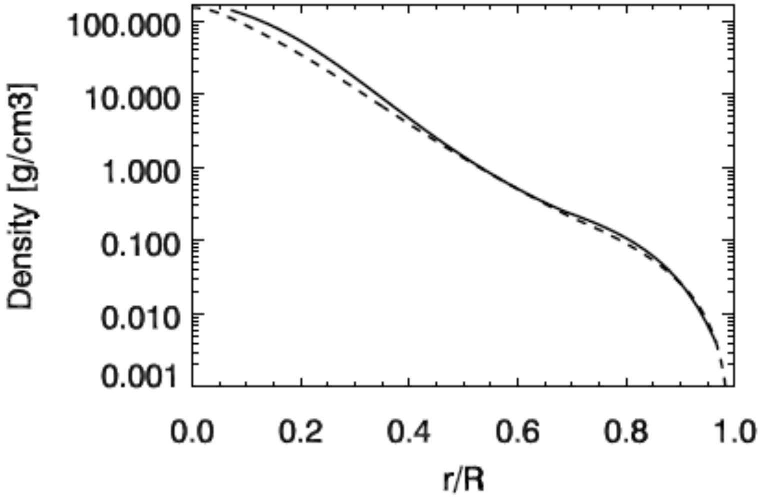
Including a Stable Layer Below

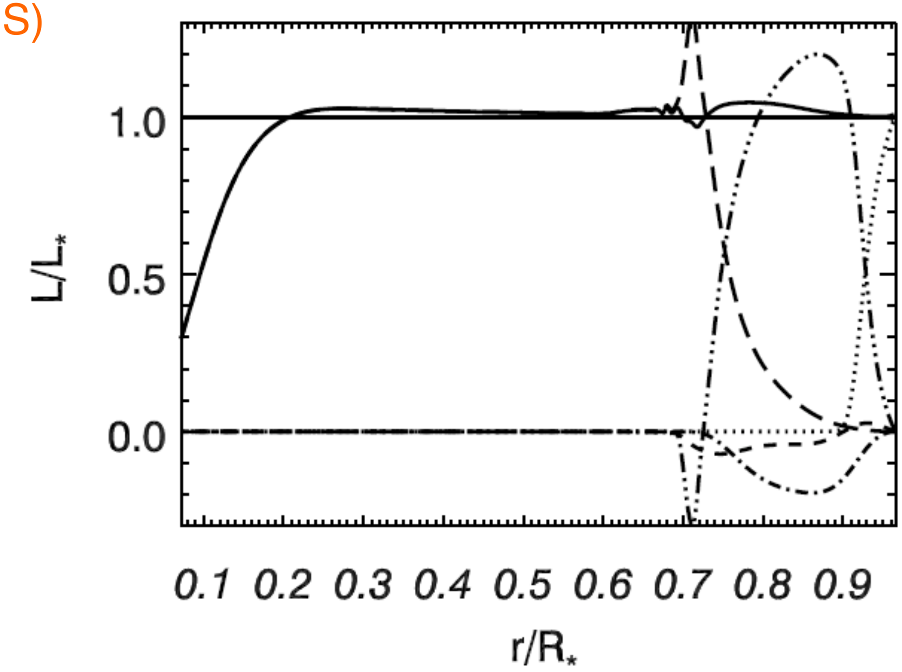
Realistic Solar Stratification Background State





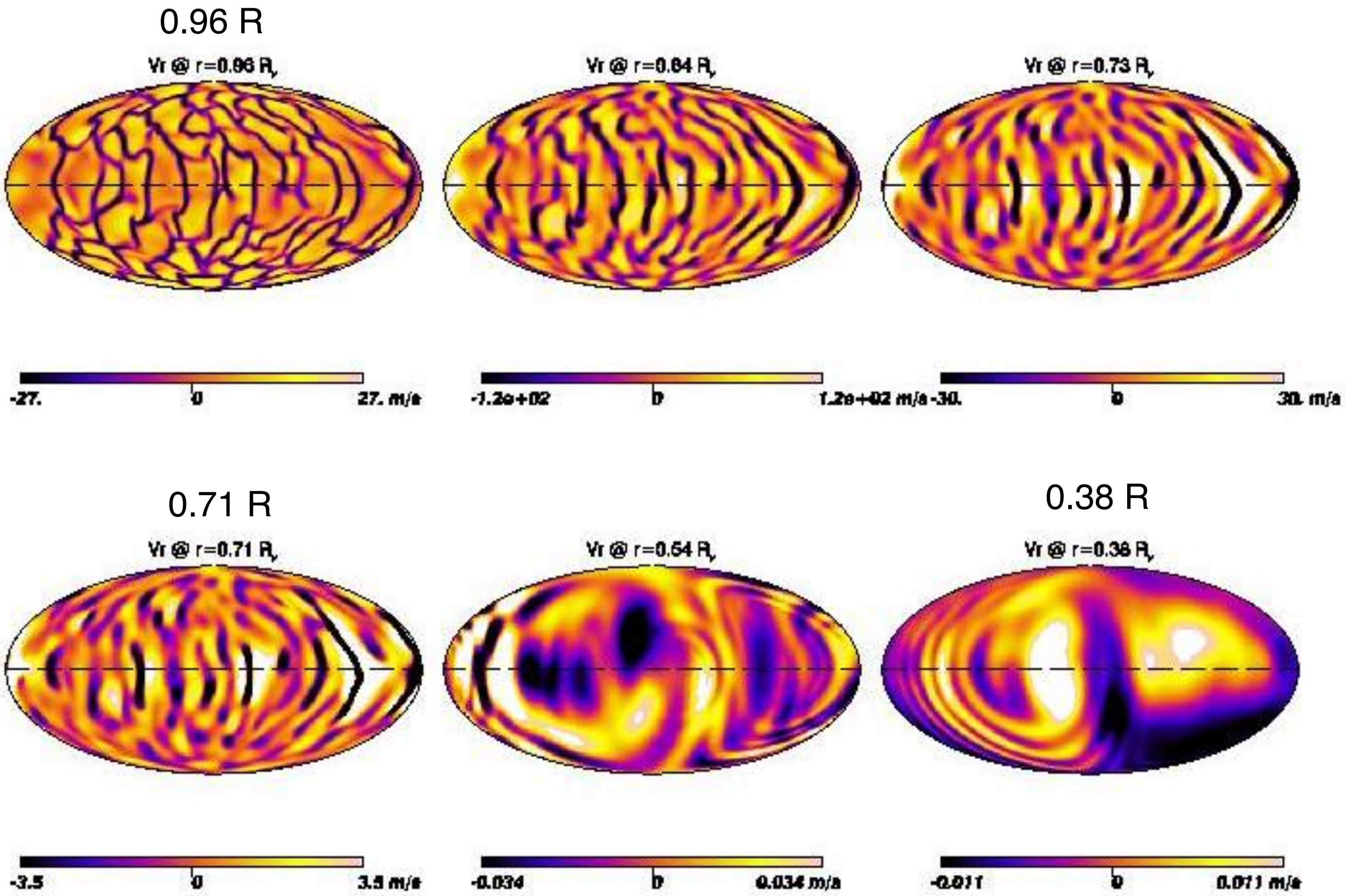
Cesam 1-D model (Brun et al. 2002 ~ model S)





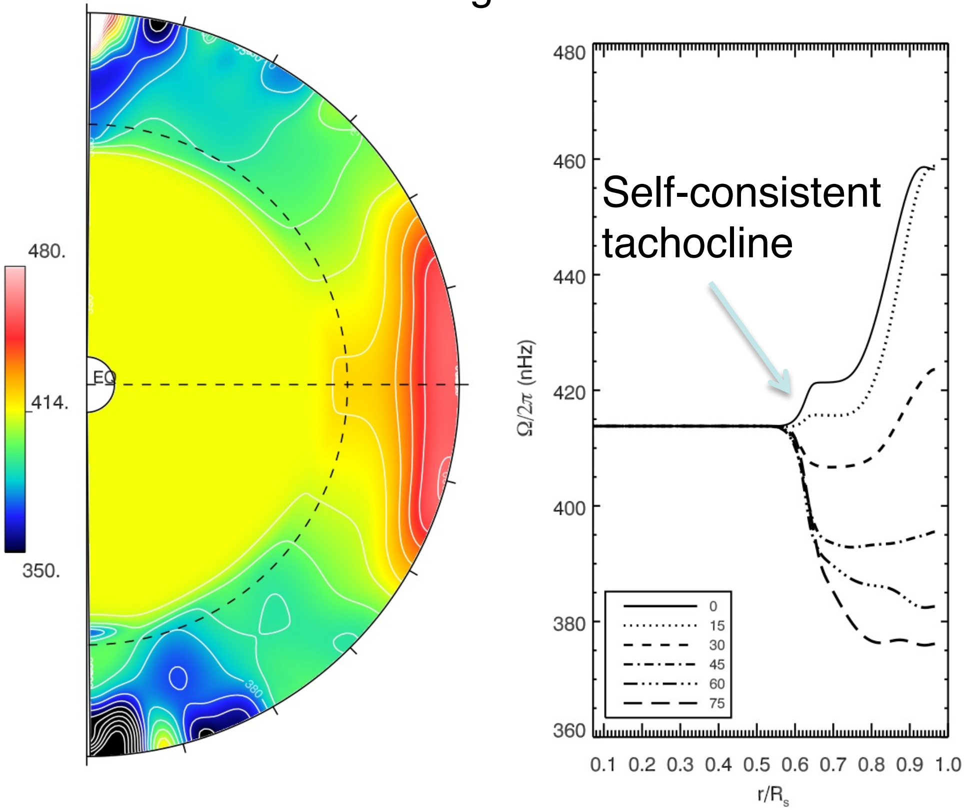
Brun, Miesch, Toomre, 2011, ApJ , 742

Convection and Waves



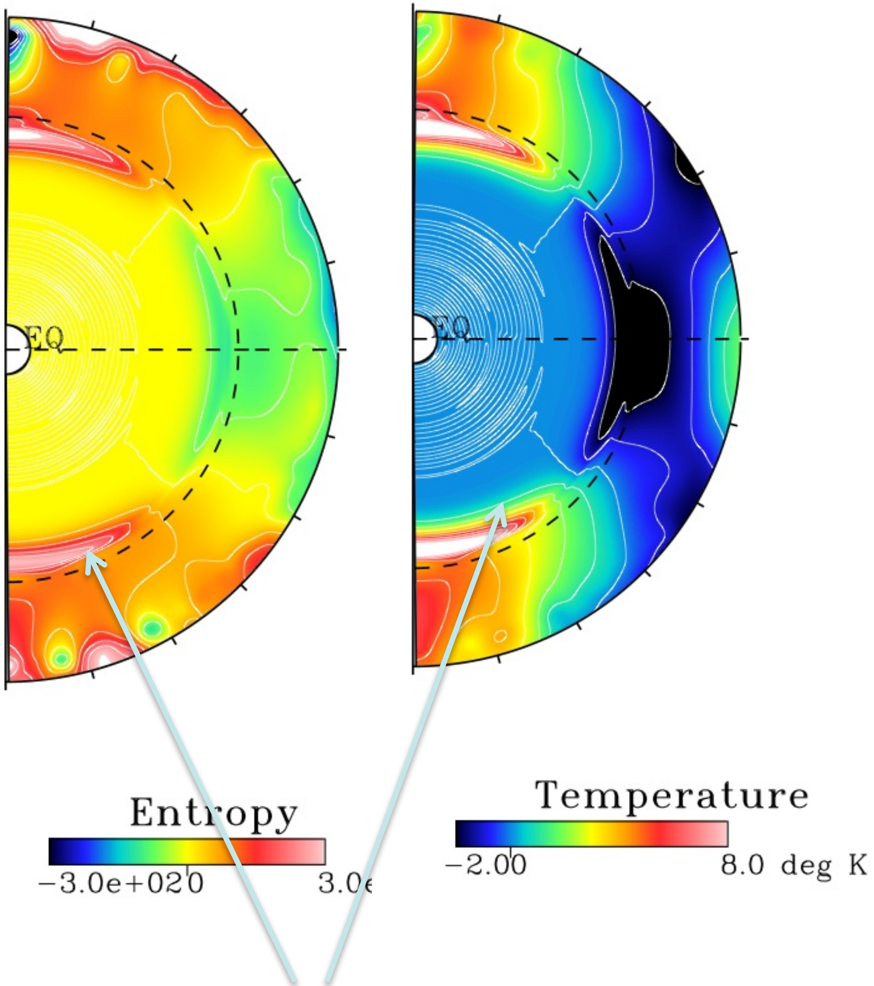
Omega Profile & Thermal Perturbations

Omega



Self-consistent tachocline

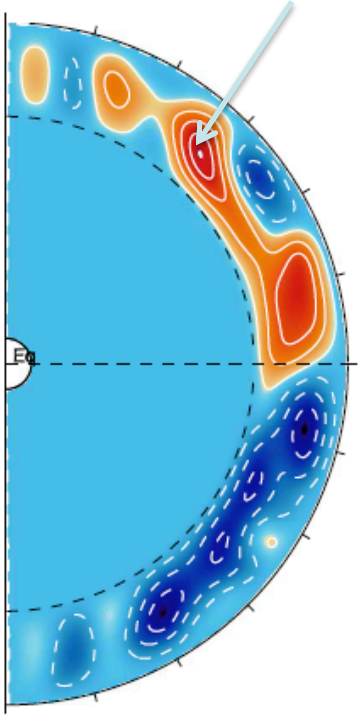
Warm poles, cool equator

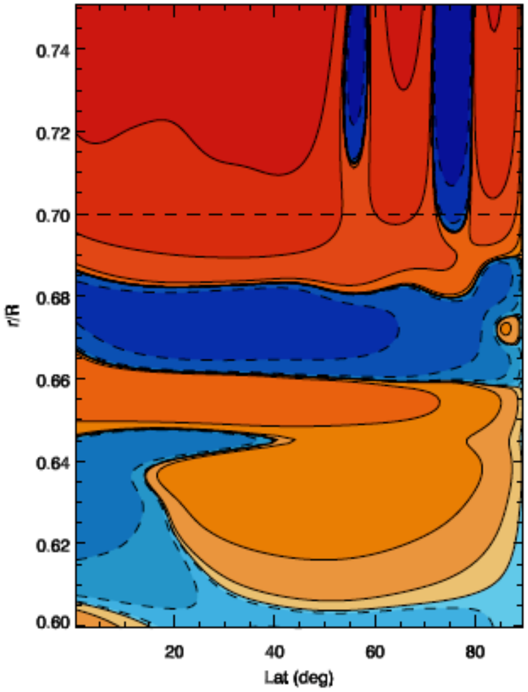


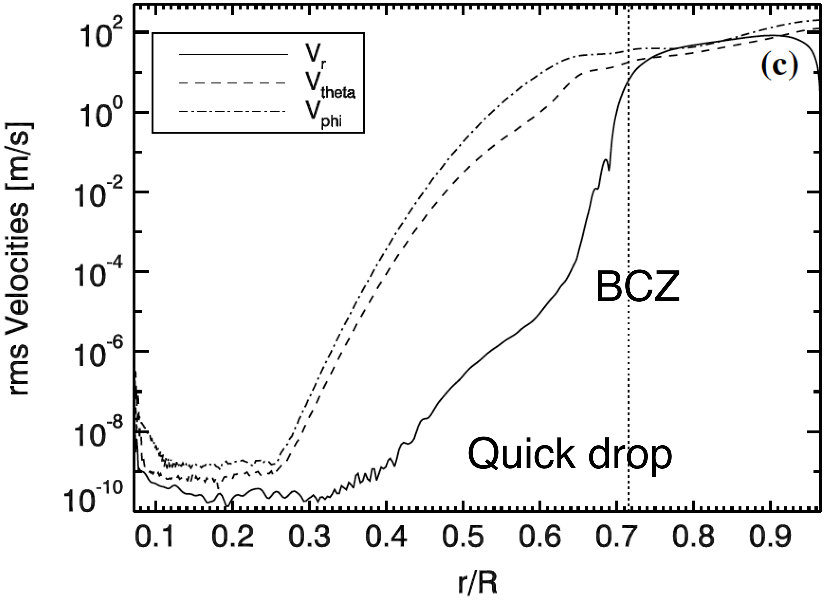
LARGER fluctuations at bcz

Meridional Circulation

Almost unicellular flow







BCZ

Quick drop

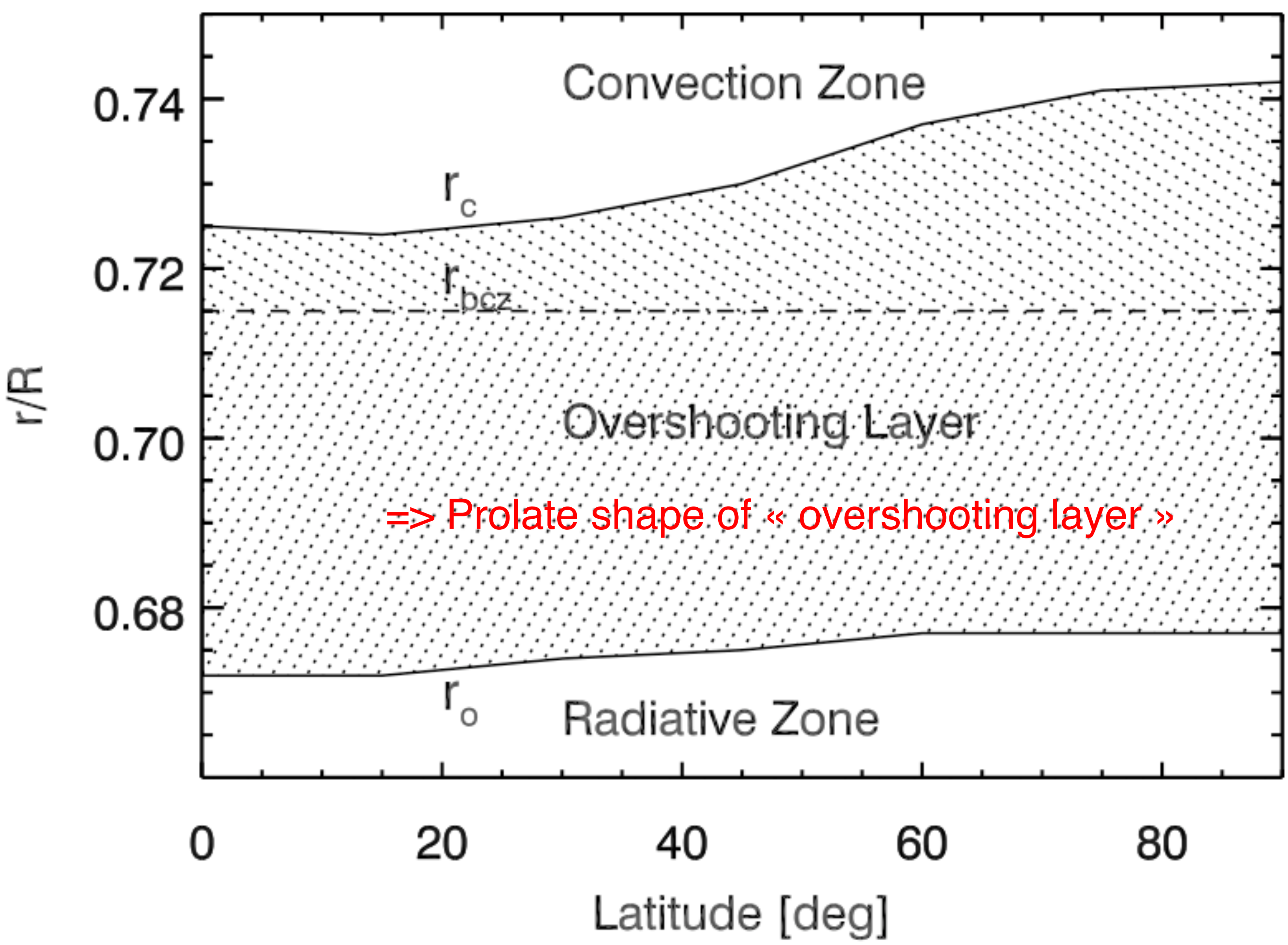
Penetrationof MC flow
< 0.02 Rsol

Drop by
3 orders of
magnitude
over 0.04 R

Brun, Miesch, Toomre, 2011, ApJ , 742

Overshooting

Radial Enthalpy Flux



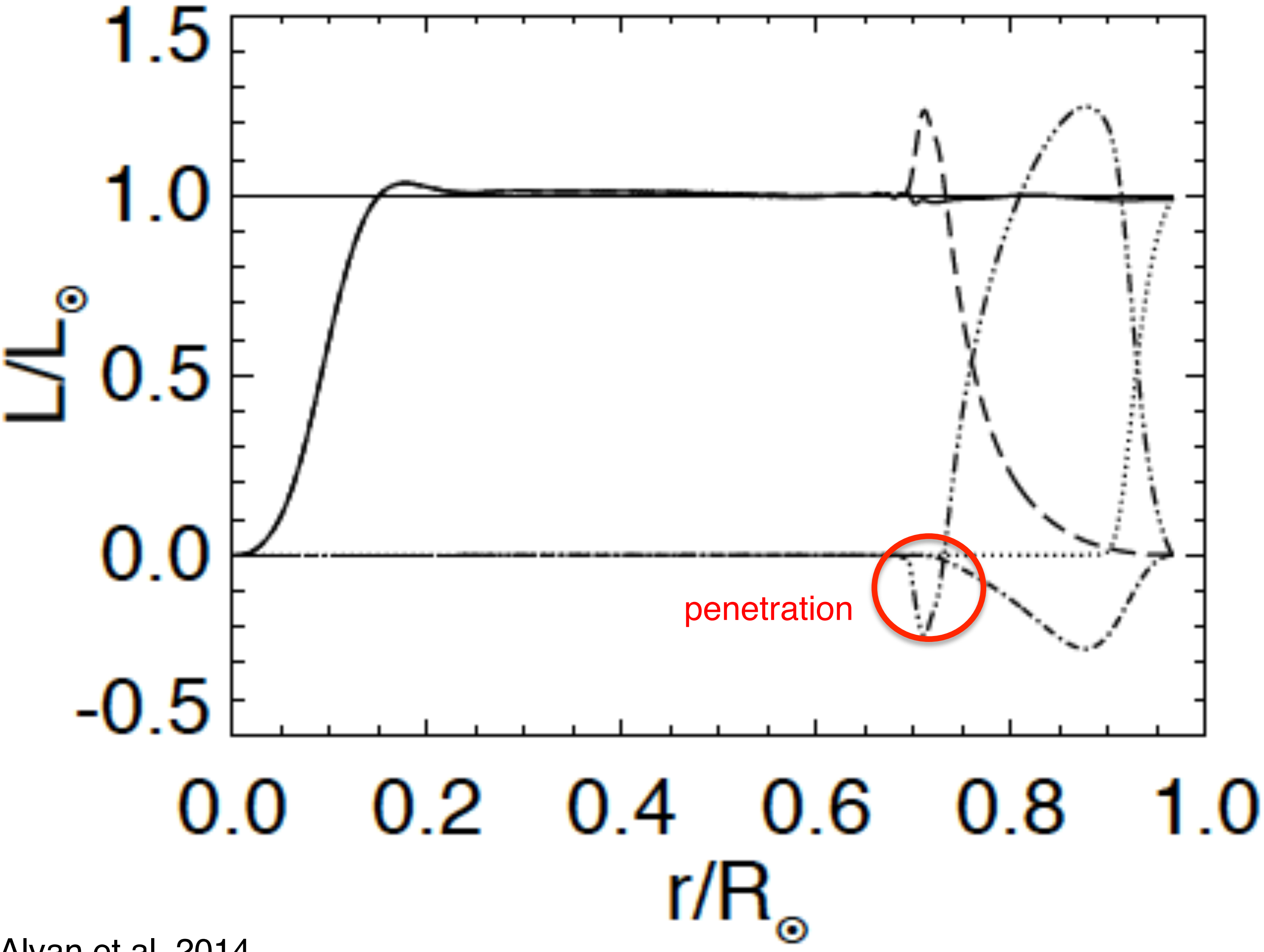
=> Prolate shape of « overshooting layer »

Enthalpy flux becomes negative

d_ov ~ 0.04 R

Last radius of « mixing »

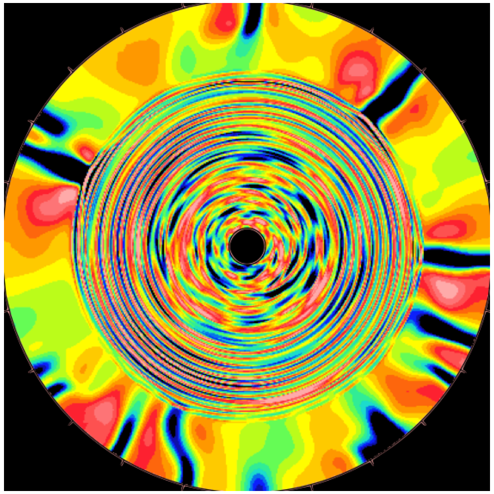
Going to r=0



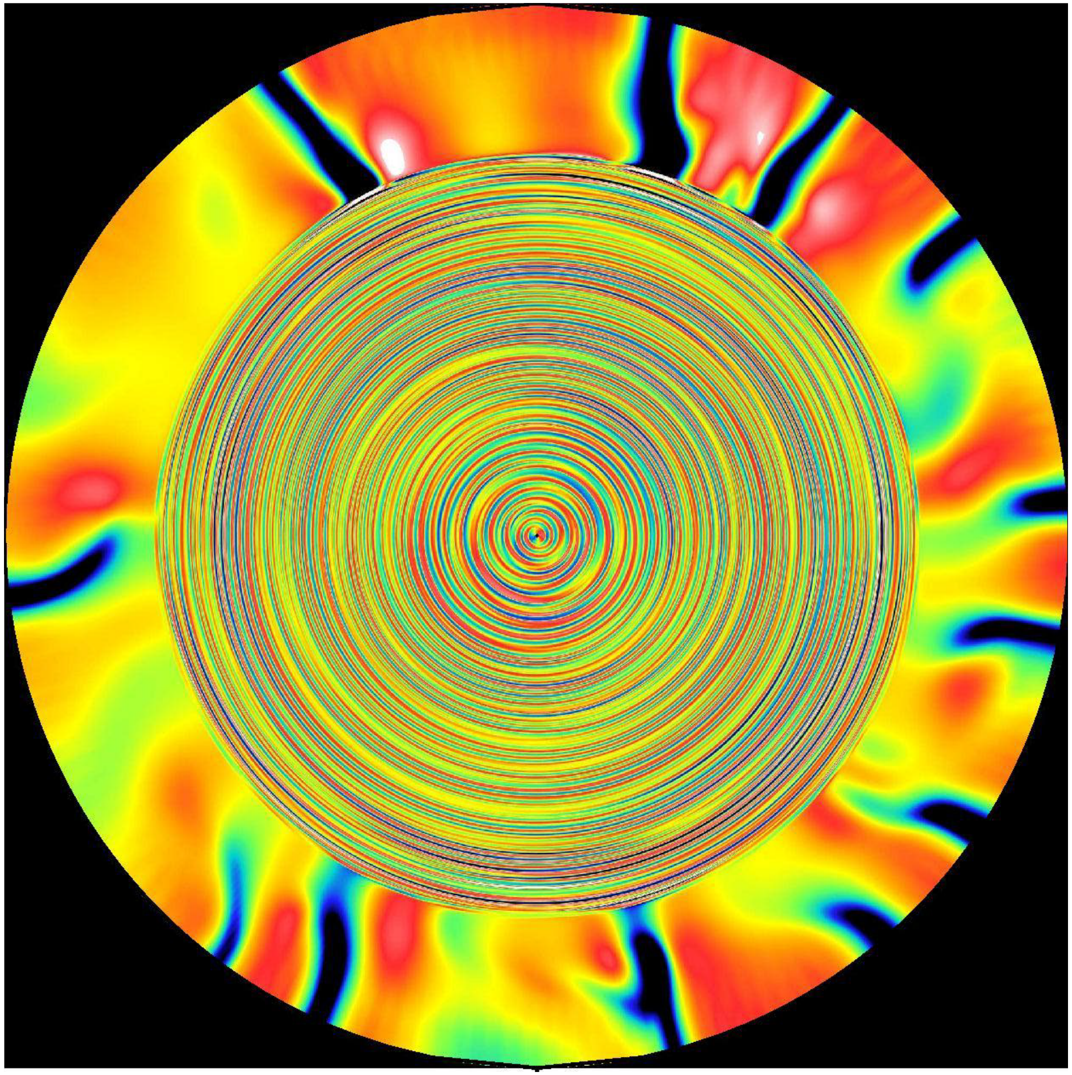
Alvan et al. 2014

Gravity waves in the Sun – improving BC’s

Inner sphere



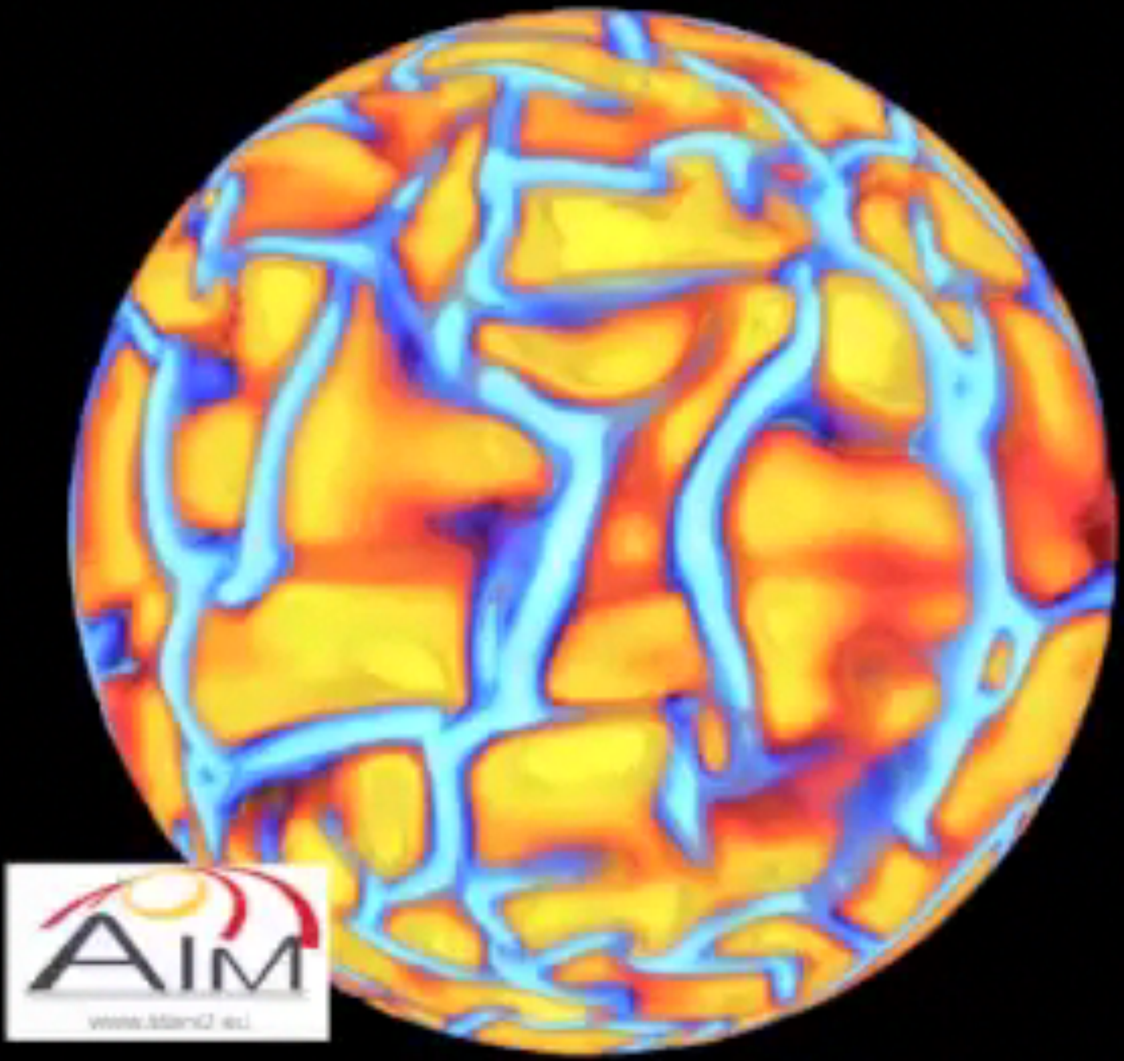
“Full sphere”



Alvan, Brun, Mathis 2014
A&A, 565, A42

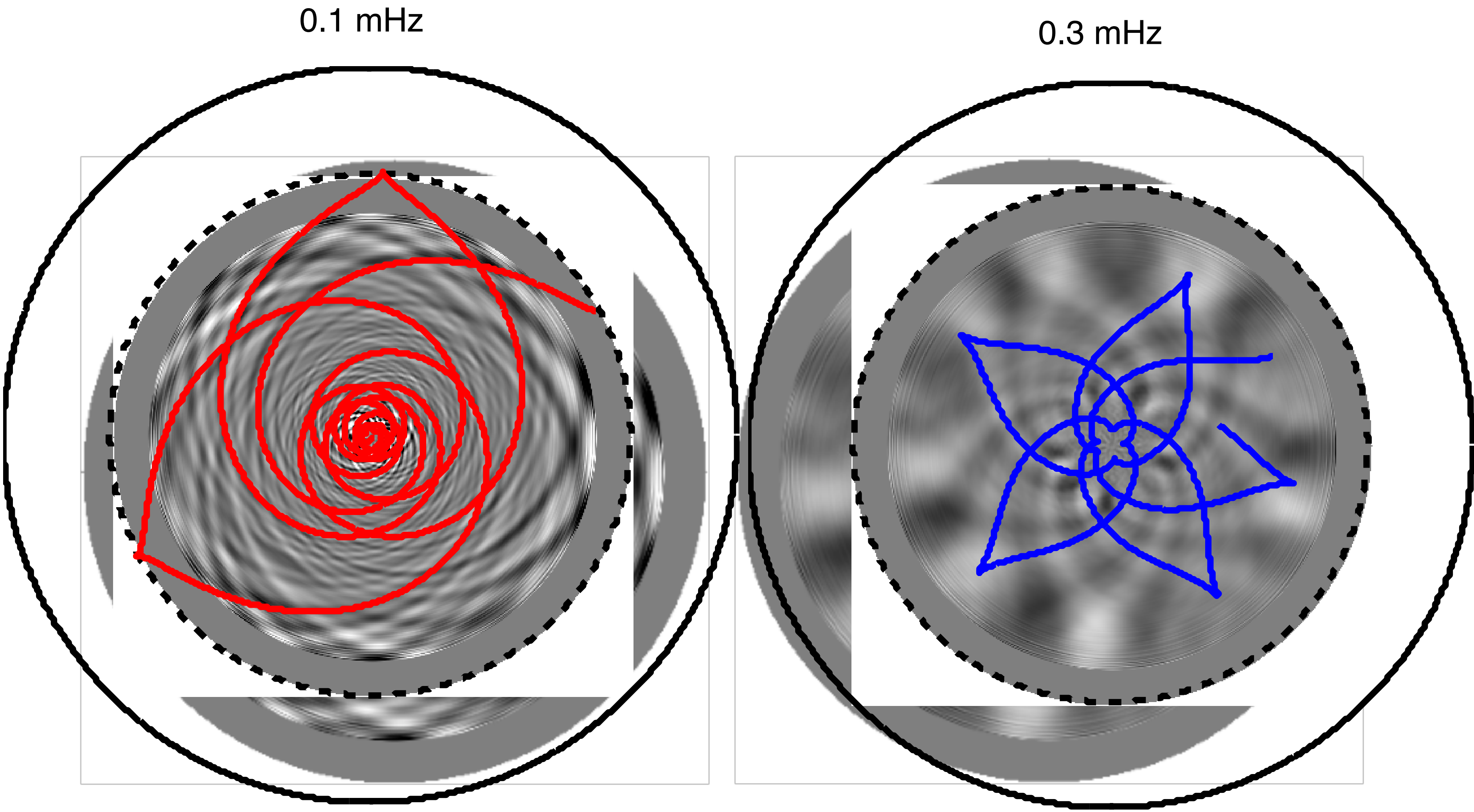
Internal Waves

3D view



Vr/sqrt(<Vr(r)^2>)

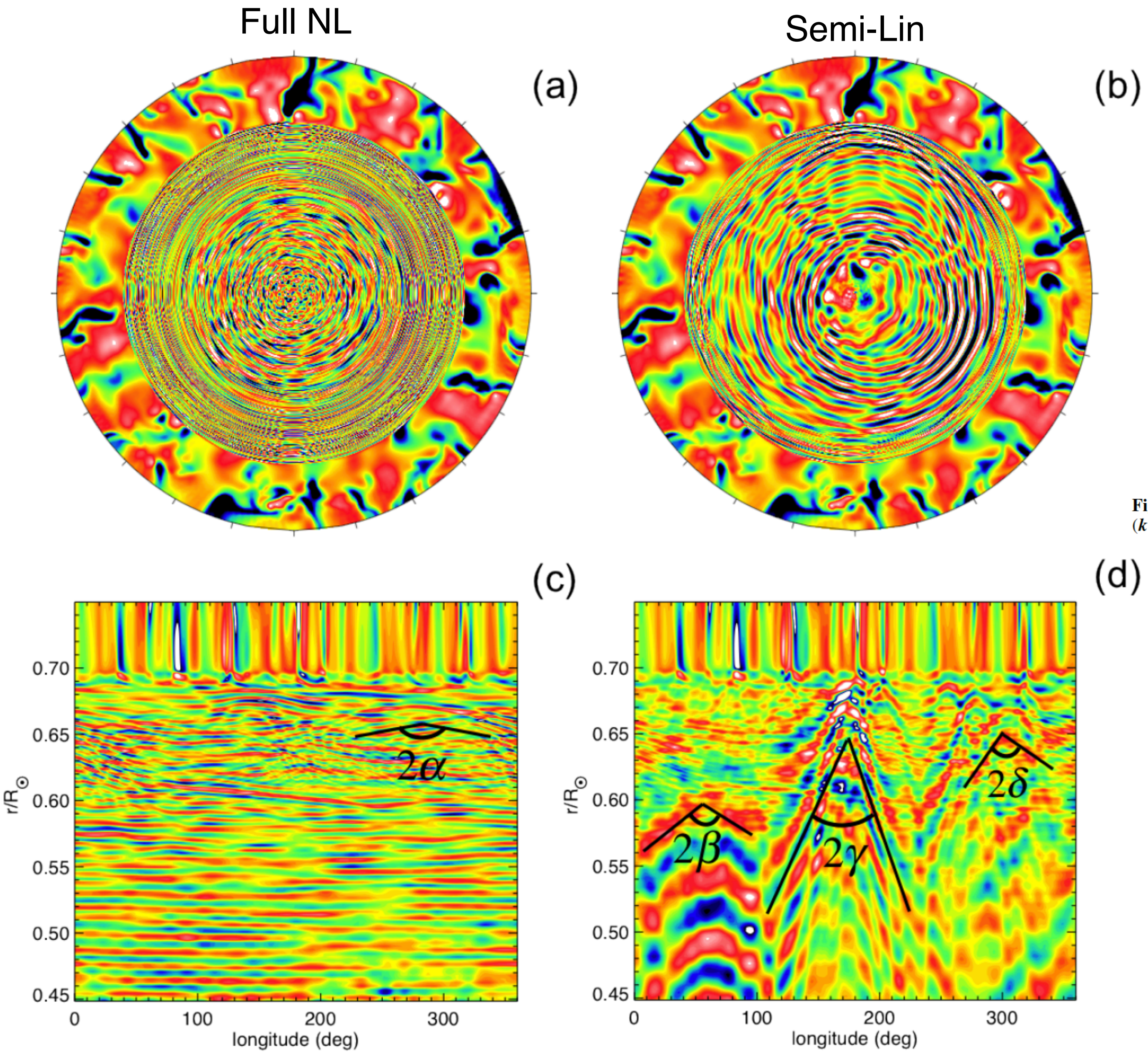
Internal Gravity Modes: Frequency Filtering



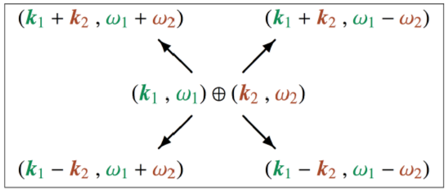
Ray path recovered

Alvan et al. 2014

Understanding Nonlinear Coupling between Waves



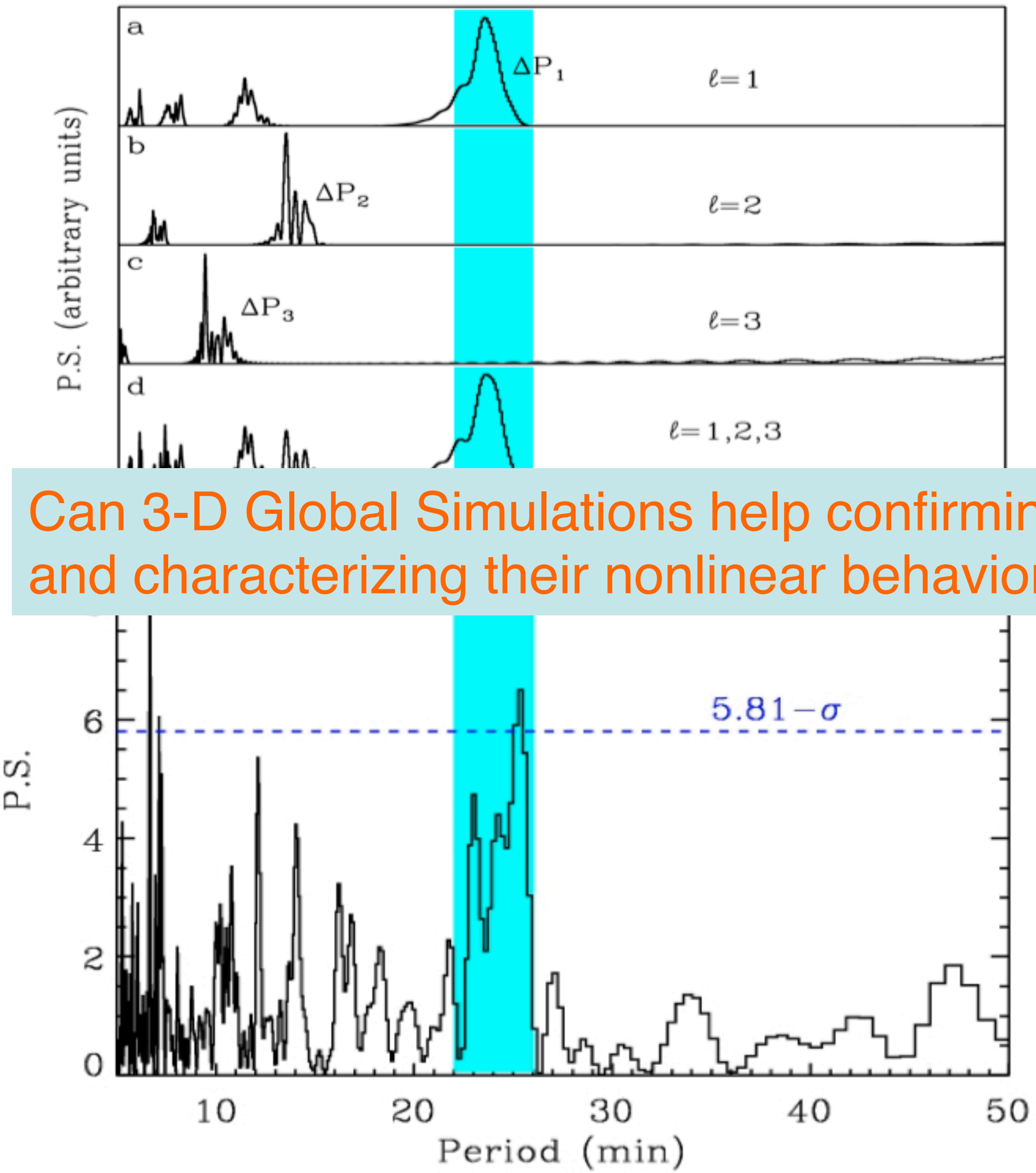
Step fct in RZ cancels
N.L. terms as in
Rogers & Glatzmaier 2005



$$\begin{tikzcd}
(\boldsymbol{k}_1+\boldsymbol{k}_2\,,\omega_1+\omega_2) & & (\boldsymbol{k}_1+\boldsymbol{k}_2\,,\omega_1-\omega_2) \\
& (\boldsymbol{k}_1\,,\omega_1)\oplus(\boldsymbol{k}_2\,,\omega_2) \arrow[ul] \arrow[ur] \arrow[dl] \arrow[dr] & \\
(\boldsymbol{k}_1-\boldsymbol{k}_2\,,\omega_1+\omega_2) & & (\boldsymbol{k}_1-\boldsymbol{k}_2\,,\omega_1-\omega_2)
\end{tikzcd}$$

Fig. 23. Diagram showing the possibilities for two waves $(\boldsymbol{k}_1,\omega_1)$ and $(\boldsymbol{k}_2,\omega_2)$ to interact and give birth to a third wave.

Solar G modes envelope detection



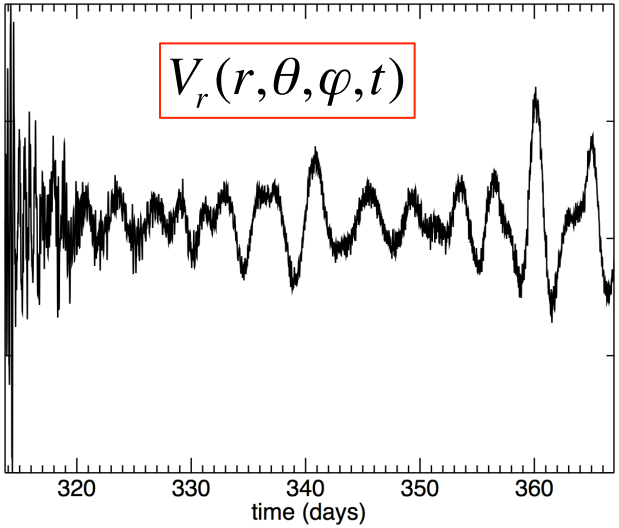
Simple model

Garcia et al. 2007, Science

GOLF observations

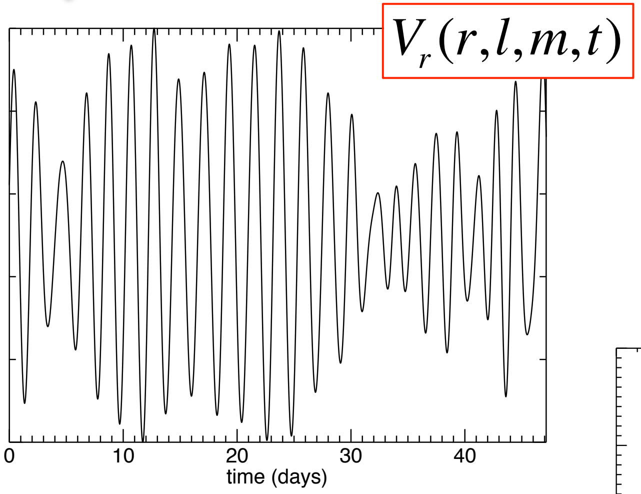
Can 3-D Global Simulations help confirming their detection and characterizing their nonlinear behavior and visibility?

From Physical Space to Spectral Space



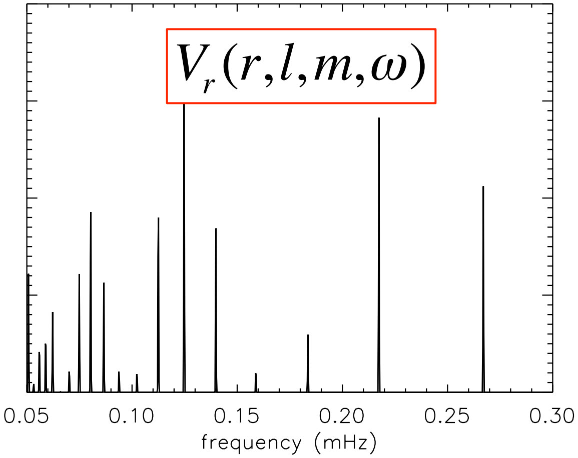
Spherical Harmonic transform

$$\theta,\varphi \rightarrow l,m$$

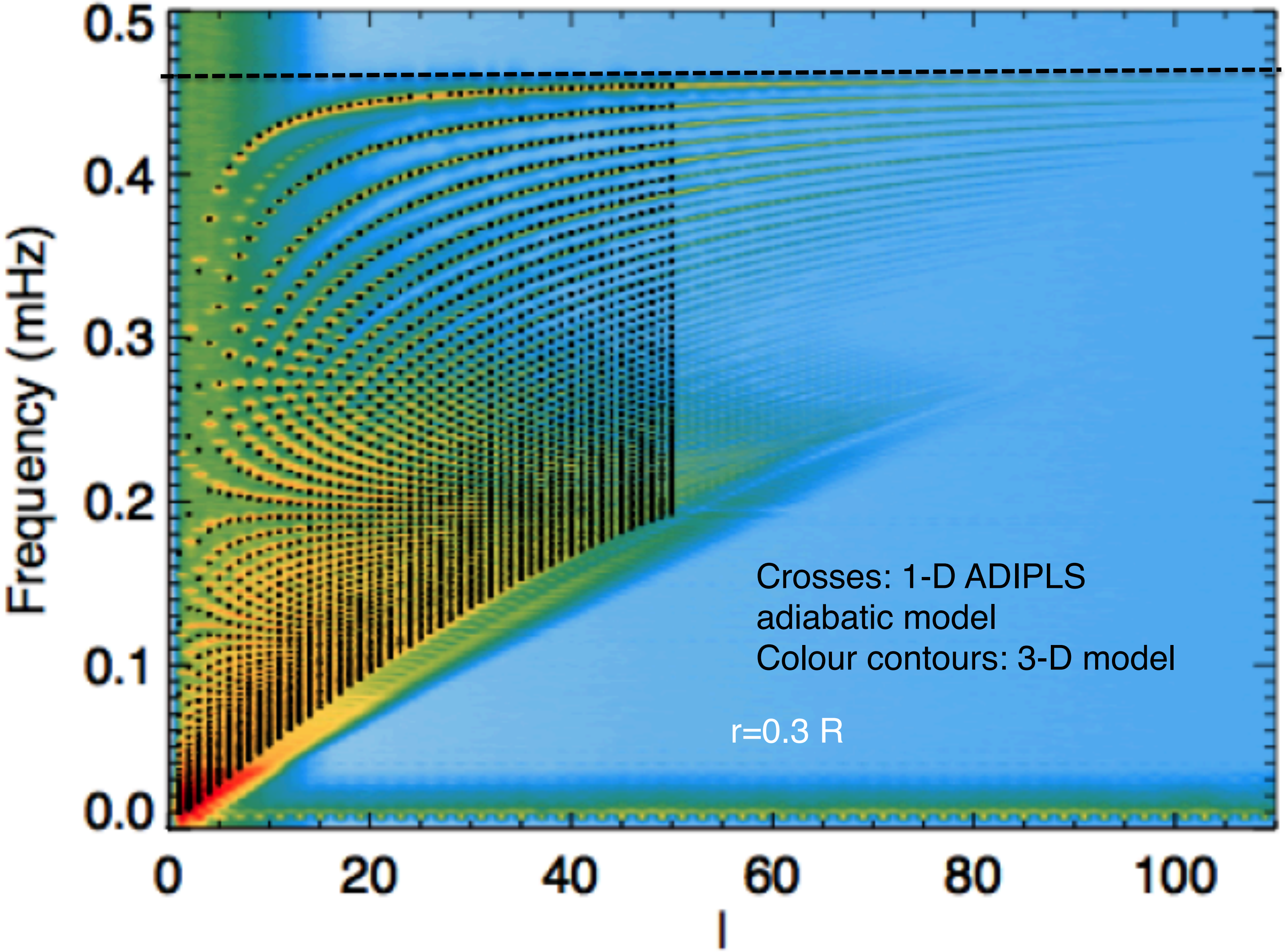


Temporal Fourier transform

$$t \rightarrow \omega$$



l-omega spectra (full sphere)

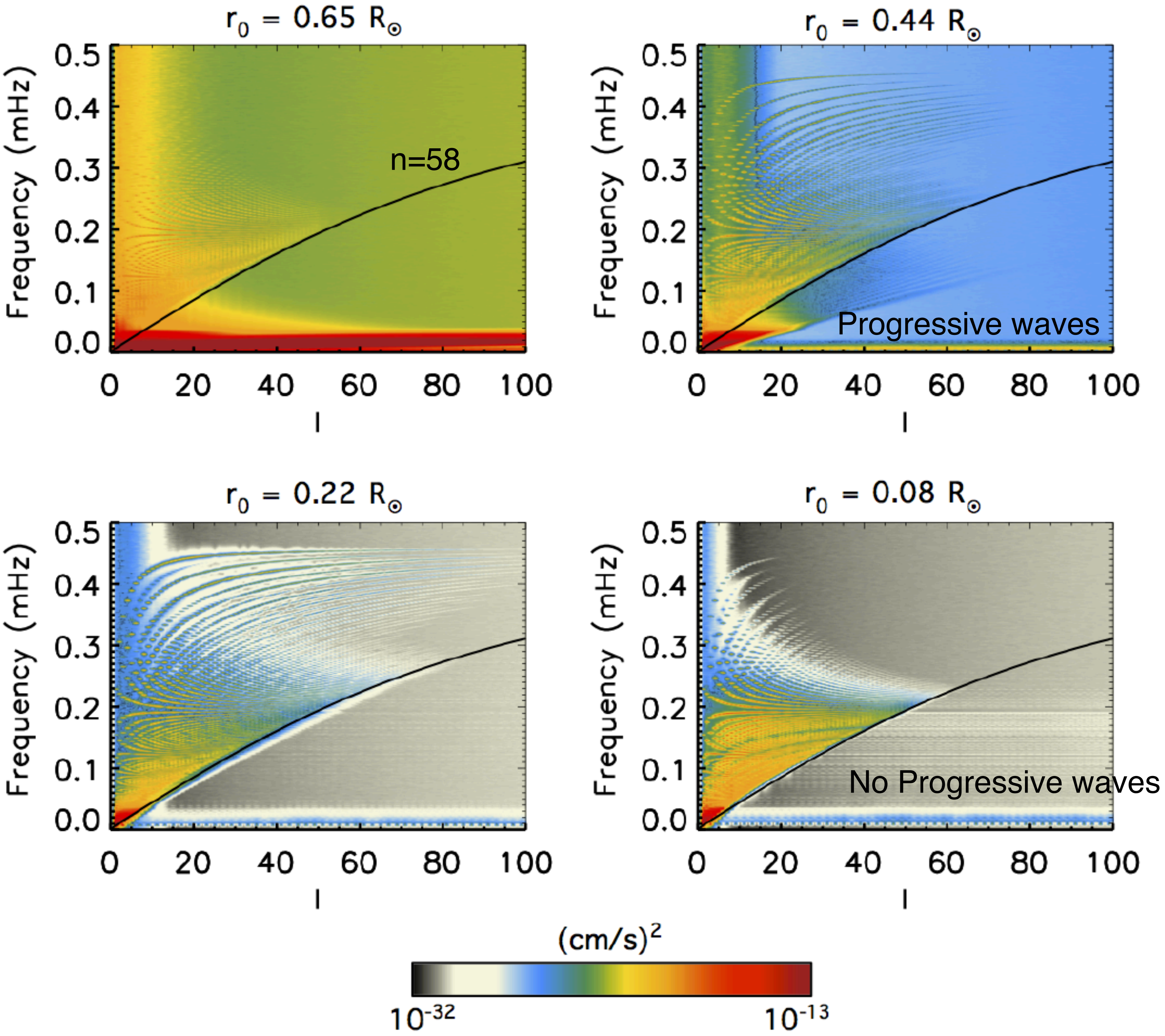


Crosses: 1-D ADIPLS adiabatic model
Colour contours: 3-D model

r=0.3 R

Alvan et al. 2014

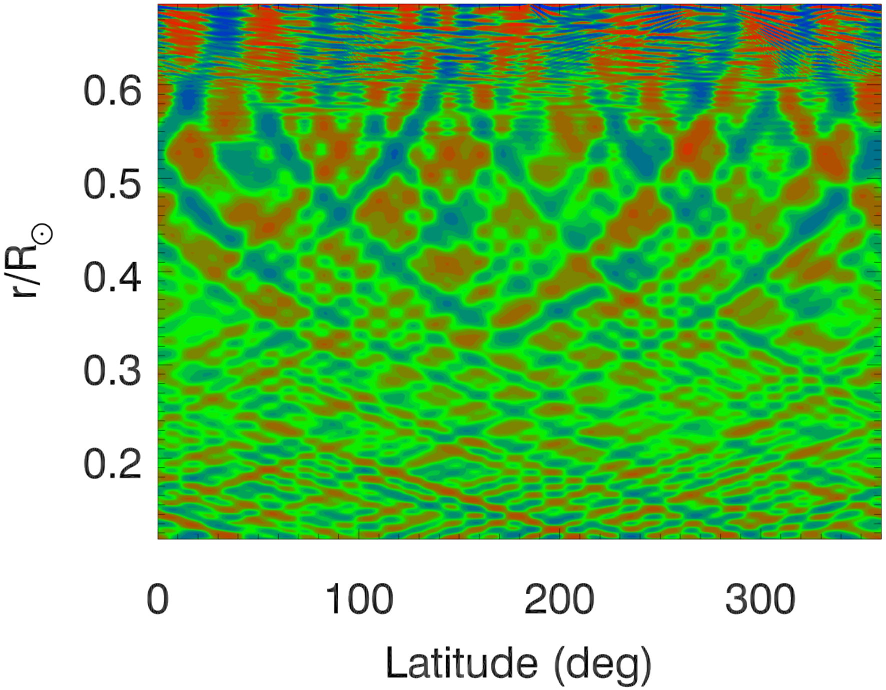
Spectra vs Depth



Progressive vs Standing Waves

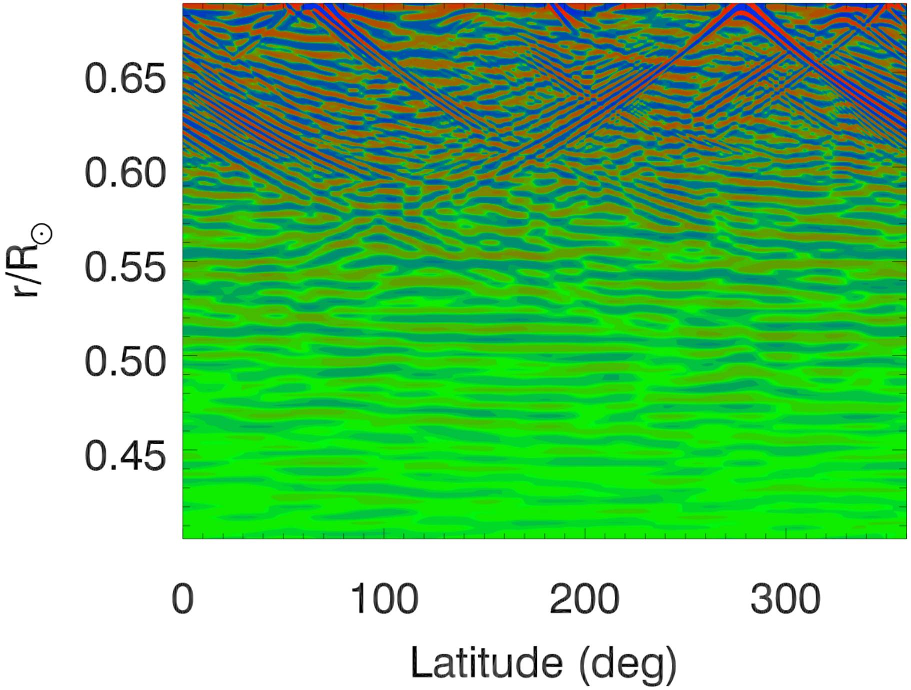
Resonant mode

0.1 mHz



Damped propagative wave

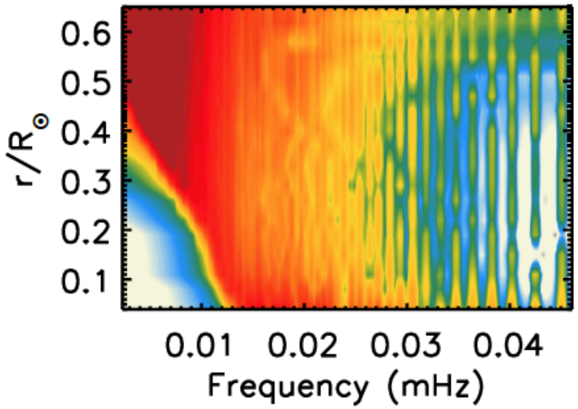
0.016 mHz



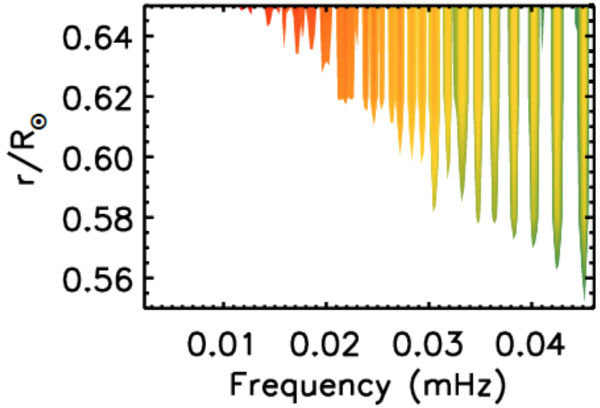
Alvan et al. 2014b in prep

Radiative Damping of the waves

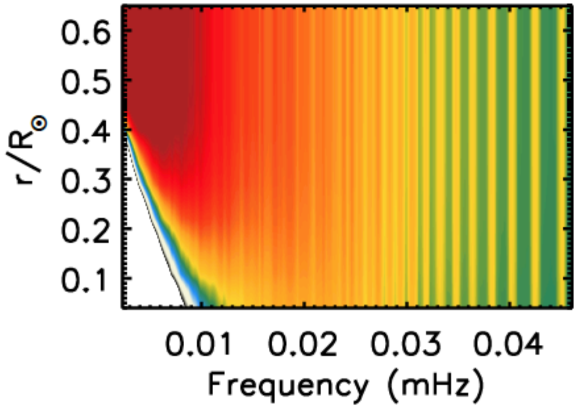
Zahn et al. 1997



$$\tau\,(r,\ell,\omega) = [\ell(\ell+1)]^{\frac{3}{2}} \int_r^{r_{\rm cz}} \kappa \frac{N^3}{\omega^4}\frac{\mathrm{d}r'}{r'^3}.$$



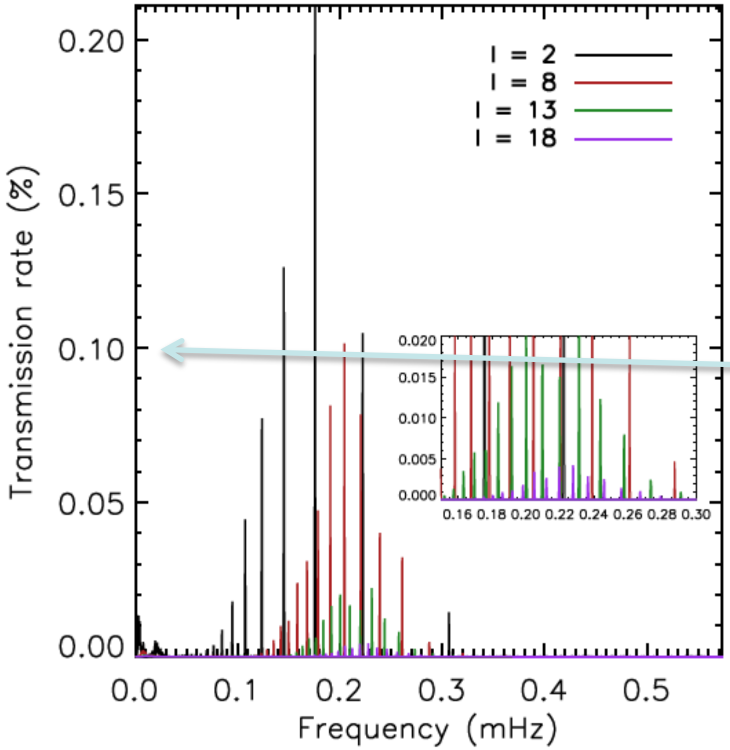
$$E_{\rm damp}(r,\omega) = E_0(\omega)\times e^{-\tau(r,2,\omega)}.$$



Proportional to $1/\omega^3$

(as in Rogers et al. 2013)

Wave's energy transmission from convective motions



about 0.1 to 0.4 % of Solar Luminosity

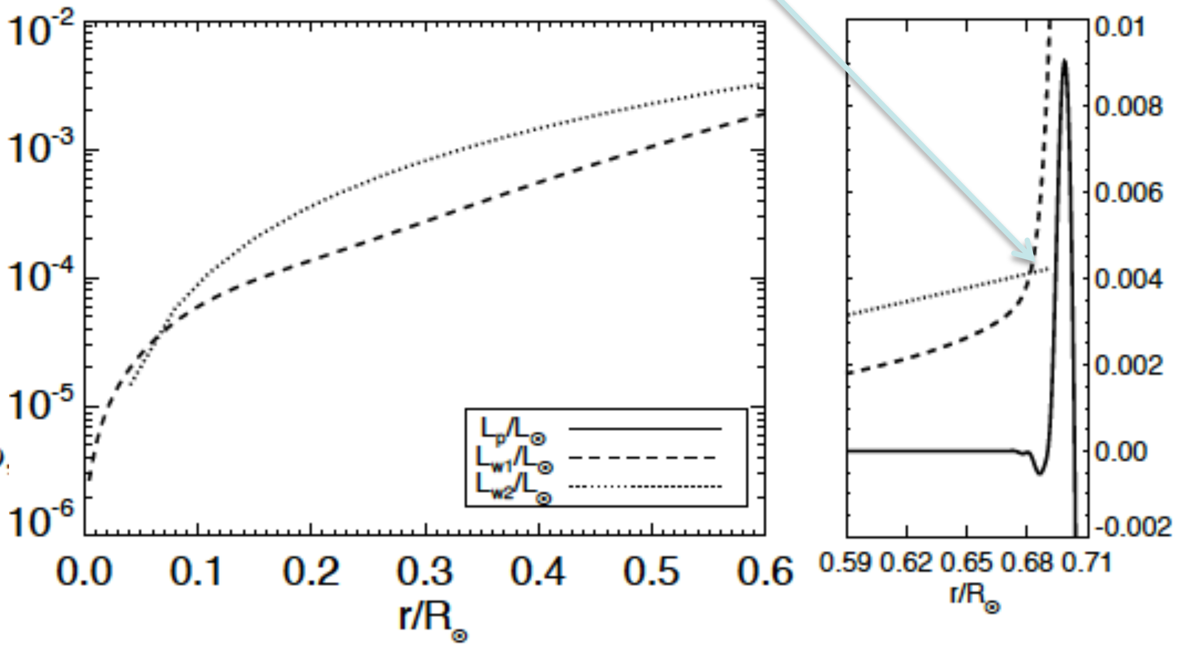
3 ways of evaluating the wave flux

$$\mathcal{F}_p = < \overline{V_r P} >$$

$$\mathcal{F}_{W1} \propto \frac{\omega_c}{N} \mathcal{F}_c,$$

$$\mathcal{F}_{W2} = \int_{\omega_c}^{N} \int_{k_h} \rho \frac{E(k_h,\omega)}{k_h \omega} V_{\rm gr}(k_h,\omega) dk_h d\omega,$$

$$V_{\rm gr}(k_h,\omega) = \frac{\sqrt{N^2-\omega^2}}{N^2} \frac{\omega^2}{\sqrt{\ell(\ell+1)}} r,$$

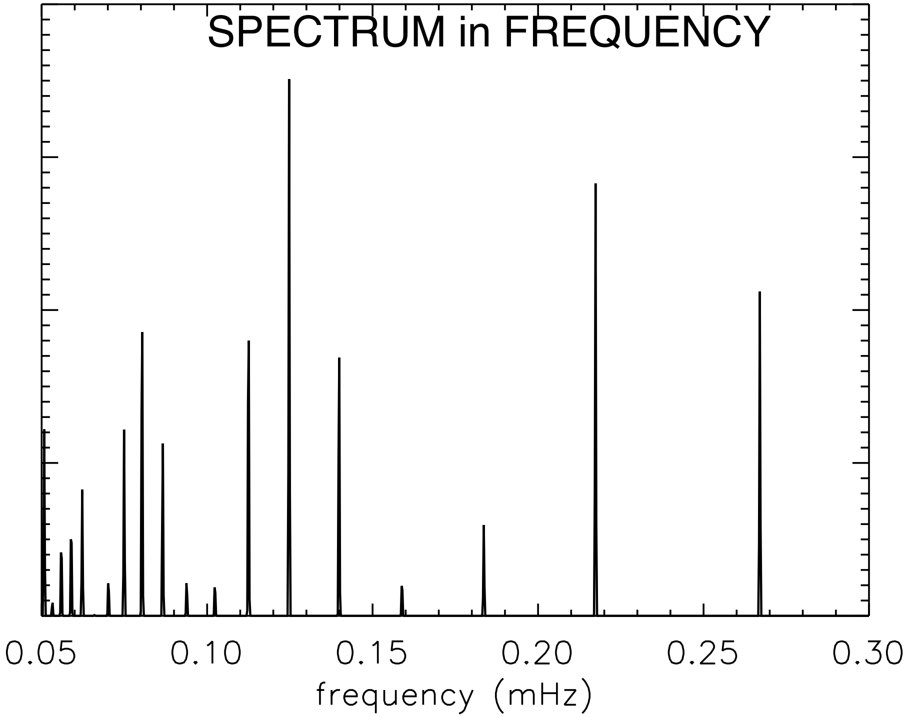


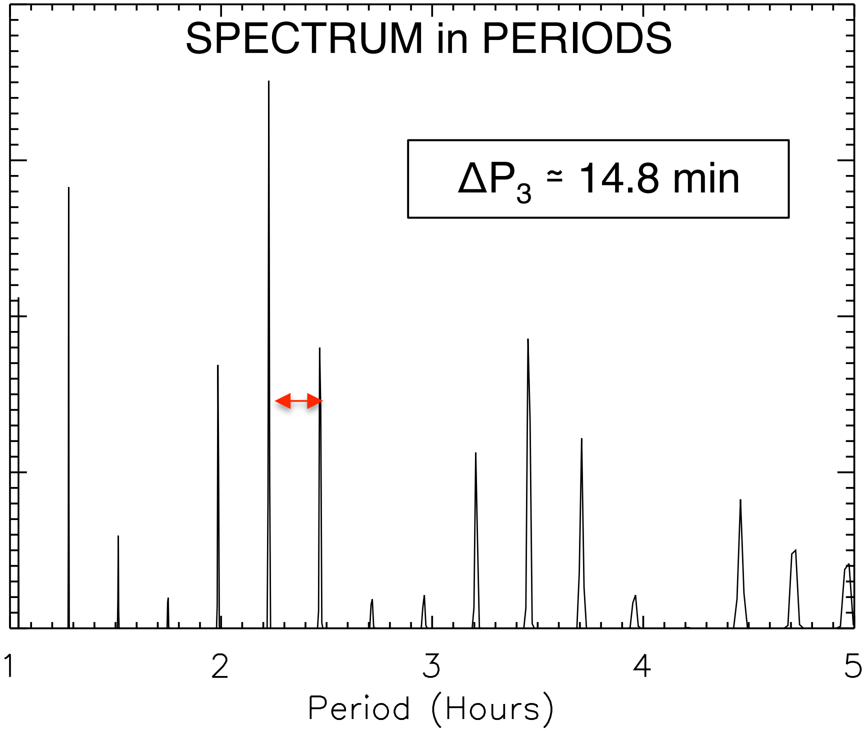
Constant Period Spacing

$$P_{n,l} = \frac{\pi}{2\sqrt{l(l+1)}\int_0^{r1} \frac{N}{r} dr}(2n + l - \delta)$$

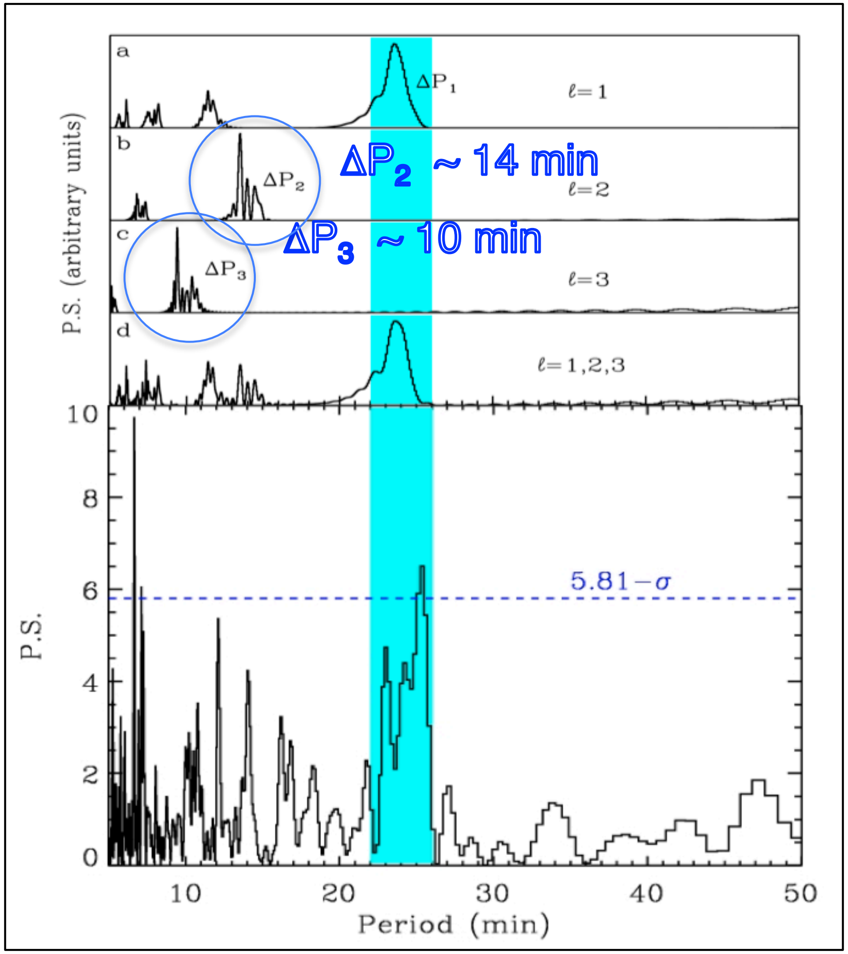
$$\delta \approx 5/6$$

$r_1(\omega)$ turning point



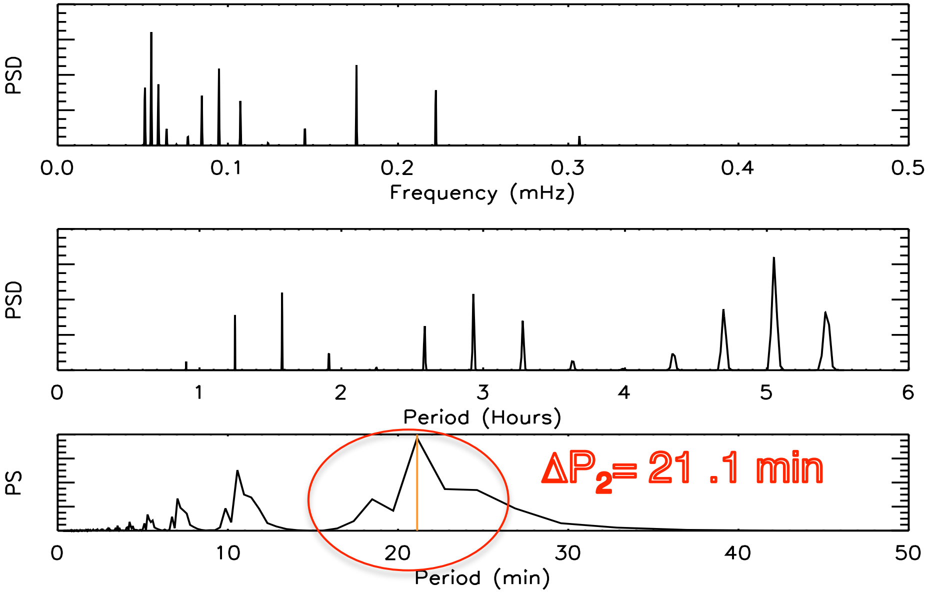


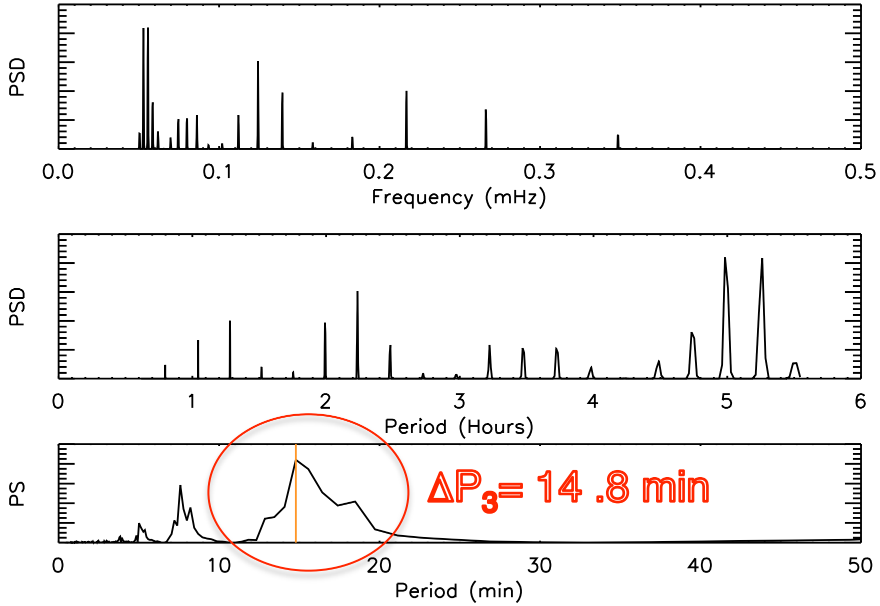
Comparing Model to Observations: Full Sphere Case



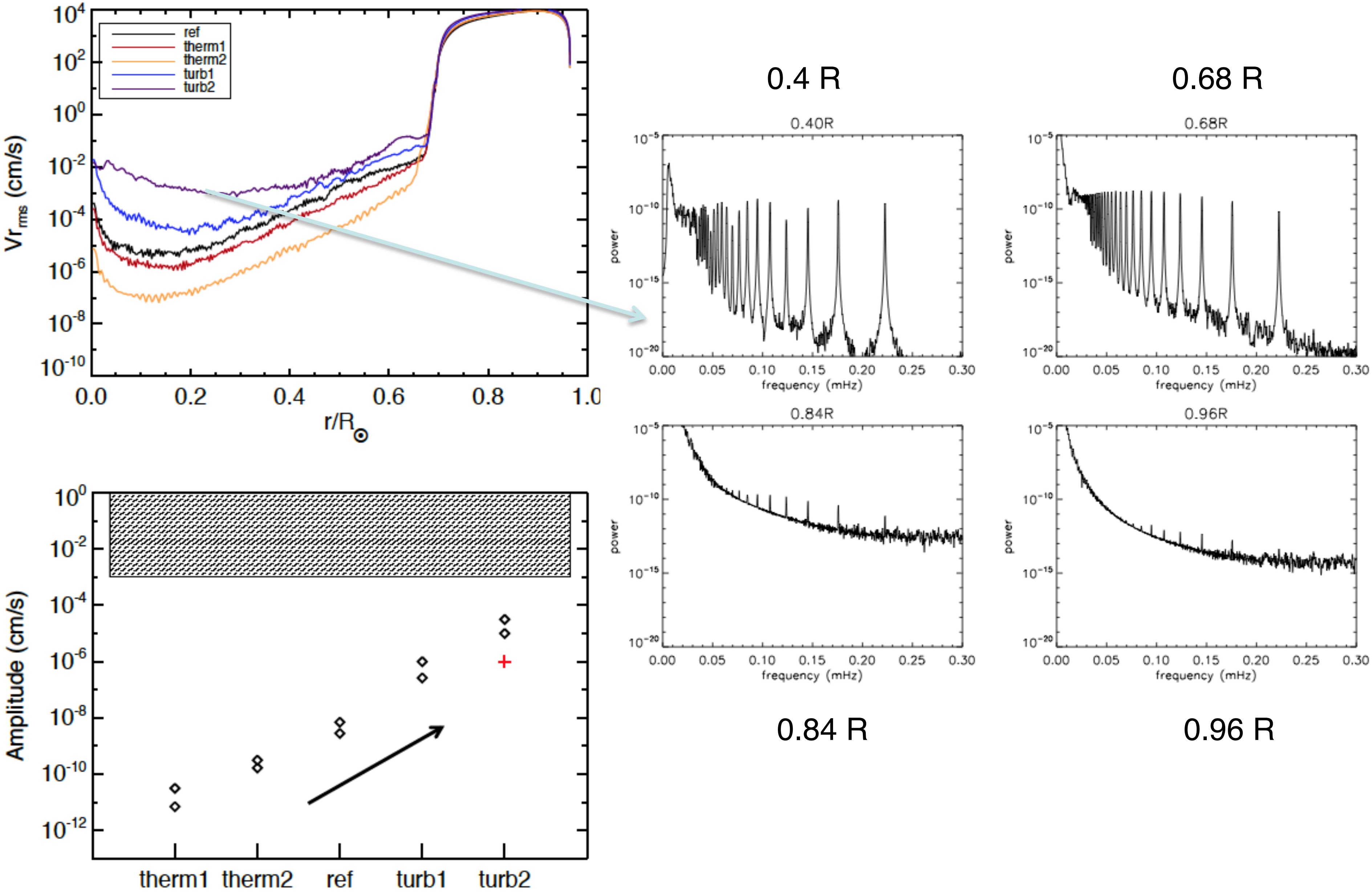
Garcia et al. 2007

$$\Delta P_l = \frac{\pi}{\sqrt{l(l+1)}\int_0^{rc} \frac{N}{r} dr}$$





Mode Visibility through Convective Layer?



0.4 R

0.68 R

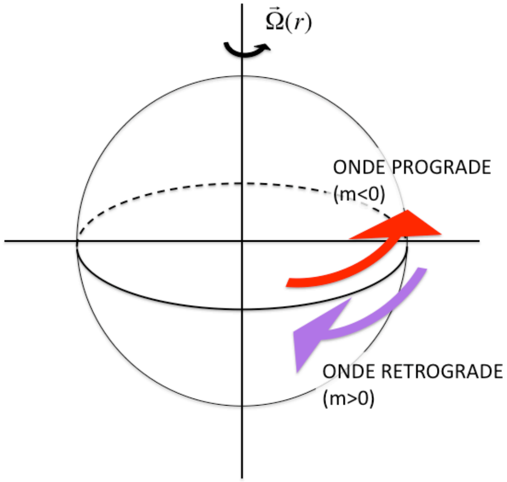
0.84 R

0.96 R

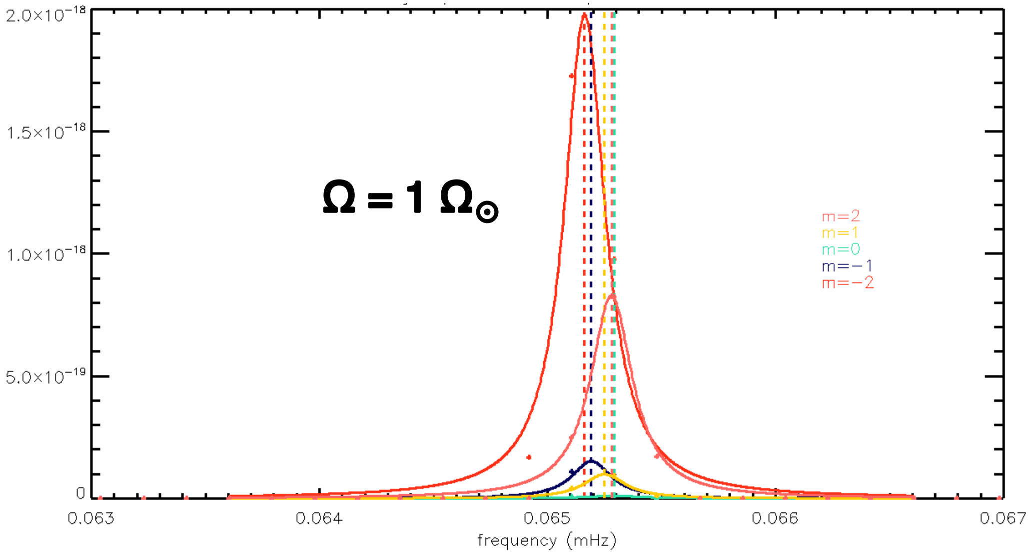
EFFECTS of ROTATION

$F\left(V(r,l,m,t)\right) =$ Retrograde wave **(m>0)**

$F\left(\overline{V(r,l,m,t)}\right) =$ Prograde wave **(m<0)**



Rotational Splitting



Asymptotic law

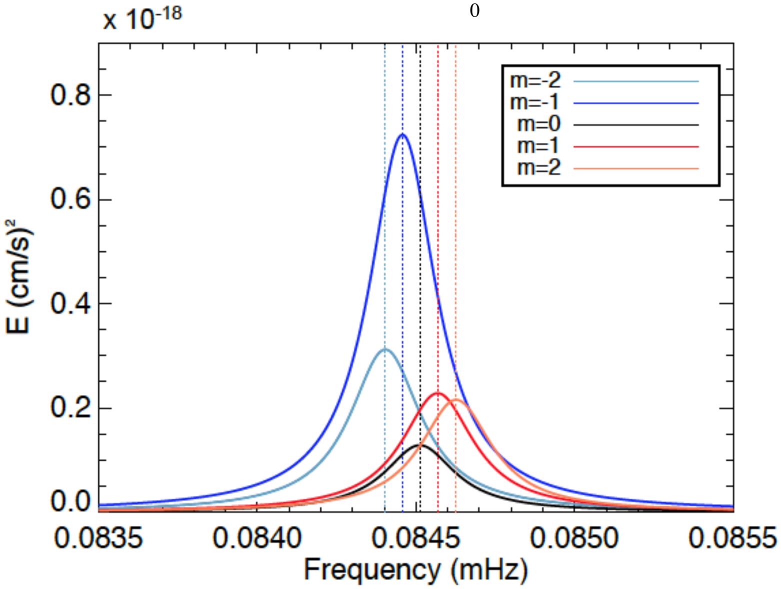
$$\nu_{n,l,m} = \nu_{n,l,0} + m\frac{\Omega}{2\pi}\left(1 - \frac{1}{l(l+1)}\right)$$

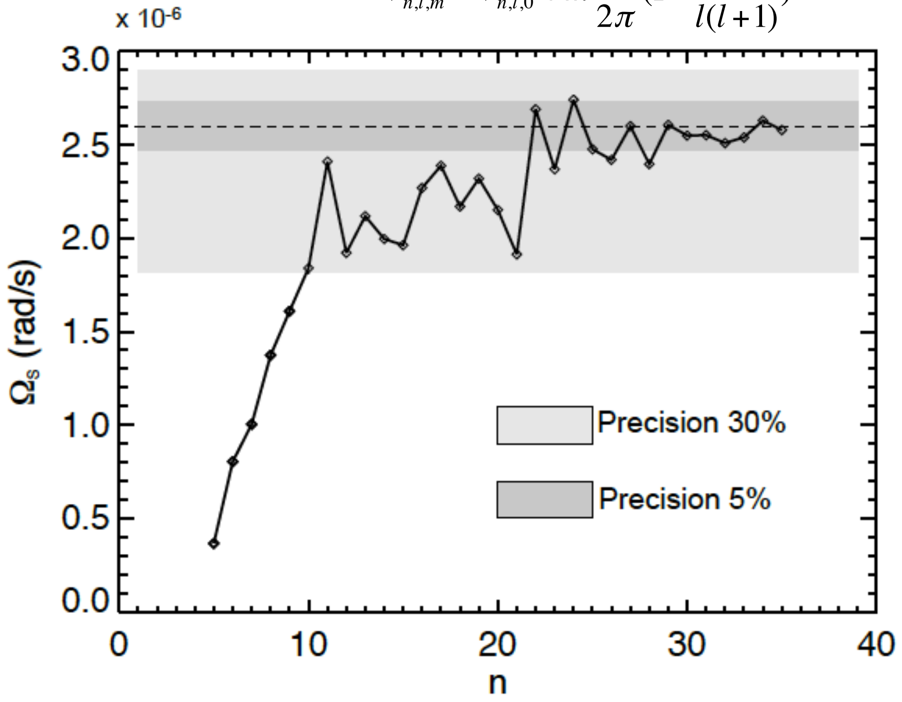
Rotational Splitting (m azimuthal wave nb)

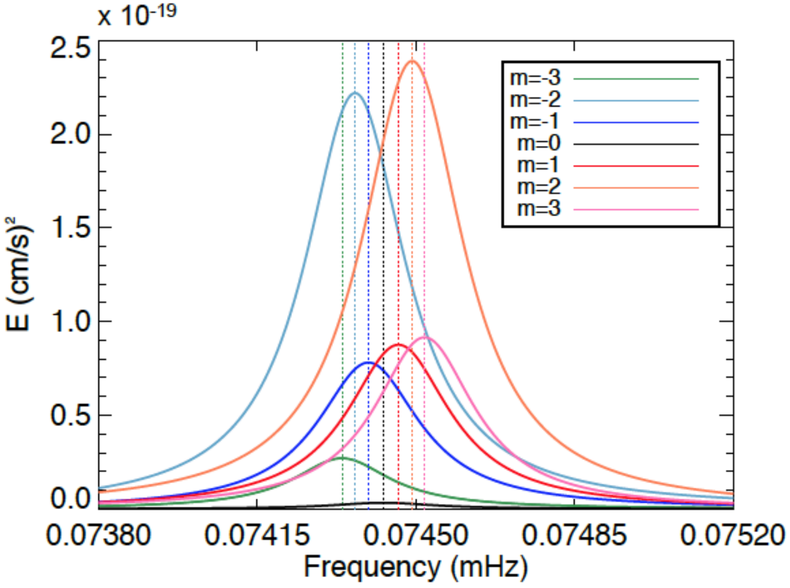
$$\delta_{n\ell m} = -m(1-\beta_{n\ell})\Omega_S .$$

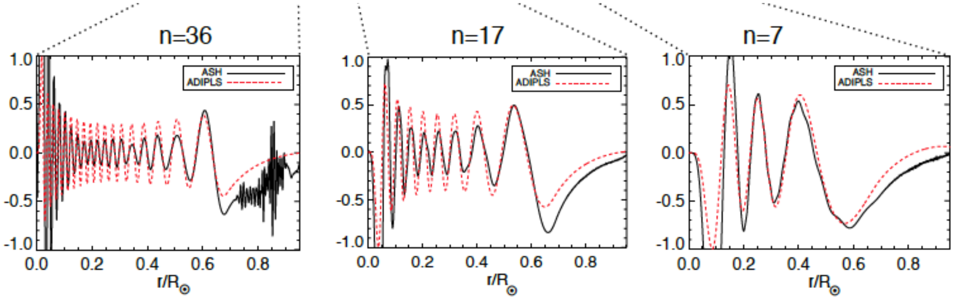
$$\nu_{n,l,m} = \nu_{n,l,0} + m\beta_{n,l}\int_0^R K_{n,l}(r)\Omega(r)\,dr$$

$$\nu_{n,l,m} = \nu_{n,l,0} + m\frac{\Omega}{2\pi}\left(1-\frac{1}{l(l+1)}\right)$$





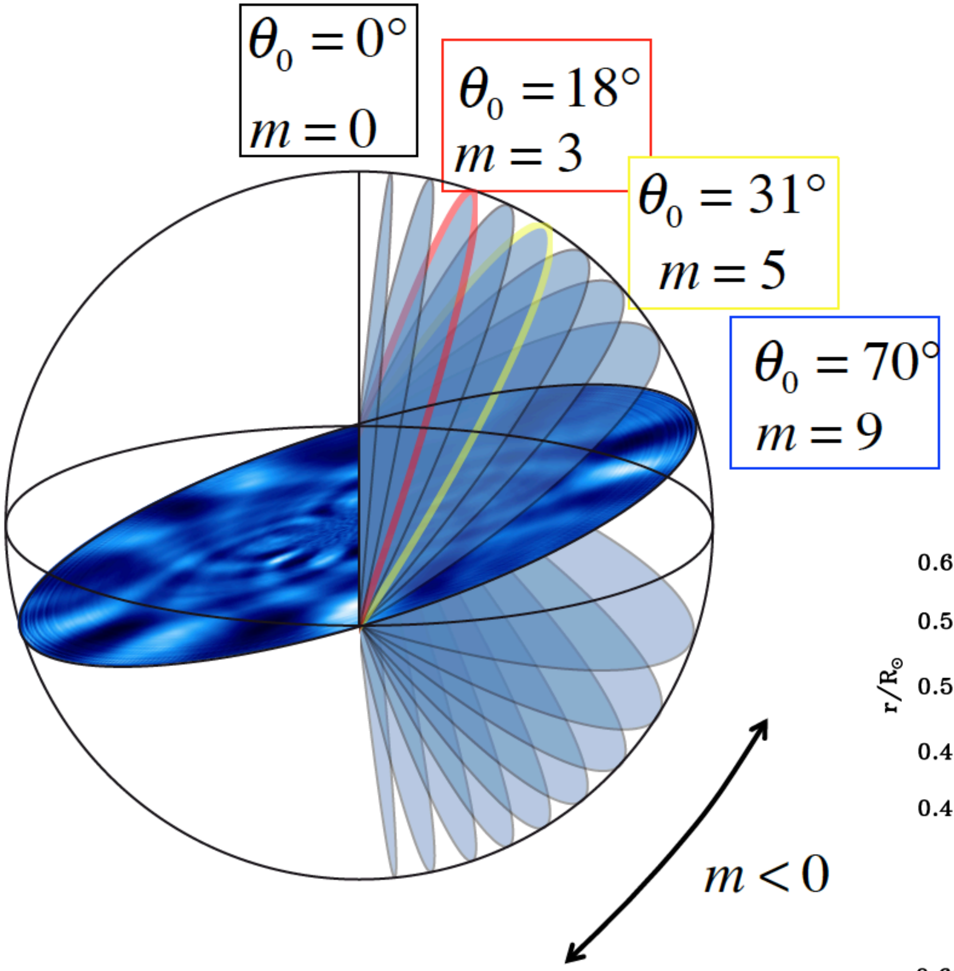




Mode radial eigenfunction (n order)

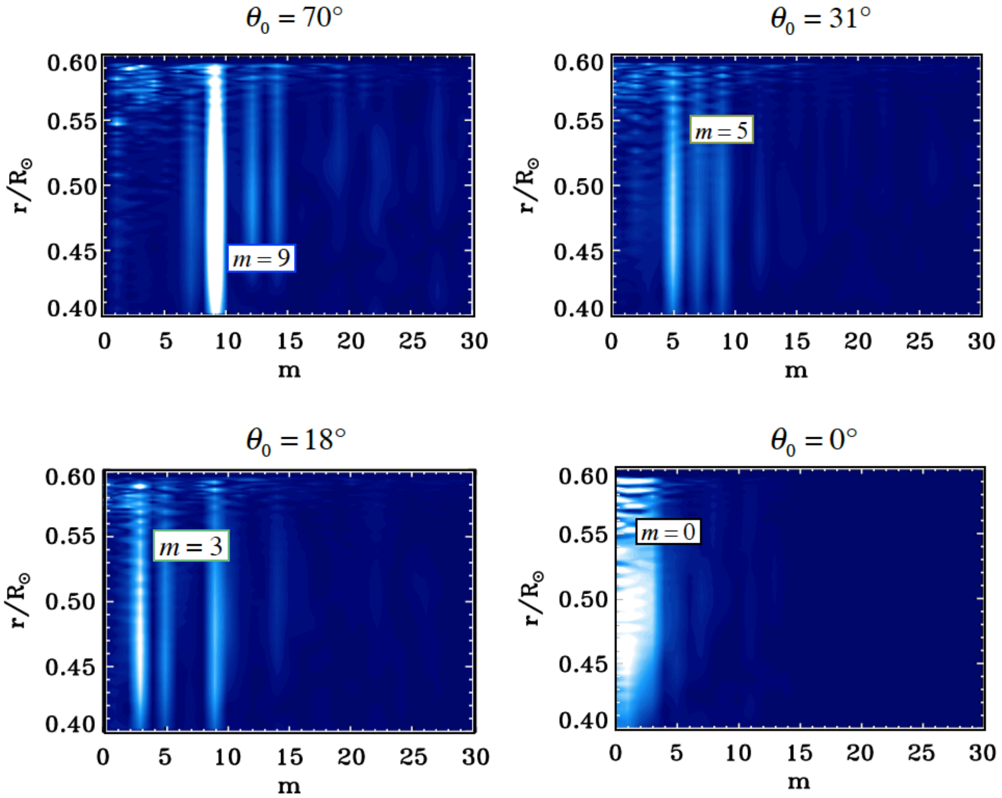
Last version of Adipls code gives β & K

Inclination of Propagation Planes for Gravity Waves



Checking in ASH code

l=9 mode



Gough 1993, Alvan et al. 2014b (in prep)

GRAVITO-INERTIAL Modes : radial differential rotation

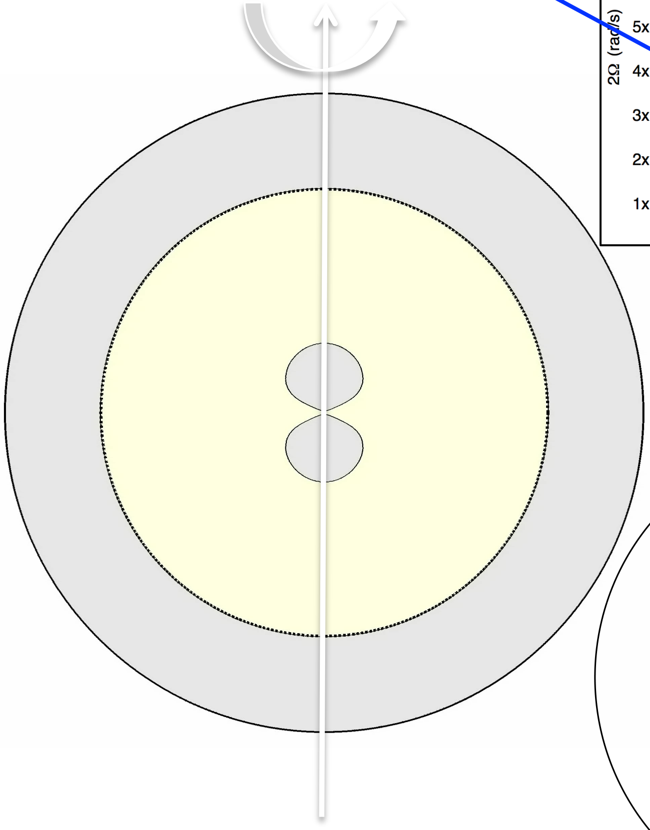
Dispersion relation:

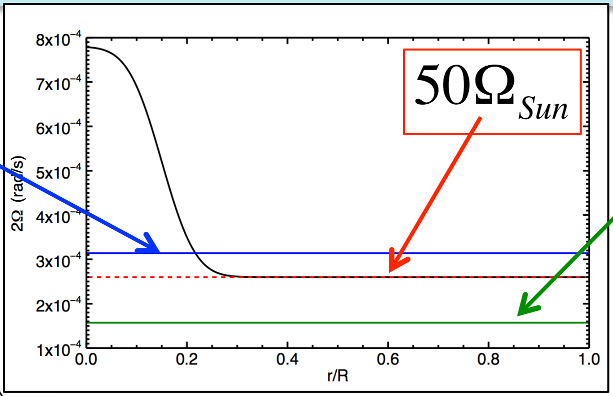
$$\Omega = \Omega(r)$$

$$\left(N^2 + \boxed{4\Omega^2\sin^2\theta + 2\Omega q\sin^2\theta} - \omega^2\right)k_h^2 \boxed{-(4\Omega^2\sin 2\theta + \Omega q\sin 2\theta)k_r k_h} + \left(\boxed{4\Omega^2\cos^2\theta} - \omega^2\right)k_r^2 = 0$$

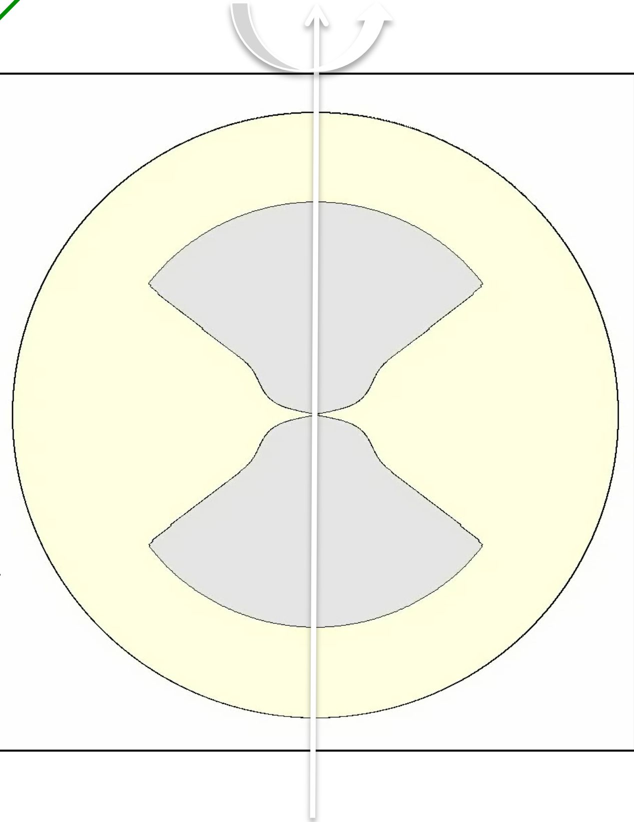
$$q = r\frac{\partial\Omega}{\partial r}$$

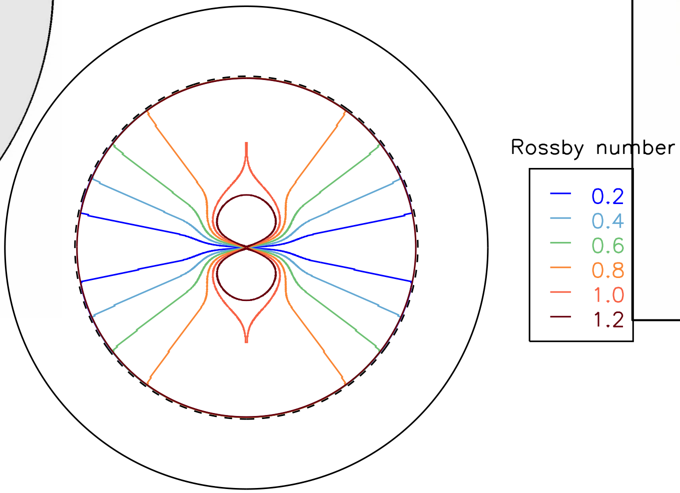
Super-inertial mode





Sub-inertial mode





Mathis, Alvan, Brun (2014)

Influence of Solid Rotation on Gravito-Inertial Wave Propagation

3-D ray path

$m = 0$

Super-inertial

$Ro = 1.2$

Sub-inertial

$Ro = 0.72$

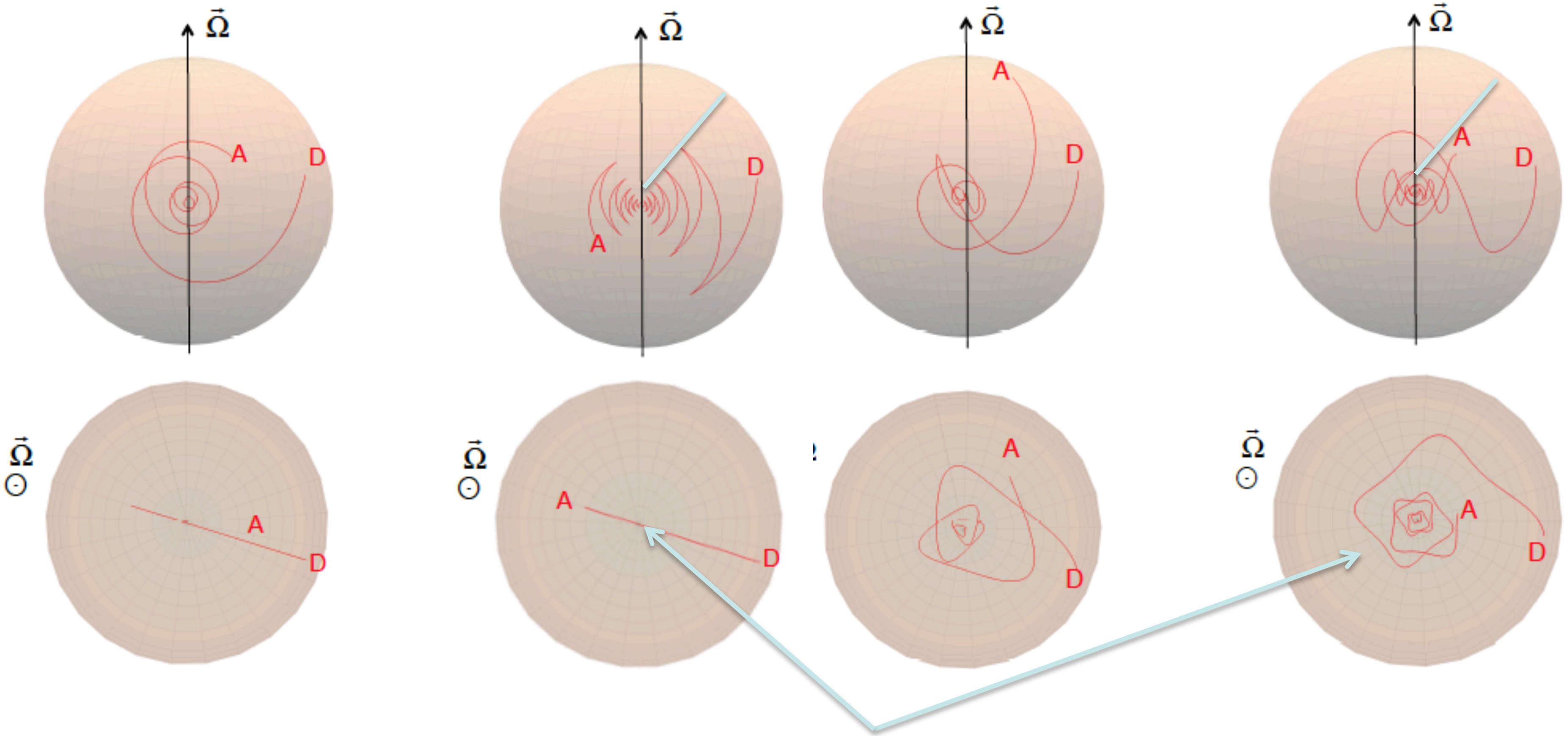
$m \neq 0$

Super-inertial

$Ro = 1.2$

Sub-inertial

$Ro = 0.72$

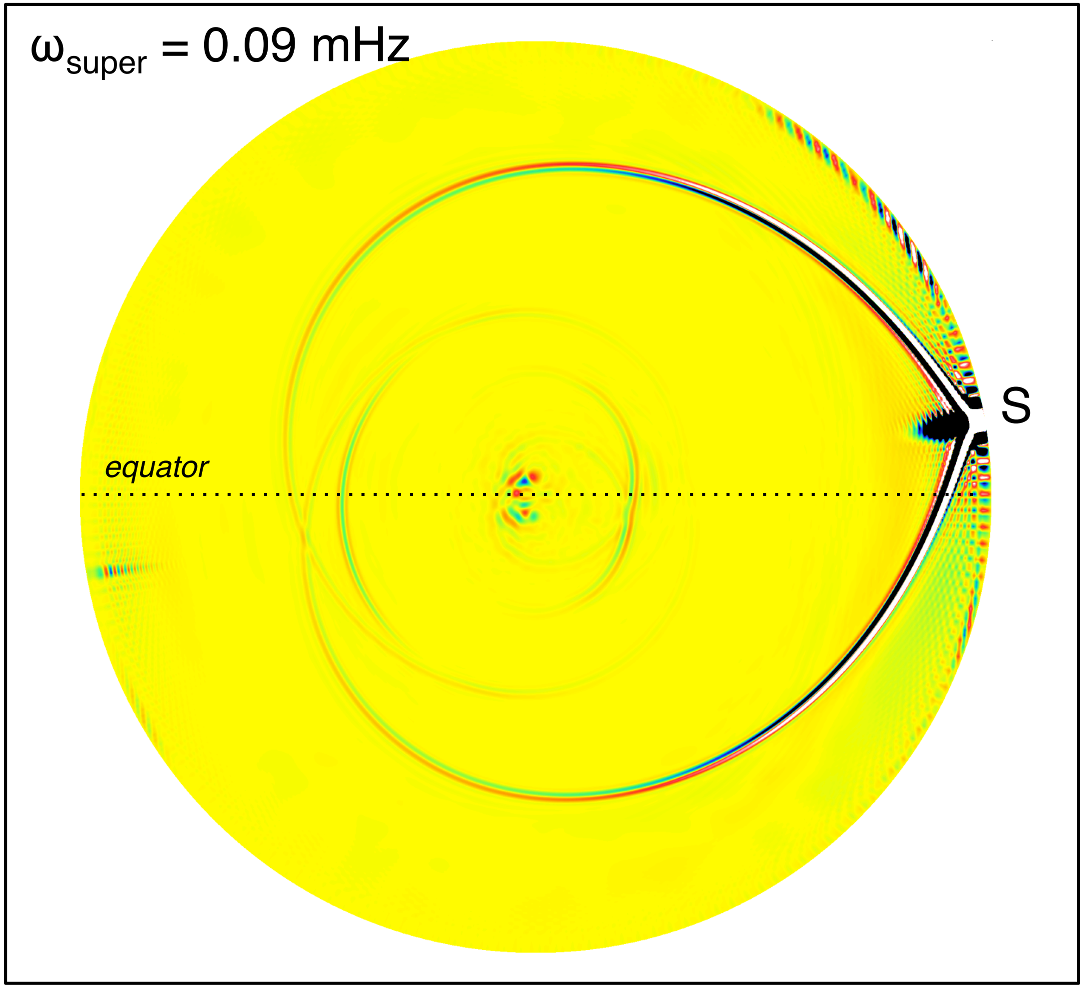


Note how the waves are no more confined in a single plane

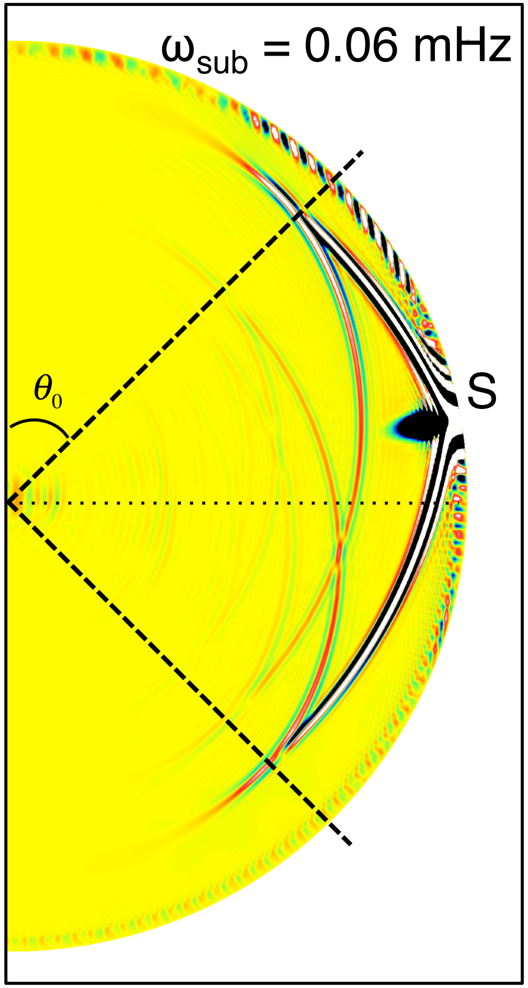
Trapping of waves in ASH

$2\Omega = 0.083$ mHz

$$\theta_c = \arccos\left(\frac{\omega}{2\Omega}\right)$$

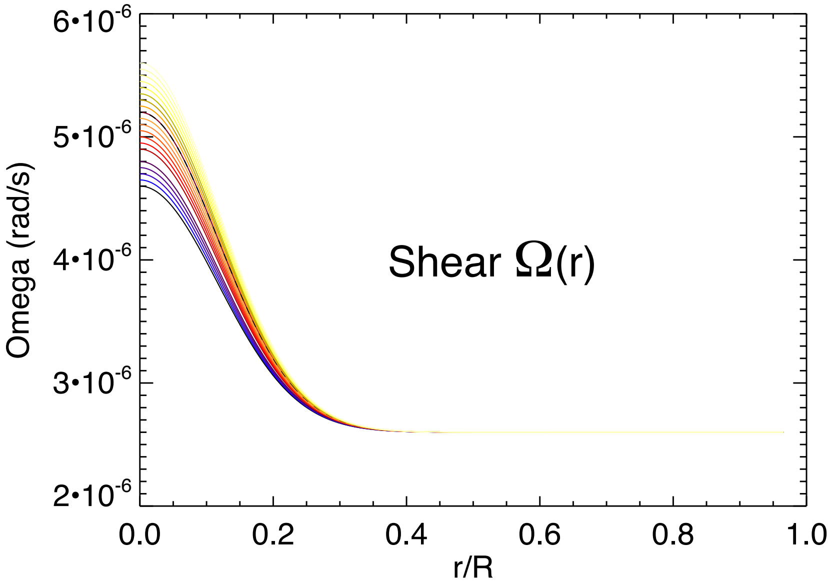


Super-inertial

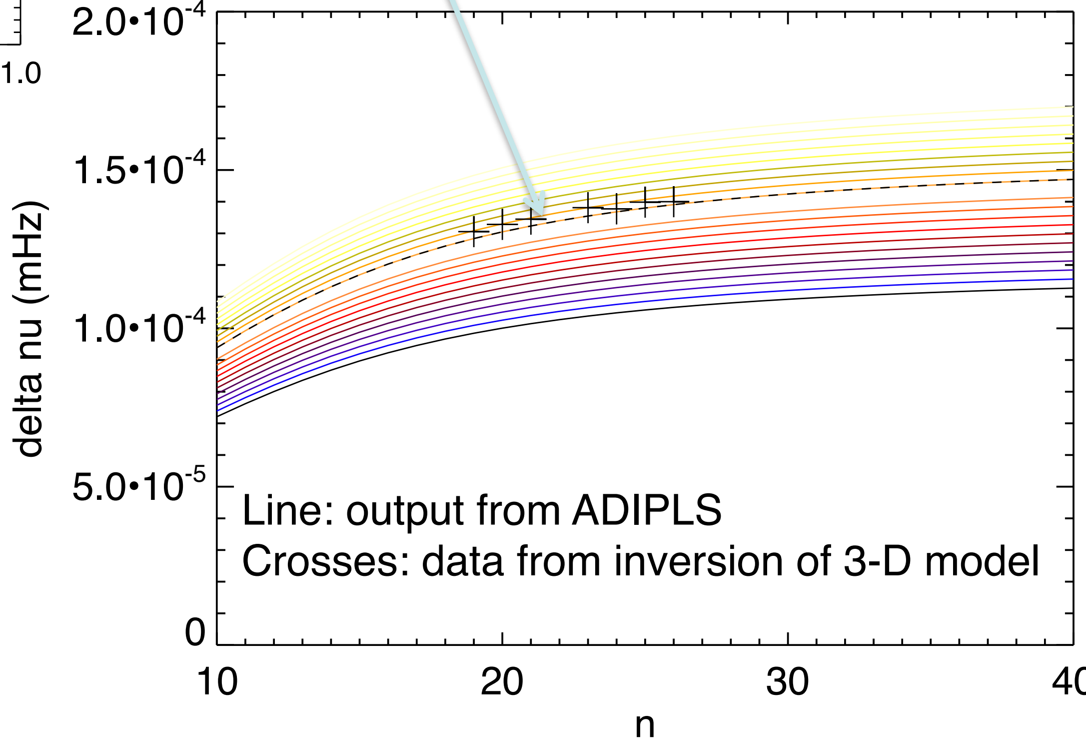


Sub-inertial

Introducing Differential Rotation in the Radiative Interior



Inversion of radially dependent rotation profile



Line: output from ADIPLS
Crosses: data from inversion of 3-D model

Conclusion

- 3-D model of the Whole Sun/stars using realistic stratification are now tractable

- When coupling a radiative interior to a convective envelope the agreement with observations improve, for instance we get a correct differential rotation and a tachocline of shear

- The pummeling of downward plumes excite a large range of internal waves

- Detailed analysis revealed that they are indeed gravity waves

- Comparison with ADIPLS adiabatic oscillations code confirms the good agreement

- Damping seems different in 3-D code vs linear analysis, likely due to nonlinearity

- Comparison with observations indicate that even better stratification is necessary If one wants to guide the observers

- trapping of waves recovered by simple model
- Same analysis/models underway for more massive stars with core convection

ASH is now a full sphere 3-D anelastic code using either finite difference or Tchebyshev polynomials in radius scaling up to 100,000 cores.